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# SENSITIVITY AND TOLERANCE ANALYSIS FOR DEVICES USED IN THE WIRELESS TRANSFER OF THE ELECTRIC POWER

Georgiana ZAINEA (REZMERIȚĂ)[1], Mihai IORDACHE[2], Florin REZMERIȚĂ[3]

*This paper presents an improved method of the tolerance and sensitivity analysis for devices used in wireless transfer of the electrical power. The tolerance analysis method is based on the Fast Monte Carlo Analysis (FMCA) which uses the circuit functions in order to generate the results of the tolerance analysis. For the computation of the sensitivities with respect to any circuit parameter, the derivatives of the circuit functions are used. The method developed for the tolerance and sensitivity analysis has been implemented in a computing program. This program can perform the tolerance analysis for any linear and/or piecewise-linear nonlinear analog circuits.*

**Keywords**: sensitivity analysis, tolerance analysis, circuit function, Fast Monte Carlo Analysis

## 1. Introduction

In the last few years, many applications like autonomous electronic devices (laptops, mobile phones, medical equipments etc.) or electric (hybrid) cars, whose batteries need to be recharged, have focused the researchers' attention on the wireless power transfer [1-5].

Witricity (WIreless elecTRICITY) represents an experimental technology used in order to transfer electricity/power between electrical sources and receivers without wires. The transfer is made over distances at which the electromagnetic field is strong enough to allow a reasonable power transfer. This is possible if both the emitter and receiver achieve magnetic resonance. Wireless transmission is useful in cases where instantaneous or continuous energy is needed but interconnecting wires are inconvenient, hazardous or impossible.

The term Witricity was introduced by Professor Marin Soljačić from MIT who, together with his team, started to work on the subject in 2005. Their first papers on this subject, "Coupled-mode theory for general free-space resonant scattering of waves" [1], and "Wireless power transfer via strongly coupled

[1] PhD eng., Electrical Engineering Departament, University POLITEHNICA of Bucharest, Romania, e-mail: z.georgiana.n@gmail.com
[2] Prof., Electrical Engineering Departament, University POLITEHNICA of Bucharest, Romania
[3] PhD eng., Electrical Engineering Departament, University POLITEHNICA of Bucharest, Romania, e-mail: florin.rezmerita@yahoo.com

magnetic resonances" [2] were published in 2007. An experimental demonstration with a 60 W bulb was powered with this technology over a distance of 2 m with an efficiency of 45%. The coils resonated together at 9.9 MHz.

Many of the Witricity technology aspects can be found in Andre Kurs Master of Science Thesis [1] or Bachelor of Science Thesis of Robert Moffatt [2], members of the initial team who developed this technology.

At the beginning the technology was proposed for small power application like cordless battery charging of laptops and mobile phones, but the papers that succeeded, suggested practical implementations in the electric vehicle field [4] and medical area [5] as a way to recharge the sensors and implanted devices.

The coupling through magnetic resonance implies the coupled systems to work at their resonance frequency. In [2] the energy is transferred between two simple coils which form resonators due to the parasitic capacitance of their turns. In [6] the concept is improved by connecting the coils to capacitors. The coupling is made through magnetic field and the electric field is reduced.

Some remarks have to be made:

- The interaction between the source and device is strong enough so that the interactions with non-resonant objects can be neglected, and an efficient wireless channel for power transmission is built;
- Magnetic resonance is particularly suitable for applications because, in general, the common materials do not interact with magnetic fields;
- It seems that the power transfer is not visibly affected when humans and various objects, such as metals, wood, electronic devices, are placed between the two coils at more then few centimetres from each of them, even in cases when they completely obstruct the line of sight between source and device;
- Some materials (such as aluminium foil and humans) just shift the resonant frequency, which can in principle be easily corrected with a feedback circuit.

Obviously, the efficiency of the wireless power transfer depends on the configuration of the two magnetic coupled resonators (series-series, parallel-parallel, series-parallel and parallel-series), on the values of the two circuit parameters, and on the resonant frequency.

The optimal design of witricity circuits must guarantee all required operating and reliability parameters with minimal manufacturing costs. After choosing the appropriate topology and performing optimization studies, this goal is strongly closed to the market price of the components. All the circuit components are manufactured around a target value of their main parameter (the rated value) with an allowable tolerance. Smaller tolerance requires higher technological accuracy, which involves the increasing of the component costs and vice versa. In order to choose as many cheap components as possible, keeping the circuit performance, one must decide which components are critical and how much is the required value of the tolerance. Such a decision is possible only through a rigorous sensitivity analysis followed by tolerance analyses. The problem related to sensitivities and tolerances became necessary in connection with the development of electronic circuits and their serial production [7]. Therefore, using quality and expensive devices for the critical components and cheaper devices for the no critical ones, the cost minimization can be achieved, by keeping the performance specifications of the whole equipment [8].

In this paper we propose an improved method of the tolerance and sensitivity analysis for the devices used in the wireless power transfer. The tolerance analysis method is based on the Fast Monte Carlo Analysis (FMCA) which uses the circuit functions in order to generate the results of the tolerance analysis. In this way we can compute the sensitivity and the tolerance of any transfer functions of the resonators with respect to all resonator parameters. In function of the sensitivity and tolerance values we can select the optimal values of the resonator parameters which assure the maximum efficiency in the wireless power transfer.

The normalized sensitivity of a network function $H(s)$ with respect to one parameter $x_k$ has the following expression:

$$S_{x_k}^{H} = \frac{\partial H}{\partial x_k} \cdot \frac{x_k}{H}. \tag{1}$$

The sensitivity shows the influence of small deviations of this parameter on the network function, if all the other parameters remain unchanged [9, 10]. The parameter of interest can be any resistance, inductance, capacitance, the transfer parameters of the controlled sources and the parameters describing the characteristics of the nonlinear components. Higher values of the sensitivities designate the critical components.

Many methods to compute the circuit sensitivities were developed. A brute-force method is to vary the parameter of interest slightly, $\Delta x_k \to 0$, calculate the change in *H*, then the ratio $\Delta H/\Delta x_k$. The accuracy of such a method can be poor because of the round-off numerical errors given by the differences between two nearly equal numbers [10]. Thus, the computational effort is high because of many repeated analyses and multiple sensitivity evaluations at each frequency. Other methods have been promoted, as the incremental-network approach, the network approach, as well as symbolic-network-function approach [10-16]. It is generally recognized that the most powerful methods are based on symbolic algorithms.

Many techniques of tolerance analysis were developed during the last decades. A good technique must be able to find the worst case given by the specified tolerances of the circuit elements [8, 11, 13-16]. A tolerance analysis is commonly based on stochastic models because of the random distribution of the parameters within their tolerance domain. The well known approach Monte Carlo analysis requires repeated circuit simulations in which the parameter value samples are chosen with a normal (Gaussian) or uniform distribution [8, 11, 13]. A higher number of samples gives more accurate results, but involves increasing the computational effort. Nevertheless, it has been adopted by many commercial analysis programs, like SPICE.

Other techniques deal with mathematics interval methods [14], root-sum-square or extreme-value analysis [15, 16], but the efficiency of each method depends on the circuit complexity or on the operation mode.

We developed the Monte Carlo and extreme-value approaches, exploiting symbolic and partial symbolic computation methods.

## 2. Methods for tolerance analysis of the electric circuit components

The outcome of a circuit-design process is a circuit whose parameters are fixed values. The vector of parameter values is referred to as the nominal parameter vector:

$$\mathbf{x}_0 = \left[x_{1,} x_2, ..., x_n\right]^{\mathrm{t}} \tag{2}$$

Unfortunately, due to the imperfect manufacturing process of the coil, the actual values of the circuit parameters will usually deviate from the nominal values as given in (2). The statistical distribution of parameter values associated with manufactured products is referred to as tolerance. If the nominal parameter values are given by (2), tolerance implies the combined effect of *n* parameter changes $\Delta x_k,\ k = \overline{1,n}$, on the performance *f*, resulting in the change

$$\Delta f = \sum_{k=1}^{n} \frac{\partial f}{\partial x} \Delta x_k + \frac{1}{2} \sum_{k=1}^{n} \sum_{j=1}^{n} \frac{\partial f}{\partial x_k \partial x_j} \Delta x_k \Delta x_j + ... \tag{3}$$

in which higher-order derivatives of the Taylor-series expansion of $\Delta f$ are neglected, [7,9,10].

There are two ways for quantifying the concept of tolerance:

1. By specifying the minimum and maximum values $x_m$ and $x_M$ associated with each nominal parameter $x_0$, where $x_m \le x_0 \le x_M$; 2. By assuming a certain probability-density function for the circuit parameters and specifying the mean value μ and the standard deviation σ (or the variance $\sigma^2$).

Definitions of device tolerance are based on 1 or 2. The characteristic variations of the *n* parameters are represented by the tolerance vector $\boldsymbol{p}_0 = [p_1, p_2, ..., p_n]^{\mathrm{t}}$. Each tolerance value $p_k, k = \overline{1,n}$ may have the meaning of either the worst-case value (i.e., $\Delta x_m = x_0 - x_m$ or $\Delta x_M = x_M - x_0$) or the standard deviation $\sigma_k$.

In general, manufacturers of discrete electronic devices whose parameters have Gaussian-like distributions, specify three parameter values: the typical (most probable) value $x_t$, the minimum value $x_m$ and the maximum value $x_M$. When this distribution is to be modeled by a normal distribution with mean value μ and the standard deviation σ, empirical relations are used.

Therefore, $x_m$ and $x_M$ are assumed to correspond to the 3σ limits of the normal probability-density curve.

Any tolerance analysis technique requires the sensitivity computation and for this reason an efficient procedure to compute the sensitivities in respect to any circuit parameter is necessary. Because our software tools TAACP – Tolerance Analysis of Analog Circuit Program [8, 11], and TFSGP – Transfer Function Symbolic Generation Program generate any network (transfer) function in full symbolic form, in order to compute the sensitivities we use the definition (1).

According to the two aforementioned ways in which tolerance can be specified, the methods for tolerance analysis can be divided into two distinct classes:

1. Worst-case analysis: by selecting the proper minimum or maximum values of the circuit parameter *x*, the worst possible upper and lower limits of the output function are evaluated. For this we can use the differential sensitivities (derivatives).

2. Statistical methods: the most used methods are Monte-Carlo method and moment method. In Monte-Carlo method the analysis of the circuit is performed for a large number of times *n*. The parameter values are randomly selected according to a given probability distribution of *x*. For reliable results, *n* should be large, so that much computation time is required. The moment method assumes a normal distribution of the random variable *x*.

The sensitivity and tolerance analysis procedure, presented in this paper, is based on the most flexible circuit analysis method – the modified nodal analysis (MMA)[9, 16].

## 3. Fast Monte Carlo method for tolerance analysis (fmcmta)

The Fast Monte Carlo Method for Tolerance Analysis (FMCMTA) analyses a circuit in such a way that all possible combinations of element tolerances are used. For all $2^n$ to be shown, an *n* bit binary counter is used. If the bit has a value equal to 0, the minimal tolerance is used and if the bit is equal to 1 the maximum tolerance is used. For tolerance computation and for generation the graphics, some similar steps to Monte Carlo Analysis need to be followed.

Step 1: The nominal value of each circuit component is defined. Values are defined for the low frequency limit (LF) and for the high frequency limit (HF), as well as for the sample rate (DF). The $N = 2^n$ binary counter is created, where *n* represents the number of circuit components.

Step 2: The symbolic tolerance matrix is created having a dimension of 2x*n* and the tolerance of each component is defined.

Step 3: For each studied circuit the numerator and denominator of the transfer function is defined.

Step 4: The transfer function absolute value is computed.

Step 5: Together with the component counter $k$, each circuit component is perturbed within the transfer function. The resulting function is rewritten. The perturbation for a component is written in the following way: $R_1*T_n(k,1)$, where $R_1$ is the value of the circuit element, $T_n$ the tolerance, $k$ the sample rate and 1 the position of the element within the tolerance matrix.

Step 6: Sensitivity is computed with the definition formula (1).

Step 7: Plotting the graphs representing sensitivities and amplitude-frequency characteristics.

If $n_f$ frequency points are considered, the tolerance analysis requires $n_f \cdot N$ successive evaluations of the network function and the same amount of values must be written in the computer memory.

As an alternative, the extreme-value strategy deals only with the minimum and maximum values of each circuit parameter. Therefore, if $n$ parameters are considered, $N = 2^n$ samples and $2^n$ AC analyses are necessary. In this manner, the worst possible upper and lower limits of the circuit response are obtained. The extreme-value analysis requires $n_f \cdot 2^m$ evaluations of the network function, but at each frequency only the maximum and minimum values have to be kept in the computer memory ($2n_f$ values). Hence less computer memory than for the statistic-based methods is required. However, the great amount of successive AC analyses can require too much computation time. For instance, if the circuit contains 12 tolerance parameters, the extreme-value strategy requires $2^{12} = 4096$ AC analyses, while a Monte Carlo strategy can give satisfactory results with fewer samples. Above this number of tolerance parameters, the extreme-value strategy becomes too costly in terms of computation effort. Although it gives better results comparing to the statistical methods, the extreme-value strategy can be exploited only for simple circuits.

## 4. Software implementation

The method described in the previous section has been implemented as an improved version of the program TAACP – Tolerance Analysis of Analog Circuit Program [8, 11] under the powerful computing environment Matlab. The TAACP window is shown in Fig. 1. It combines several capabilities, accomplishing the following performance criteria: usability, reliability, precision (error minimization), constructive flexibility, hardware and software resources requirement, and compatibility with other analysis programs. The newest version exploits better the symbolic algorithms, being better adjusted for large-scale circuits. Another version of the full-transfer function generation in the full-symbolic form was implemented in the TFSGP.

An improved interactive graphical user interface facilitates handling the program; it contains push buttons, popup menus and editable text boxes that allow performing any command action and setting analysis parameters.

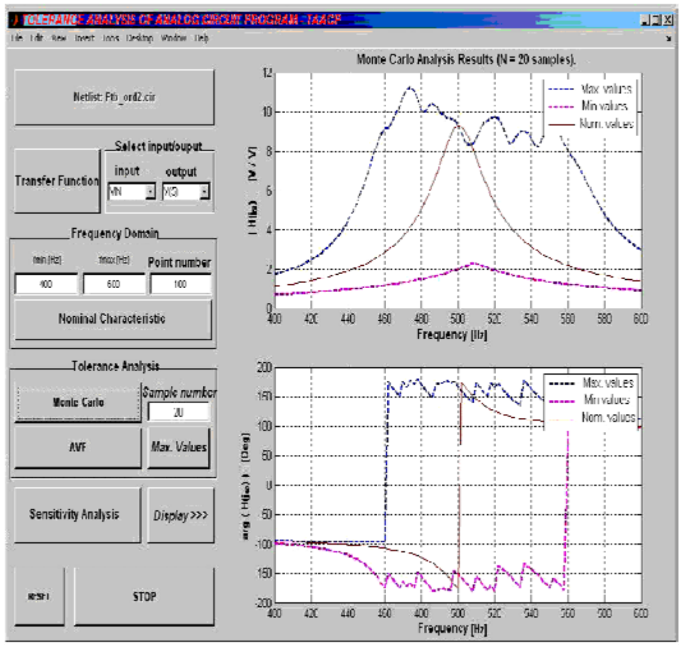


Fig. 1. Window of the TAACP

The input data is a SPICE-compatible netlist (.cir file), which can be created either by text editing or rather through the circuit diagram built using the Schematic editor of SPICE. This solution allows performing preliminary AC analyses using SPICE, as well as witness analyses.

Next to importing the netlist, the user chooses the desired input and output quantities using popup menus. Then the network function computation is launched by pushing a button. It is computed in symbolic form and evaluated for the nominal values of the parameters. Its poles and zeros are computed and represented in the complex plan, as a qualitative image of the circuit behavior.

The next step requires setting the frequency domain and the number of frequency points (using the corresponding text boxes), if they differ from those given in the netlist. Then the nominal frequency characteristics (magnitude and phase respectively) are computed and plotted. Linear, logarithmic or Bode plot style can be chosen.

The tolerance analysis is available either by a Monte Carlo approach with normal distribution of the samples (the number of samples is established by the user) or by an extreme-value strategy. The results are plotted near the nominal characteristics, showing their possible deviation range. If the number of evaluation points overcomes a reasonable value, a warning message is generated, notifying the user about the required computation time. The sensitivity analysis results are plotted both for magnitudes and phases in normalized units.

## 5. Examples

Let us consider that the two coils are equivalent with two series-series resonators inductively coupled as in Fig. 2, where $L_3$ and $L_4$ are two coaxial identical coils represented in Fig. 3, [4]. The parameters of the coils are: the radius $r$ = 150 mm, the pitch $p$ = 3 mm, the wire size $w$ = 2 mm, the distance between the coils $g$ = 150 mm, and the number of the turns $N$ = 5. Using the Q3D Extractor program [17], we get the following numerical values for the parameters of the system of the two inductively coupled coils: $C_1 = C_2 = C = 1.0404$ pF, $L_3 = 16.747$ μH, $L_4 = 16.736$ μH, $M$ = 1.4898 μH, $R_5$ = 0.12891 Ω, $R_6$ = 0.12896 Ω.

The source circuit has the resistance $R_8$ = 5.0 Ω, and the load is $R_7$ = 5.0 Ω.

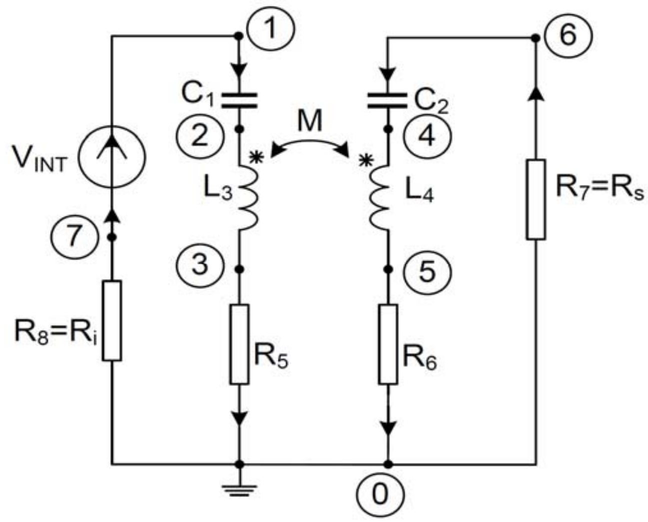


Fig. 2. System of Two Series-Series Resonators

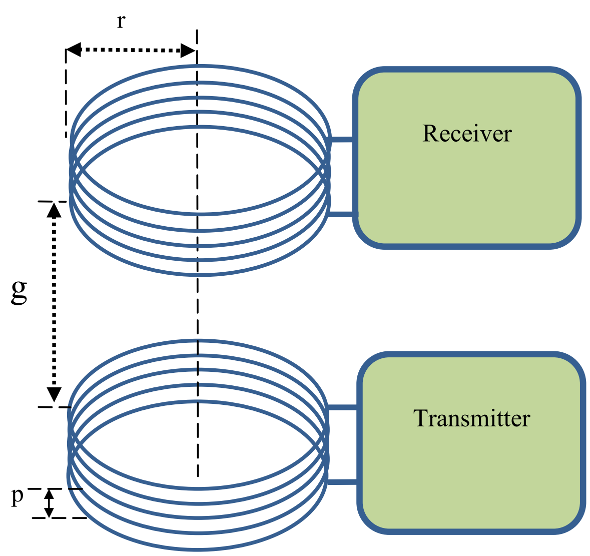


Fig. 3. Coil Geometry [4]

The netlist file corresponding to the circuit in Fig. 2 has the following structure:

**SPICE_NET*
*.AC LIN 100 100HZ 1e08HZ*
*.PRINT AC I(VIN) IP(VIN)*
*C1 1 2 1.0404e-12 TOL=1%*
*C2 6 4 1.0404e-12 TOL=1%*
*L3 2 3 16.747e-06 TOL=1%*
*L4 4 5 16.736e-06 TOL=1%*
*K34 L3 L4 0.5 TOL=1%*
*R5 3 0 0.12891 TOL=1%*
*R6 5 0 0.12896 TOL=1%*
*R7 0 7 5 TOL=1%*
*R8 0 6 5 TOL=1%*
*VIN 7 1 AC 1*
*.END*

In order to compute the network function (input impedance)

$$H(s) = Z_{1\_7\_1\_7}(s) = \frac{V_{out}(s)}{I_{in}(s)} \equiv \frac{V_1(s)}{I_{in}(s)} \tag{4}$$

the modified nodal equations in the Laplace domain are used when $I_{in}(s) = 1$.

Running TAACP the results shown in Figs. 4, 5, 6, 7, 8, and 9 are obtained.

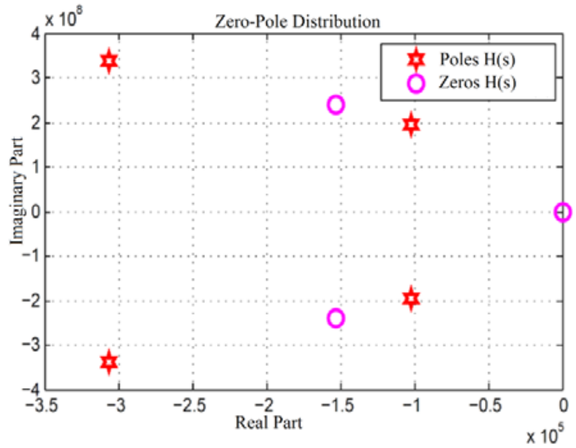


Fig. 4. Zero – pole distribution in the complex plan.

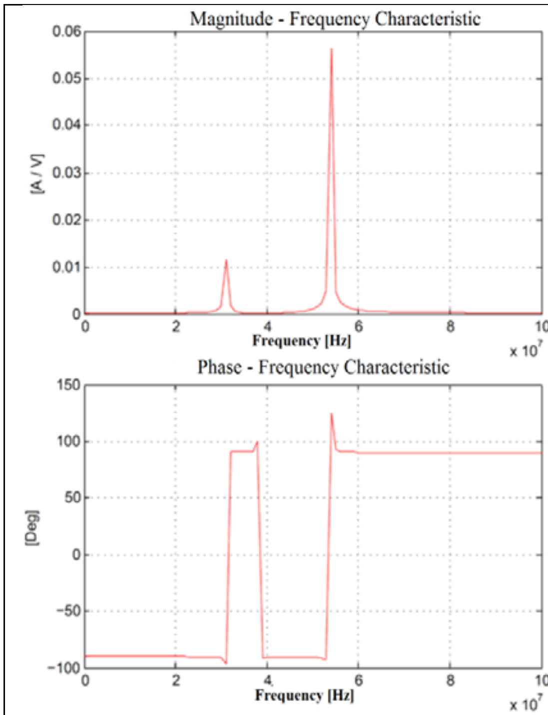


Fig. 5. Frequency characteristics for the circuit in Fig. 2 (rated conditions).

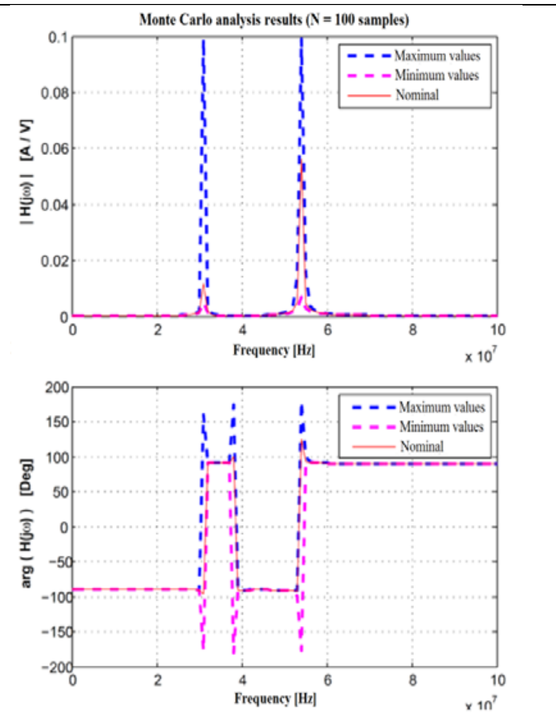


Fig. 6. Tolerance analysis of the driving-point impedance $Z_{1\_7\_1\_7}$, obtained by the program TAACP

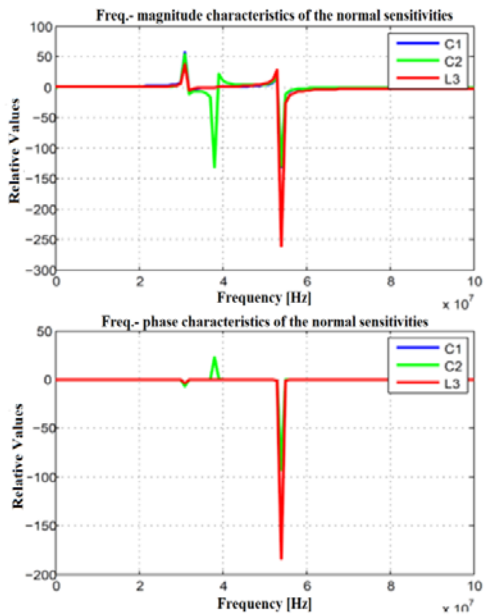


Fig. 7. Frequency characteristics of the sensitivities with respect to $C_1$, $C_2$ and $L_3$.

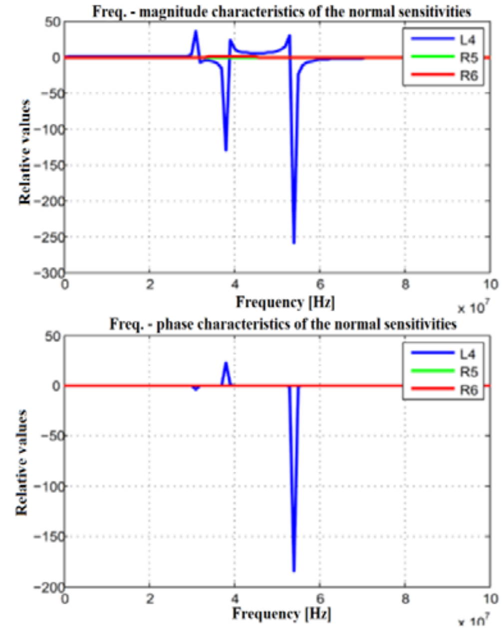


Fig 8. Frequency characteristics of the sensitivities with respect to $L_4$, $R_5$ and $R_6$.

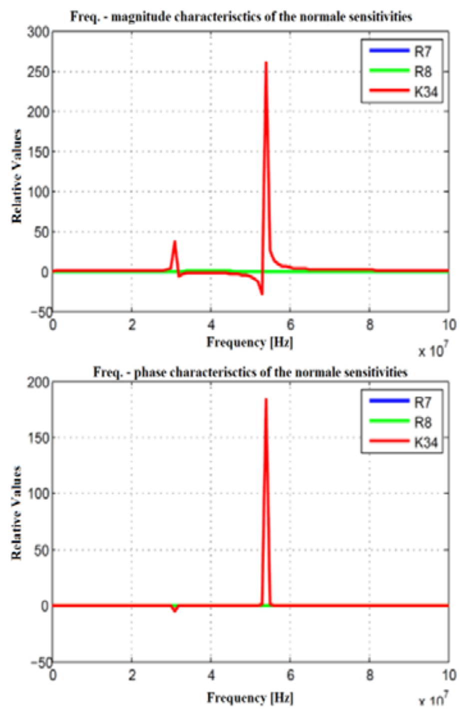


Fig .9. Frequency characteristics of the sensitivities with respect to $R_7$, $R_8$ and $K_{34}$

## 6. Conclusions

Geometrical parameters are closely related with electrical parameters (number of turns, section, length, and distance between the coils).

The study of the two methods shows that MCAM is more precise than FMCAM, fact proven by the above presented diagrams. It can be clearly seen that MCAM displays the maximum and minimum characteristics much closer to the nominal characteristic than the FMCAM. The MCAM is much more precise giving a more accurate view of the circuit output.

Fig. 4 shows the location of poles and zeros and we notice that the real parts of all the poles are negative, which proves that the circuit in Fig. 2 is stable. The nominal frequency characteristics of the input impedance are shown in Fig. 5. Fig. 6 presents the frequency characteristics obtained with the Fast Monte Carlo Method (100 samples). Here you can see the variation limits of the input module impedance with respect to the parameter tolerances of the circuit elements.

The frequency characteristics of sensitivities in connection with the circuit parameters are set for a group of three parameters - Figs. 7 (relative to C1, C2 and L3), Figs. 8 (L4, R5 and R6), and Figs. 9 (relative to R7, R8 and K34). From these characteristic we can see that the parameters L3, L4 and K34 are critical having much higher sensitivity than the other parameters (especially at the frequencies that provide highs and lows of the circuit function). Therefore especial attention should be paid to the calculation of these parameters.

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