U.P.B. Sci. Bull., Series D, Vol. 74, Iss. 1, 2012 ISSN 1454-2358

HBMOA APPLIED TO DESIGN A WATER DISTRIBUTION NETWORK FOR A TOWN OF 50000 INHABITANTS

Sanda-Carmen GEORGESCU[1]

A modified form of Honey Bees Mating Optimization Algorithm (HBMOA) has been used to optimize the design of a water distribution network for a town with 50,000 inhabitants. HBMOA is an evolutionary algorithm, highly ranked among the most known and effective algorithms in the literature. Within HBMOA, the search procedure is inspired by the process of mating in a real honey bee colony. Optimizing the design of a hydraulic network means to obtain the smallest network cost, under imposed hydraulic constraints. The water distribution network designed in this paper corresponds to a gravity distribution scheme, consisting of a looped network with 42 junctions, 78 pipes and a single source (a reservoir). All pipes were selected among 10 available pipe sizes, with diameters ranging from 50 mm, to 400 mm, meaning that there are 10^{78} design options. In this paper, 90000 design options have been investigated, by running HBMOA for 50 times, with 30 global iterations per run and 60 design solutions per global iteration. The hydraulic analysis was performed 60 times at each global iteration, for each design solution, to compute the objective function, which includes network cost and penalties for exceeding velocity limits on network pipes.

The best (suboptimal) design solution, meaning the lowest network cost and minimal penalties, was obtained at the 46th HBMOA run. The hydraulic analysis performed for the network using the pipe diameters of that best solution gave the flow rate distribution on network's pipes; it has been verified that the imposed condition of minimum 19.5 m head is respected at each node, for an input of 39.5 m head and 249 l/s flow rate at the reservoir, ensuring a base demand from 5 to 23 l/s in 29 specific nodes. All computations, concerning HBMOA coupled with hydraulic analysis, were performed using a personal numerical code, written and ran in GNU Octave (a free software clone of MATLAB).

Keywords: Honey Bees Mating Optimization Algorithm, water distribution network, GNU Octave, MATLAB.

1. Introduction

Various stochastic methods for combinatorial optimization can be applied to obtain the least-cost design of looped or combined water distribution networks. Stochastic optimization refers to the minimization or maximization of the objective function (performance function), in the presence of randomness within the search process.

[1] Associate Prof., Power Engineering Faculty, University “Politehnica” of Bucharest, Romania

Honey Bees Mating Optimization Algorithm, abbreviated HBMOA [1], is a modern evolutionary algorithm, successfully used to optimize the design of water distribution networks that involve huge nonlinear systems of equations. HBMOA has been implemented to hydraulic networks design optimization not only in its classical form [1], but also through two improved forms, namely a firstly modified form (denoted as HBMOA-M1) obtained by improving one important classical hypothesis [2]÷[4], and a new modified form that increases clearly the computational efficiency [3] & [5] (denoted as HBMOA-M2), issued from HBMOA-M1 after improving supplementary two classical hypotheses.

By considering the Hanoi water distribution network as basic test-case, the performances of both modified HBMOA forms have been compared by Popa and Georgescu [3] with the performances of other stochastic methods, like the Ant Colony Optimization Algorithm [6] & [7], the Simulated Annealing Algorithm [8], and various formulations based on Genetic Algorithms, like the one proposed by Popa and Tudor [9]: HBMOA-M2 formulation [3] gave the smallest cost of Hanoi network.

Due to its performances, the HBMOA-M2 formulation will be used within the present paper to optimize the design of the water distribution network for a town with 50,000 inhabitants: a hydraulic network consisting of one reservoir, 36 loops, 42 junctions and 78 pipes.

2. HBMOA modified form description

The search algorithm within Honey Bees Mating Optimization is inspired by the process of mating in a real honey bee colony. The queen bee q, drones d and brood have their own genome composed of genes. When modelling the mating process, the genome is attached to one bee/solution (here, the network design solution) of the optimization problem. One genome is mathematically described by a list of numerical values, where each value is attached to a gene/ decision variable that represents an unknown of the problem. Depending on the values of genes/unknowns from such a list, the objective function of the problem (here, the network cost, with or without penalties linked to water velocity in pipes) has a smaller or greater value, so the genome of the associated bee/solution is stronger or weaker.

In this paper, a solution (bee) has a number of unknowns (genes) equal to the total number of pipelines that form the studied hydraulic network: there are 78 pipes. The network can be build using 10 available pipe sizes, ranging from 50 mm, to 400 mm-diameter. There is a marker, of integer value ranging from 1 to 10, attached to each pipe size, as in Table 1. Thus, each pipeline can have a diameter D_j and a marker c_j, where $j = 1 \div 78$. The values of the unknowns (genes) are the integer values of c_j attached to each pipe upon its size, meaning that each solution has a number of 78 unknowns, where each unknown can have an integer value c_j from 1 to 10. So, there are 10^{78} design options for the studied looped hydraulic network! The unknowns (genes) vector, of components c_j, will be further denoted by $\mathbf{C}$.

The network cost F is computed as: $F = \sum_{j=1\div 78} p_j L_j$, where p_j is the unitary price of each pipe, expressed in fictive monetary units by meter (U.M./m), as in Table 1, and L_j is the pipe length, in meters.

Table 1

Available pipe sizes, attached marker and pipe's unitary price

D [mm]	50	75	100	125	150	200	250	300	350	400
c	1	2	3	4	5	6	7	8	9	10
p [U.M./m]	10	20	30	40	50	60	70	80	90	100

The objective function is computed as: $f = \left(F + 1000\, n_v\right)/1500000$, where 1000 is the value of penalties coefficient, n_v is the number of pipes where the water velocity exceeds the imposed limits (namely, pipes where velocity value is below 0.4 m/s, or where velocity value is above 1.7 m/s), and 1,500,000 is a coefficient that adjusts objective function values around 1. The above penalties were selected to avoid too low velocity values (involving water quasi-stagnation), or too high velocity values (involving too high hydraulic losses values) on network pipes. The best performance, or minimal objective function f_{min} corresponds to the lowest network cost F_{min} with none or minimal penalties.

At the first global iteration of HBMOA, a given number of bees, $N_{in} = 30$, is generated; those bees are viewed as potential network design solutions. To each bee's genome, a vector $\mathbf{C}_i$ is attached, where $i = 1 \div N_{in}$; each vector $\mathbf{C}_i$ has 78 components c_j of values randomly generated from 1 to 10. So, for each bee, a network cost F_i and subsequently an objective function f_i can be computed. Then, that initial population of bees is ranked increasingly upon the objective function values; the best solution (the one with the best performance/ lowest objective function value) is selected as initial queen bee q, and its objective function will be denoted as: $f_q = \min_{i=1\div 30} \left(f_i\right) = f_1$. Further, a number N_d of solutions, ranked after the queen, forms a list of drones d, which may mate with the queen during the first mating-flight, while the rest of initially generated solutions are ignored. In this paper, $N_d = 29$, so there are no ignored solutions.

Each drone d_i has an objective function denoted as f_{d_i}, where $i = 2 \div (N_d + 1) = = 2 \div 30$. Besides its genome, which is the strongest, the queen is characterised by her speed V, as well as by her spermatheca capacity N_s (which is kept constant during all mating-flights, and equals the maximum number of drones that can mate with the queen during such a flight); in this paper, $N_s = N_d$.

HBMOA consists of the following five steps [1] & [3]:

① *Mating-flight* that represents a global iteration k, during which the current queen bee q (best solution) selects randomly some drones, and by mating, each drone's genome is stored in her spermatheca. Before leaving the hive, at time $t = 0$, the queen is initialized with some amount of speed, generated in the range $0.5 \div 1$. The mating-flight process scheme consists of:

- Selecting randomly a drone d_i where $i = 2 \div (N_d + 1)$, from drones' list;
- Computing the probability of mating between that drone d_i and the queen q, using a function of Boltzmann type: $\text{Prob}(q, d_i) = \exp\left(-\left|f_q - f_{d_i}\right| / V(t)\right)$, where $\text{Prob}(q, d_i)$ is the probability of successfully mating; f_q and f_{d_i} are the objective functions of queen bee q and drone d_i; $V(t)$ is queen's speed at time t;
- Generating a random number $r \in (0;1)$; if $\text{Prob}(q, d_i) > r$, then the drone d_i successfully mated with the queen, and his genome is added to the spermatheca;
- Even if the mating succeeds or not, queen's speed decays upon time t as: $V(t+1) = \alpha\, V(t)$, where $\alpha \in (0;1]$ is a decay coefficient, usually close to 1. In this paper, $V(0) = 1$ and $\alpha = 0.97$;
- Iterating the above process, either until queen's spermatheca is full (the maximum capacity N_s is reached), or until her speed decays down to a minimum given value V_{min}; in this paper, $V_{min} = 0.2$;

② *Creation of new brood* (trial solutions), by crossovering queen's own genome $\mathbf{C}_q = \mathbf{C}_1$ with drones' genomes $\mathbf{C}_{d_i} = \mathbf{C}_i$ for $i = 2 \div (N_d + 1)$: after queen returning to the hive, a given number N_b of new bees (b) appears. In this paper, there is a maximum number $N_b = 29$ of new bees that can appear. Thus, a drone genome is randomly selected from the spermatheca, and is combined with queen's genome, leading to the genome of a new bee: $\mathbf{C}_{b_i}$. The new genome creation is made here with a single heuristic crossover operator used in Genetic Algorithms, as: $\mathbf{C}_{b_i} = \mathbf{C}_q + \text{round}\left(r\left(\mathbf{C}_q - \mathbf{C}_{d_i}\right)\right)$, for $i = 2 \div (N_d + 1)$, where the drone d_i is the solution randomly selected from the spermatheca to generate the new solution (new bee) b_i and "round" refers to rounding towards the nearest integer;

③ *Improvement of brood's fitness* (trial solutions) by worker bees: after creating the total number N_b of new bees, the worker bees start to take care of the brood. In this paper, workers role is implemented by a single mutation operator, which is applied to a new bee for N_m times, thus simulating the *feeding* with royal jelly, to improve bee's performance (N_m is an imposed number of mutations, equal to the number of worker bees: in this paper, $N_m = N_d$). After selecting randomly a new bee b_i, the mutation operator chooses randomly one gene c_j from bee's genome $\mathbf{C}_{b_i}$, and modifies his current value c_{j_i}. By generating randomly the numbers $r_1, r_2 \in (0;1)$ and by denoting c_{min} and c_{max} as limits of gene's values (here, $c_{min} = 1$ and $c_{max} = 10$), the following non-uniform mutation operator allows modifying the value of the gene selected for mutation:

$$c_{j_{i_{new}}} = \begin{cases} \text{round}\left(c_{j_i} + r_2\left(c_{max} - c_{j_i}\right)\right), & \text{if } r_1 < 0.5 \\ \text{round}\left(c_{j_i} - r_2\left(c_{j_i} - c_{min}\right)\right), & \text{if } r_1 \geq 0.5 \end{cases} \tag{1}$$

④ *Adaptation of workers fitness* in accordance with the amount of improvement (performance) achieved on brood. Here, the objective function f_i of each solution modified by mutation is computed;

⑤ *Replacement of the least fittest queen* by a new queen (new best solution), selected among the fitter brood. If the performance of a new solution (modified by mutation) is better than the performance of the current queen, then that new solution will become new queen, replacing the old queen. In other words, after N_m imposed mutations applied on brood, the initial queen can preserve his role for the next global iteration (or mating-flight), or it can be replaced.

The above five steps are iterated to minimize the objective function corresponding to the studied optimization problem. Computations stop when the maximum number of global iterations, $k_{max} = 30$, is reached.

When passing from the current iteration k, to the next one $(k+1)$, there are several possible HBMOA formulations, which are extremely important with respect to algorithm's convergence. Within the classical HBMOA formulation [1], all brood fed by worker bees, who failed to replace the existing queen after the current iteration k, are completely destroyed and a new list of drones is randomly generated for the next iteration $(k+1)$. Within the modified HBMOA forms, HBMOA-M1 [2]÷[4] and HBMOA-M2 [3] & [5], brood fed by worker bees and not transformed in a new queen in the current iteration k, are inserted within the list of drones for the next iteration $(k+1)$. Even if it is far from the mating process in a real honey bee colony, that approach improves clearly the computational efficiency of the search algorithm, because the bees already fed within the k iteration will have a greater performance at the beginning of the $(k+1)$ iteration, than a new list of drones fully randomly formed. HBMOA-M2 form is an improvement of HBMOA-M1, by adding 2 modifications to it, namely:

- the use of tournament rule when creating new brood in step ②, by selecting randomly 3 drone's genomes from the spermatheca, and combining the best of them (the one with lowest objective function) with queen's genome; it ensures a greatest chance to available genetic material to produce better new bees;
- in step ③, for each new bee randomly selected, the mutation operator chooses randomly 3 genes from bee's genome, and modifies their current values upon (1); it ensures to new solutions a more intensive performance improvement.

All computations, concerning HBMOA-M2 formulation coupled with hydraulic analysis (needed to compute objective function values), have been performed using a personal numerical code, written and ran in GNU Octave (a free software clone of MATLAB).

3. Water distribution network description and hydraulic analysis

The studied water distribution network corresponds to a town of 50,000 inhabitants: it consists of one source (reservoir R), 36 loops, 42 junctions (nodes) and 78 pipes, labelled by $j = 1 \div 78$ (figure 1). The hydraulic system is constantly fed from the reservoir R, with a flow rate of 249 l/s. Some junctions (29 nodes) request a base demand of constant value ranging from 5 l/s to 23 l/s, as in figure 2. The studied network is flat, with a head $H_{\mathrm{R}} = 39.5$ m at its source. A minimum head $H_{min} = 19.5$ m is imposed at each node. Head losses on each pipe j were computed with Darcy-Weissbach formula: $h_{rj} = M_j Q_j^2$, where the hydraulic resistance modulus is $M_j = 0.0826 \lambda_j L_j / D_j^5$ (in s^2/m^5) and Q_j is the flow rate (in m^3/s). The friction factor λ_j was defined for fully turbulent flow, using the Prandtl-Nikuradse formula [10, page 28], for 0.2 mm pipe's wall roughness. The values of pipe length L_j are given in Table 2 (Section 4). The flow rate distribution Q_j for $j = 1 \div 78$ is a solution of the nonlinear system consisting of 36 head losses balance equations on each loop (labelled from 1 to 36 in figure 1), and 42 continuity equations written for each node, other than the source node R.

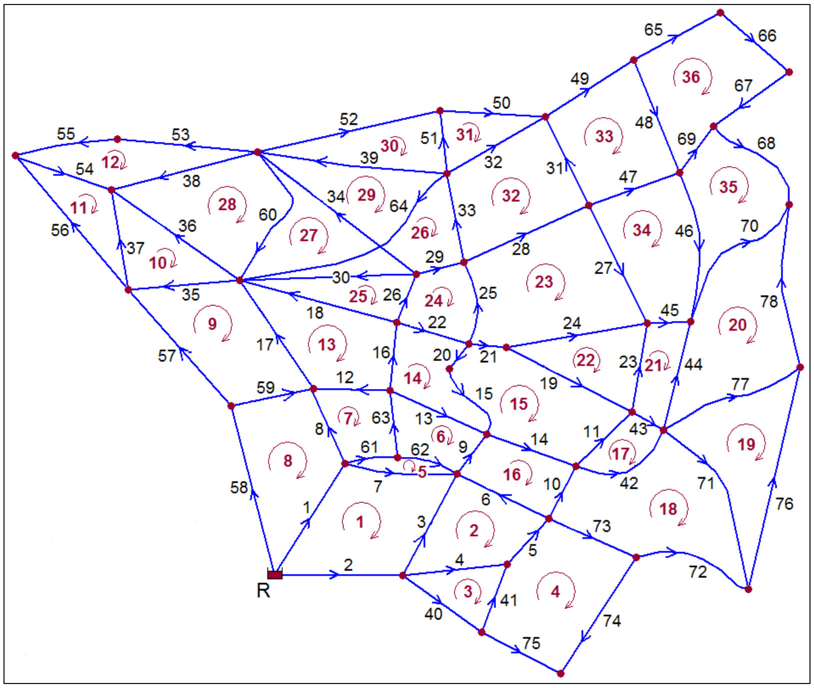

Fig. 1. Water distribution network scheme: reservoir R; flow direction on pipes (in blue), labelled by j from 1 to 78 (in black); loops labelled from 1 to 36 (in bold red), in clockwise direction

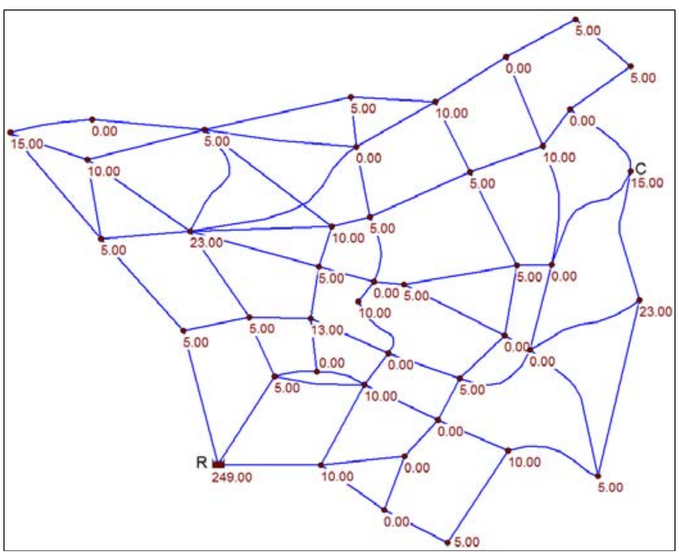

Fig. 2. Flow rate at the reservoir R and base demand at each junction, in l/s

For example, the head losses balance equation corresponding to the loop no. 1 is:

$$M_1 Q_1 |Q_1| + M_7 Q_7 |Q_7| - M_3 Q_3 |Q_3| - M_2 Q_2 |Q_2| = 0 \qquad (2)$$

The continuity equation $Q_1 + Q_2 + Q_{58} - 0.249 = 0$, written at the reservoir R, has been used to verify the solution of the nonlinear system of 78 equations with 78 unknowns Q_j ($j = 1 \div 78$). The hydraulic analysis has been performed in GNU Octave using the built-in function called *fsolve*.

For each solution/bee given by HBMOA-M2, a certain network design solution was yielded; for each design solution, the hydraulic analysis was performed for the whole network, firstly to compute the flow rate distribution and the velocity distribution on network's pipes, then to verify that the condition of minimum 19.5 m head is reached at each node for an input of 39.5 m head at R.

4. Numerical results

In this paper, 90000 design options have been investigated, by running HBMOA-M2 for 50 times, with $k_{max} = 30$ global iterations per run and 60 design solutions per global iteration. Thus, the hydraulic analysis was performed for 60 times at each global iteration, for each design solution, in order to compute the objective function, which includes network cost and penalties for exceeding velocity limits on network pipes.

The measured overall elapsed time to perform a single run of HBMOA-M2, with all 1800 subsequent hydraulic analysis (1800 calls of the *fsolve* built-in function in GNU Octave) was of 15,5 minutes, on a two-processors workstation with 16 GB RAM, meaning about 31 seconds per global iteration. It must be pointed out that for some design solutions (for 27% of all cases), computations related to the hydraulic analysis did not converge to a solution Q_j ($j = 1 \div 78$) and the solver stopped prematurely: *fsolve* stopped because it exceeded the maximum number (7800) of function evaluation, before it minimized the objective to the requested tolerance. For such a case, the corresponding HBMOA design solution was ignored and was not ranked among the convergent cases.

The best suboptimal design solution, meaning the lowest network cost and minimal penalties, was obtained at the 46th run of HBMOA-M2. The hydraulic analysis performed for the network built using the pipe diameters of that best solution gave the flow rate distribution on network's pipes; the corresponding values of D_j and Q_j are inserted in Table 2. For that best design solution, the condition of minimum 19.5 m head is reached at each node, for an input of 39.5 m head at the reservoir. The consumer C from figure 2 has the lowest head: 19.54 m.

Table 2

Pipes lengths, computed pipes diameters and flow rates for the suboptimal solution

j	L_j [m]	D_j [mm]	Q_j [l/s]	j	L_j [m]	D_j [mm]	Q_j [l/s]
1	380	400	182.26	40	380	75	3.41
2	400	250	35.32	41	450	75	1.9
3	350	75	5.53	42	400	75	1.27
4	350	125	16.38	43	200	250	27.37
5	250	150	18.28	44	520	50	1.12
6	300	50	0.88	45	250	150	11.11
7	400	100	10.54	46	480	50	1.01
8	380	50	1.61	47	520	100	6.78
9	240	250	19.4	48	600	100	4.93
10	300	75	2.86	49	560	200	15.41
11	320	250	27.76	50	430	200	17.65
12	360	50	1.23	51	360	200	19.15
13	420	100	8.11	52	340	125	3.51
14	320	300	31.18	53	350	250	15.07
15	260	75	3.67	54	250	50	0.85
16	180	350	130.32	55	340	125	15.07
17	350	250	20.39	56	450	50	0.77
18	280	100	8.26	57	550	75	3.86
19	400	50	1.03	58	600	200	31.42
20	270	150	13.67	59	210	125	22.56
21	240	150	16.6	60	500	50	0.62
22	420	400	61.24	61	200	350	165.11
23	300	50	1.42	62	240	100	12.45
24	380	100	10.57	63	300	350	152.66
25	230	200	30.97	64	340	75	2.55
26	200	250	55.83	65	380	150	10.48
27	520	75	4.12	66	400	100	5.48
28	470	200	16.42	67	300	50	0.48
29	340	300	41.54	68	450	50	1.18
30	480	50	0.83	69	250	50	0.71
31	440	50	0.52	70	520	150	13.24
32	430	125	7.23	71	550	100	5.87
33	420	350	51.09	72	450	50	1.05
34	440	100	3.45	73	540	150	14.54
35	410	100	3.17	74	500	100	3.49
36	550	100	6.49	75	420	50	1.51
37	300	50	1.25	76	600	75	1.92
38	400	50	1.41	77	550	150	21.65
39	500	250	22.16	78	530	50	0.57

The minimum objective function attached to the best suboptimal solution (at the 46th run) has the value $f_{min} = 0.754$; the corresponding network cost is $F_{min} = 1120100$ U.M. That best solution contains a minimum number of penalties applied for exceeding velocity limits on $n_v = 11$ pipes, labelled by $j = \{31; 42; 52; 53; 56; 59 \div 61; 67; 69; 78\}$, namely: the velocity is below 0.4 m/s on 9 pipes (down to a minimum value of 0.24 m/s for $j = 67$), and the velocity is above 1.7 m/s on 2 pipes (1.72 m/s for $j = 61$; 1.84 m/s for $j = 59$).

For all 50 runs, the values of the objective function f_q attached to the queen bee solution of each run can be evaluated statistically. All 50 solutions corresponded to an unrepeatable cost of the water network, ranged from $F_{min} = 1120100$ to $F_{max} = 1428500$ U.M., with a mean cost $\bar{F} = 1335254$ U.M. The objective function (which includes penalties for exceeding upper and lower velocity limits) spread from $f_{min} = 0.754$ to $f_{max} = 0.980$ with a mean value $\bar{f} = 0.928$, a standard deviation $\sigma_f = 0.0408$, and a 95% confidence interval for the mean objective function, $\bar{f} - 1.96\sigma_f / \sqrt{50} < \mu_{\bar{f}} < \bar{f} + 1.96\sigma_f / \sqrt{50}$, of $0.892 < \mu_{\bar{f}} < 0.964$.

A water distribution network can be also designed using the classical approach based on economic criteria, where an optimal diameter is proposed for a certain range of flow rates in water pipes. Such optimal diameter of pipes corresponds to the minimum of the curve summarising investment and operational costs, calculated for a range of possible diameters [11, page 127]. Designing the studied water distribution network based on economic criteria is beyond the purpose of this paper.

5. Conclusions

In this paper, a modified form of the Honey Bees Mating Optimization Algorithm (HBMOA) has been applied to design the water distribution network for a town with 50,000 inhabitants. The studied network corresponds to a gravity distribution scheme, consisting of a looped network with 42 junctions, 78 pipes and a reservoir. All pipes were selected among 10 available pipe sizes, with diameters ranging from 50 mm, to 400 mm, meaning that there are 10^{78} design options; in this paper, 90000 design options have been investigated.

The objective function of the problem contains the network cost and penalties for exceeding velocity limits on network pipes (penalties were applied for any velocity value below 0.4 m/s, or above 1.7 m/s). The best design solution corresponds to a network cost of 1120100 fictive monetary units. Even for this best solution, penalties were applied on 11 pipes. It must be pointed out that on 6 pipes, where velocity values are too low, the velocity cannot be increased by decreasing pipe's diameter, due to the fact that for those pipes, the smallest available pipe size (of 50 mm diameter) is used.

All computations, concerning HBMOA coupled with hydraulic analysis have been performed in GNU Octave, using a personal numerical code.

R E F E R E N C E S

[1]. *O. B. Haddad, A. Afshar and M. A. Mariño*, "Honey-Bees Mating Optimization (HBMO) algorithm: A new heuristic approach for water resources", in Water Resour. Manag., **vol. 20**, 2006, pp. 661-680.

[2]. *T. Niknam, J. Olamaie and R. Khorshidi*, "A hybrid algorithm based on HBMO and Fuzzy Set for multi-objective distribution feeder reconfiguration", in World Applied Sciences Journal, **vol. 4**, no. 2, 2008, pp. 308-315.

[3]. *R. Popa and Sanda-Carmen Georgescu*, "Water distribution networks optimal design using the Honey Bees Mating Optimization Algorithm", in Proceedings of the 4th International Conference on Energy and Environment CIEM2009, November 12-14, Bucharest, 2009, Paper S6_10.

[4]. *Sanda-Carmen Georgescu and R. Popa*, "Application of Honey Bees Mating Optimization Algorithm to pumping station scheduling for water supply", in University "Politehnica" of Bucharest Scientific Bulletin, Series D: Mechanical Engineering, **vol. 72**, no. 1, 2010, pp. 77-84.

[5]. *Sanda-Carmen Georgescu, R. Popa and A.-M. Georgescu*, "Pumping stations scheduling for a water supply system with multiple tanks", in University "Politehnica" of Bucharest Scientific Bulletin, Series D: Mechanical Engineering, **vol. 72**, no. 3, 2010, pp. 129-140.

[6]. *Liana Vu□ă and R. Popa,* "Optimal design model for hydraulic networks, considering water quality aspects" (in Romanian), in Proceedings of IWM2008, Bucharest, 2008, pp. 233-248.

[7]. *M. López-Ibáñez, T. D. Prasad and B. Paechter*, "Ant Colony Optimization for optimal control of pumps in water distribution networks", J. Water Res. Pl.-ASCE, **vol. 134**, no. 4, 2008, pp. 337-346.

[8]. *Maria da Conceição Cunha and J. Sousa*, "Water distribution network design optimization: Simulated Annealing approach", in J. Water Res. Pl.-ASCE, **vol. 125**, no. 4, 1999, pp. 215-221.

[9]. *R. Popa and Angela Tudor*, "Optimal design of looped water distribution networks using Genetic Algoritms" (in Romanian), in Hidrotehnica, **vol. 45**, no. 10, 2000, pp. 275-286.
[10]. *A.-M. Georgescu and Sanda-Carmen Georgescu*, Hidraulica rețelelor de conducte şi maşini hidraulice (Hydraulic Networks and Hydraulic Machineries), Ed. Printech, Bucharest, 2007.
[11]. *N. Trifunović*, Introduction to Urban Water Distribution, UNESCO-IHE Lecture Note Series, Taylor & Francis/Balkema, London, 2006.