CALCULUS OF JOINT FORCES IN DYNAMICS OF A PLANAR PARALLEL ROBOT

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Articolul stabileşte relaţii matriceale pentru dinamica inversă a unui robot paralel planar. Trei picioare identice care conectează platforma mobilă sunt localizate în acelaşi plan vertical. Pornind de la o cinematică inversă cunoscută, dinamica mecanismului este rezolvată printr-un procedeu bazat pe principiul lucrului mecanic virtual. În final, se obţin ecuaţii compacte şi grafice de simulare pentru unele forţe şi momente din legături.

Recursive matrix relations for the inverse dynamics of a planar parallel robot are established in this paper. Three identical legs connecting to the moving platform are located in the same vertical plane. Starting from a known inverse kinematics, the dynamics of the mechanism is solved using an approach based on the principle of virtual work. Finally, compact matrix equations and graphs of simulation for some forces and torques in joints are obtained.

Keywords: Joint force; Kinematics; Dynamics; Parallel robot

1. Introduction

Equipped with revolute or prismatic actuators, the parallel robots have a robust construction and can move bodies of large dimensions with high velocities and accelerations [1]. Parallel manipulators have received more and more attention from researches and industries. Among these, the class of manipulators known as Stewart-Gough platform focused great attention (Stewart [2]; Merlet [3]). The prototype of Delta parallel robot (Clavel [4]; Tsai and Stamper [5]) and the Star parallel manipulator (Hervé and Sparacino [6]) are both equipped with three motors, which train on the mobile platform in a three-degrees-of-freedom translational motion.

A mechanism is said to be a *planar robot* if all the moving links in the mechanism perform the planar motions. Bonev, Zlatanov and Gosselin [7] describe several types of singular configurations by studying the direct kinematics model of a 3-RPR planar parallel robot with actuated base joints. Pennock and Kassner [8] present a kinematical study of a planar parallel robot, where a moving platform is connected to a fixed base by three links, each leg consisting of two binary links and three parallel revolute joints.

2. Kinematics analysis

A recursive method is introduced in the present paper, to reduce significantly the number of equations and computation operations by using a set of matrices for

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the calculus of *internal joint forces* in the inverse dynamics of a 3-*RPR* planar parallel robot.

The planar parallel robot of three degrees of freedom is a symmetrical mechanism composed of three planar kinematical chains $A_1A_2A_3$, $B_1B_2B_3$ and $C_1C_2C_3$, having variable length and identical topology, all connecting the fixed base $A_1B_1C_1$ to the moving platform $A_3B_3C_3$ (Fig. 1). Together, the mechanism consists of seven moving links, six revolute joints, three prismatic joints and three revolute actuators installed on the fixed base [9].

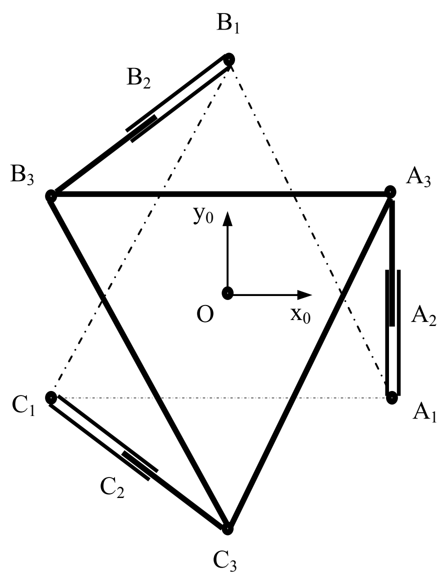

Fig. 1 General layout of the planar parallel robot

For the purpose of analysis, we attach a Cartesian frame $Ox_0y_0z_0$ to the fixed base with its origin located at the centre O, the z_0 horizontal axis perpendicular to the base. A mobile reference frame $Gx_Gy_Gz_G$ is attached to the moving platform, with the origin located just at the centre G of the triangle (Fig. 2).

One of three active legs (for example leg A) consists of a fixed revolute joint A_1 and a moving cylinder **1** of length l_1, mass m_1 and tensor of inertia $\hat{J}_1$, which has rotation about z_1^A axis with the angle φ_{10}^A. A prismatic joint is as well as a piston **2**, having a relative motion with the displacement λ_{21}^A. It has the length l_2, mass m_2 and tensor of inertia $\hat{J}_2$. Finally, a revolute joint is introduced at a planar moving platform, which is schematised as an equilateral triangle with edge $l = r\sqrt{3}$, mass m_3 and inertia tensor $\hat{J}_3$ linked at the $A_3 x_3^A y_3^A z_3^A$ frame, which rotates with the angle φ_{32}^A about z_3^A.

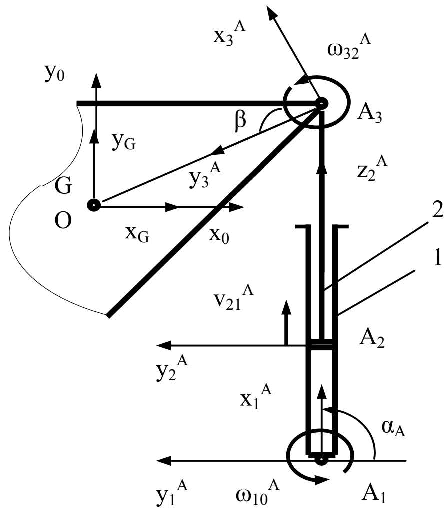

Fig. 2 Kinematical scheme of first leg A of the mechanism

At the central configuration, we consider that all legs are symmetrically extended with the angles $\alpha_A = \pi/2, \alpha_B = -5\pi/6, \alpha_C = -\pi/6, \beta = \pi/6$ of orientation of three fixed pivots.

We call the matrix $a_{k,k-1}^{\varphi}$, for example, the orthogonal transformation 3×3 matrix of relative rotation with the angle $\varphi_{k,k-1}^A$ of link T_k^A around z_k^A axis. Starting from the reference origin O and pursuing the independent legs $OA_1A_2A_3$, $OB_1B_2B_3$, $OC_1C_2C_3$, we obtain the following transformation matrices [10]

$$q_{10} = q_{10}^{\varphi} a_{\alpha}^{i},\ q_{21} = \theta,\ q_{32} = q_{32}^{\varphi} a_{\beta} \theta^T, \quad (q = a, b, c), \quad (i = A, B, C), \tag{1}$$

where we denote:

$$a_{\alpha}^{i} = rot(z, \alpha_i),\ a_{\beta} = rot(z, \beta),\ \theta = rot(y, \frac{\pi}{2}) = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$q_{k,k-1}^{\varphi} = rot(z, \varphi_{k,k-1}^{i}),\ q_{k0} = \prod_{s=1}^{k} q_{k-s+1,k-s} \ (k = 1, 2, 3). \tag{2}$$

In the inverse geometric problem, the position of the mechanism is completely given through the coordinates x_0^G, y_0^G of the mass centre G and the orientation angle ϕ of the mobile central frame $Gx_G y_G z_G$. The orthogonal known rotation matrix of the platform from $Ox_0y_0z_0$ to $Gx_Gy_Gz_G$ reference system is $R = rot(z,\phi)$.

We suppose that the position vector of G centre $\vec{r}_0^{\,G} = [x_0^G \quad y_0^G \quad 0]^T$ and the orientation angle ϕ, which are expressed by following analytical functions

$$\frac{x_0^G}{x_0^{G*}} = \frac{y_0^G}{y_0^{G*}} = \frac{\phi}{\phi^*} = 1 - \cos\frac{\pi}{3}t\,, \tag{3}$$

can describe the general absolute motion of the moving platform in its *vertical plane*.

From the conditions concerning the orientation of the platform

$$\overset{\circ}{q}{}_{30}^{T} q_{30} = R\,,\ \overset{\circ}{q}_{30} = a_\beta a_\alpha^i,\ (q = a,b,c) \tag{4}$$

we obtain the following relations between three angles $\varphi_{10}^i + \varphi_{32}^i = \phi, (i = A, B, C)$.

Pursuing the kinematical modeling developed in [9], six independent variables $\varphi_{10}^A, \lambda_{21}^A, \varphi_{10}^B, \lambda_{21}^B, \varphi_{10}^C, \lambda_{21}^C$ will be determined by the analytical equations

$$(l_0 + l_2 + \lambda_{21}^i)\cos(\varphi_{10}^i + \alpha_i) = x_0^G - x_{10}^i + r\sin(\phi + \alpha_i + \beta)$$
$$(l_0 + l_2 + \lambda_{21}^i)\sin(\varphi_{10}^i + \alpha_i) = y_0^G - y_{10}^i - r\cos(\phi + \alpha_i + \beta)\ (i = A,\ B,\ C)\,. \tag{5}$$

Now, we compute in terms of the angular velocity of the platform and velocity of centre G the relative velocities $\omega_{10}^A, v_{21}^A, \omega_{32}^A$, starting from following *matrix conditions of connectivity* [11], [12]

$$\omega_{10}^A\, \vec{u}_j^{\,T} a_{10}^T\, \tilde{u}_3 \{\vec{r}_{21}^{\,A} + a_{21}^T \vec{r}_{32}^{\,A} + a_{21}^T a_{32}^T \vec{r}_3^{\,GA}\} + v_{21}^A \vec{u}_j^{\,T} a_{10}^T \vec{u}_1 + \omega_{32}^A\, \vec{u}_j^{\,T} a_{30}^T \tilde{u}_3 \vec{r}_3^{\,GA} = \vec{u}_j^{\,T} \dot{\vec{r}}_0^{\,G}\,,\ (j = 1,2)$$
$$\omega_{10}^A + \omega_{32}^A = \dot{\phi}\,. \tag{6}$$

Considering some successive independent virtual motions of the planar mechanism, virtual displacements and velocities should be compatible with the motions imposed by all kinematical constraints and joints at a given instant in time. Concerning the first leg A, the characteristic *virtual velocities* are expressed as functions of the pose of the mechanism by the general kinematical equations (6), where we add the contributions of successive virtual translations during some fictitious displacements of the prismatic joint A_2 and of the revolute joints A_1 and A_3, for example, as follows:

$$v_{10}^{Axv} \vec{u}_j^{\,T} \vec{u}_1 + v_{10}^{Ayv} \vec{u}_j^{\,T} \vec{u}_2 + \omega_{10}^{Av} \vec{u}_j^{\,T} a_{10}^T \tilde{u}_3 \{\vec{r}_{21}^{\,A} + a_{21}^T \vec{r}_{32}^{\,A} + a_{21}^T a_{32}^T \vec{r}_3^{\,GA}\} + v_{21}^{Av} \vec{u}_j^{\,T} a_{10}^T \vec{u}_1 + v_{21}^{Ayv} \vec{u}_j^{\,T} a_{10}^T \vec{u}_2 +$$
$$+ \omega_{21}^{Av} \vec{u}_j^{\,T} a_{20}^T \tilde{u}_1 \{\vec{r}_{32}^{\,A} + a_{32}^T \vec{r}_3^{\,GA}\} + v_{32}^{Ayv} \vec{u}_j^{\,T} a_{20}^T \vec{u}_2 + v_{32}^{Azv} \vec{u}_j^{\,T} a_{20}^T \vec{u}_3 + \omega_{32}^{Av} \vec{u}_j^{\,T} a_{30}^T \tilde{u}_3 \vec{r}_3^{\,GA} = \vec{u}_j^{\,T} \vec{v}_0^{\,Gv}\ (j = 1,\, 2)$$
$$\omega_{10}^{Av} - \omega_{21}^{Av} + \omega_{32}^{Av} = \omega_0^v\,. \tag{7}$$

Now, let us assume that the robot has successively some virtual motions determined by following sets of velocities:

$$v_{10a}^{Axv} = 1\,, v_{10a}^{Bxv} = 0\,, v_{10a}^{Cxv} = 0\,, v_{10a}^{iyv} = 0\,, \omega_{10a}^{iv} = 0\,, v_{21a}^{iyv} = 0\,, \omega_{21a}^{iv} = 0\,, v_{32a}^{iyv} = 0\,, v_{32a}^{izv} = 0$$

$$v_{10a}^{ixv}=0\,,v_{10a}^{Ayv}=1\,,v_{10a}^{Byv}=0\,,v_{10a}^{Cyv}=0\,,\omega_{10a}^{iv}=0\,,v_{21a}^{iyv}=0\,,\omega_{21a}^{iv}=0\,,v_{32a}^{iyv}=0\,,v_{32a}^{izv}=0$$
$$v_{10a}^{ixv}=0\,,v_{10a}^{iyv}=0\,,\omega_{10a}^{iv}=0\,,v_{21a}^{Ayv}=1\,,v_{21a}^{Byv}=0\,,v_{21a}^{Cyv}=0\,,\omega_{21a}^{iv}=0\,,v_{32a}^{iyv}=0\,,v_{32a}^{izv}=0$$
$$v_{10a}^{ixv}=0\,,v_{10a}^{iyv}=0\,,\omega_{10a}^{iv}=0\,,v_{21a}^{iyv}=0\,,\omega_{21a}^{Av}=1\,,\omega_{21a}^{Bv}=0\,,\omega_{21a}^{Cv}=0\,,v_{32a}^{iyv}=0\,,v_{32a}^{izv}=0$$
$$v_{10a}^{ixv}=0\,,v_{10a}^{iyv}=0\,,\omega_{10a}^{iv}=0\,,v_{21a}^{iyv}=0\,,\omega_{21a}^{iv}=0\,,v_{32a}^{Ayv}=1\,,v_{32a}^{Byv}=0\,,v_{32a}^{Cyv}=0\,,v_{32a}^{izv}=0$$
$$v_{10a}^{ixv}=0\,,v_{10a}^{iyv}=0\,,\omega_{10a}^{iv}=0\,,v_{21a}^{iyv}=0\,,\omega_{21a}^{iv}=0\,,v_{32a}^{iyv}=0\,,v_{32a}^{Azv}=1\,,v_{32a}^{Bzv}=0\,,v_{32a}^{Czv}=0$$
$$(i=A,B,C)\,. \tag{8}$$

These virtual velocities are required into the computation of virtual power and virtual work of all forces applied to the component elements of the robot.

As for the relative accelerations $\varepsilon_{10}^{A},\,\gamma_{21}^{A},\,\varepsilon_{32}^{A}$ of the robot, new conditions of connectivity can be obtained through the derivative of above equations (6):

$$\varepsilon_{10}^{A}\,\vec{u}_j^{\,T}a_{10}^{T}\,\tilde{u}_3\{\vec{r}_{21}^{\,A}+a_{21}^{T}\vec{r}_{32}^{\,A}+a_{21}^{T}a_{32}^{T}\vec{r}_{3}^{\,GA}\}+\gamma_{21}^{A}\,\vec{u}_j^{\,T}a_{10}^{T}\vec{u}_1+\varepsilon_{32}^{A}\,\vec{u}_j^{\,T}a_{30}^{T}\tilde{u}_3\vec{r}_3^{\,GA}=\vec{u}_i^{\,T}\ddot{\vec{r}}_0^{\,G}-$$
$$-\,\omega_{10}^{A}\omega_{10}^{A}\vec{u}_j^{\,T}a_{10}^{T}\tilde{u}_3\,\tilde{u}_3\{\vec{r}_{21}^{\,A}+a_{21}^{T}\vec{r}_{32}^{\,A}+a_{21}^{T}a_{32}^{T}\vec{r}_3^{\,GA}\}-\omega_{32}^{A}\omega_{32}^{A}\vec{u}_j^{\,T}a_{30}^{T}\tilde{u}_3\tilde{u}_3\vec{r}_3^{\,GA}-$$
$$-\,2\omega_{10}^{A}v_{21}^{A}\vec{u}_j^{\,T}a_{10}^{T}\tilde{u}_3\vec{u}_1\;-2\omega_{10}^{A}\omega_{32}^{A}\vec{u}_j^{\,T}a_{10}^{T}\tilde{u}_3a_{21}^{T}a_{32}^{T}\tilde{u}_3\vec{r}_3^{\,GA}\,,\qquad (j=1,2) \tag{9}$$
$$\varepsilon_{10}^{A}+\varepsilon_{32}^{A}=\ddot{\phi}\;\;.$$

3. Dynamics modelling

In the context of the real-time control, neglecting the friction forces and considering the gravitational effects, an important objective of the dynamics is first to determine the input torques or forces which must be exerted by the actuators in order to produce a given trajectory of the end-effector, but also to calculate all *internal joint forces or torques*.

Upon to now, several methods have been applied to formulate the dynamics of parallel mechanisms, which could provide the same results concerning these actuating torques or forces. First method applied to formulate the dynamics modelling is using the Newton-Euler procedure [13], the second one applies the Lagrange's equations and multipliers formalism [14] and the third approach is based on the principle of virtual work [15].

Knowing the position and kinematics state of each link as well as the external forces acting on the planar 3-RPR parallel robot, in the present paper we apply the principle of virtual work for the inverse dynamic problem in order to establish some definitive recursive matrix relations for the calculus of forces in the joints.

Planar evolution of the moving platform is controlled by three electric motors that generate three couples of moments $\vec{m}_{10}^{A}=m_{10}^{A}\vec{u}_3\,,\vec{m}_{10}^{B}=m_{10}^{B}\vec{u}_3\,,\vec{m}_{10}^{C}=m_{10}^{C}\vec{u}_3$. The force of inertia $\vec{f}_{k0}^{\,inA}=-m_k^{A}\left[\vec{\gamma}_{k0}^{\,A}+\left(\tilde{\omega}_{k0}^{A}\tilde{\omega}_{k0}^{A}+\tilde{\varepsilon}_{k0}^{A}\right)\vec{r}_k^{\,CA}\right]$ and the resulting moment of inertia forces $\vec{m}_{k0}^{\,inA}=-[m_k^{A}\tilde{r}_k^{CA}\vec{\gamma}_{k0}^{\,A}+\hat{J}_k^{A}\vec{\varepsilon}_{k0}^{\,A}+\tilde{\omega}_{k0}^{A}\hat{J}_k^{A}\vec{\omega}_{k0}^{\,A}]$ of an arbitrary rigid body T_k^{A}, for example, are determined with respect to the centre of joint A_k. On the other hand, the wrench of two vectors $\vec{f}_k^{*A}$ and $\vec{m}_k^{*A}$ evaluates the influence of the action of the weight and of other external and internal forces applied to the same element T_k^A of the robot.

Two significant recursive relations generate the vectors

$$\vec{F}_k^A = \vec{F}_{k0}^A + a_{k+1,k}^T \vec{F}_{k+1}^A, \quad \vec{M}_k^A = \vec{M}_{k0}^A + a_{k+1,k}^T \vec{M}_{k+1}^A + \tilde{r}_{k+1,k}^A a_{k+1,k}^T \vec{F}_{k+1}^A \tag{10}$$

with the notations $\vec{F}_{k0}^A = -\vec{f}_{k0}^{inA} - \vec{f}_k^{*A}$, $\vec{M}_{k0}^A = -\vec{m}_{k0}^{inA} - \vec{m}_k^{*A}$.

As example, starting from (10), we develop a set of six recursive matrix relations for the leg A:

$$\vec{F}_3^A = \vec{F}_{30}^A, \vec{F}_2^A = \vec{F}_{20}^A + a_{32}^T \vec{F}_3^A, \vec{F}_1^A = \vec{F}_{10}^A + a_{21}^T \vec{F}_2^A$$

$$\vec{M}_3^A = \vec{M}_{30}^A, \vec{M}_2^A = \vec{M}_{20}^A + a_{32}^T \vec{M}_3^A + \tilde{r}_{32}^A a_{32}^T \vec{F}_3^A, \vec{M}_1^A = \vec{M}_{10}^A + a_{21}^T \vec{M}_2^A + \tilde{r}_{21}^A a_{21}^T \vec{F}_2^A. \tag{11}$$

The fundamental principle of the virtual work states that a mechanism is under dynamic equilibrium if and only if the virtual work developed by all external, internal and inertia forces vanish during any general virtual displacement, which is compatible with the constraints imposed on the mechanism. Total virtual work contributed by the inertia forces and moments of inertia forces, by the wrench of known external forces and by some joint forces, for example, can be written in a compact form, based on the relative virtual velocities.

Applying *the fundamental equations of the parallel robots dynamics* [16], [17], following compact matrix relations results

$$f_{10x}^A = \vec{u}_1^T a_{10}^T \vec{F}_1^A + \vec{u}_3^T \{v_{21a}^{Av} \vec{F}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + v_{21a}^{Bv} \vec{F}_2^B + v_{21a}^{Cv} \vec{F}_2^C\} \tag{12}$$

for the first *joint force*,

$$f_{10y}^A = \vec{u}_2^T a_{10}^T \vec{F}_1^A + \vec{u}_3^T \{v_{21a}^{Av} \vec{F}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + v_{21a}^{Bv} \vec{F}_2^B + v_{21a}^{Cv} \vec{F}_2^C\} \tag{13}$$

for the second *joint force* acting in the external revolute joint A_1,

$$f_{21y}^A = \vec{u}_2^T \vec{F}_2^A + \vec{u}_3^T \{v_{21a}^{Av} \vec{F}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + v_{21a}^{Bv} \vec{F}_2^B + v_{21a}^{Cv} \vec{F}_2^C\} \tag{14}$$

for the *joint force* acting in the prismatic joint A_2,

$$m_{21}^A = \vec{u}_1^T \vec{M}_2^A + \vec{u}_3^T \{v_{21a}^{Av} \vec{F}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + v_{21a}^{Bv} \vec{F}_2^B + v_{21a}^{Cv} \vec{F}_2^C\} \tag{15}$$

for the *joint torque* acting in the prismatic joint A_2,

$$f_{32y}^A = \vec{u}_2^T a_{32}^T \vec{F}_3^A + \vec{u}_3^T \{v_{21a}^{Av} \vec{F}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + v_{21a}^{Bv} \vec{F}_2^B + v_{21a}^{Cv} \vec{F}_2^C\} \tag{16}$$

for the first *joint force* and

$$f_{32z}^A = \vec{u}_3^T a_{32}^T \vec{F}_3^A + \vec{u}_3^T \{v_{21a}^{Av} \vec{F}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + v_{21a}^{Bv} \vec{F}_2^B + v_{21a}^{Cv} \vec{F}_2^C\} \tag{17}$$

for the second *joint force* acting in the internal revolute joint A_3.

The simulation procedure for solving the inverse dynamics of the planar parallel robot can be summarised in several basic steps.

1. For a period of $\Delta t = 3$ seconds, it is assumed that the time-history evolution of the moving platform is specified in terms of its position and orientation about the

centre G from analytical equations (3). The relations (4), (5) give the evolution of the variables $\varphi_{10}^{i}, \lambda_{21}^{i}, \varphi_{32}^{i}$ $(i = A, B, C)$.

2. Using the relations (1), (2), we compute the transformation matrices of three legs A, B, C : q_{10}, q_{21}, q_{32} and $q_{20} = q_{21}q_{10}$, $q_{30} = q_{32}q_{20}$ $(q = a, b, c)$.

3. Determine the velocities and accelerations of all links by performing the inverse kinematics analysis in terms of prescribed velocities $\dot{x}_0^G, \dot{y}_0^G, \dot{\phi}$ and accelerations $\ddot{x}_0^G, \ddot{y}_0^G, \ddot{\phi}$ of the moving platform. Specifically, for each leg, from the conditions of connectivity (6), (9) we compute the relative velocities $\omega_{10}^{i}, v_{21}^{i}, \omega_{32}^{i}$ and the relative accelerations $\varepsilon_{10}^{i}, \gamma_{21}^{i}, \varepsilon_{32}^{i}$.

4. Using the equations (7), where successively are introduced the conditions (8), we compute the virtual characteristic velocity of each element of the robot.

5. Decompose artificially the robot in several open-loop planar chains by cutting open at the moving revolute joints A_3, B_3, C_3.

7. For each moving link and platform we determine the inertia force $\vec{f}_{k0}^{inA}$ and the resulting force $\vec{F}_k^A$ exerted to the rigid body T_k^A, for example, from recursive equations (10).

8. For each moving link and platform we determine the moment of inertia forces $\vec{m}_{k0}^{inA}$ and the resulting moment $\vec{M}_k^A$ exerted at the joint A_k, from same recursive equations (10).

9. Finally, we find the joint forces $f_{10x}^{i}, f_{10y}^{i}, f_{21y}^{i}, f_{32y}^{i}, f_{32z}^{i}$ and of the joint torques m_{21}^{i} $(i = A, B, C)$ during the platform's evolution from the compact equations (12)-(17).

As application let us consider same planar parallel robot 3-RPR analysed in [9], which has the following geometrical and architectural characteristics:

$$x_0^{G*} = 0.05\, m,\ y_0^{G*} = -0.05\, m,\ \phi^* = \frac{\pi}{6},\ r = 0.3\, m, l = r\sqrt{3}$$

$$l_0 = 0.1\, m,\ l_1 = l_2 = 0.2\, m, m_3 = 3\, kg, m_4 = 1.5\, kg, m_5 = 5\, kg, \Delta t = 3\, s.$$

Using the MATLAB software, a computer program was developed to solve the inverse dynamics of the planar *RPR* parallel robot. To illustrate the algorithm, it is assumed that for a period of three seconds the platform starts at rest from a central configuration and rotates or moves along two orthogonal directions.

Assuming that there are no external forces and moments acting on the moving platform, a dynamic simulation is based on the computation of the *joint forces* $f_{10x}^{i}, f_{10y}^{i}, f_{21y}^{i}, f_{32y}^{i}, f_{32z}^{i}$ and of the *joint torques* m_{21}^{i} $(i = A, B, C)$ during the platform's evolution.

Following examples are solved to illustrate the simulation. For the first example we consider the *rotation motion* of the moving platform about z_0 horizontal axis with variable angular acceleration while all the other positional parameters are held equal to zero (Fig. 3), (Fig. 4), (Fig. 5), (Fig. 6), (Fig. 7), (Fig. 8).

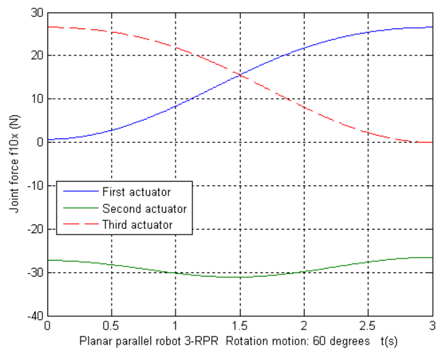

Fig. 3 Joint forces f_{10x}^{i} from three legs

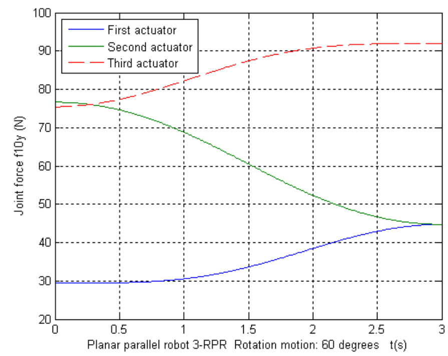

Fig. 4 Joint forces f_{10y}^{i} from three legs

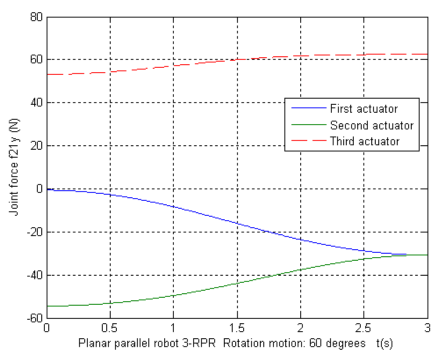

Fig. 5 Joint forces f_{21y}^{i} from three legs

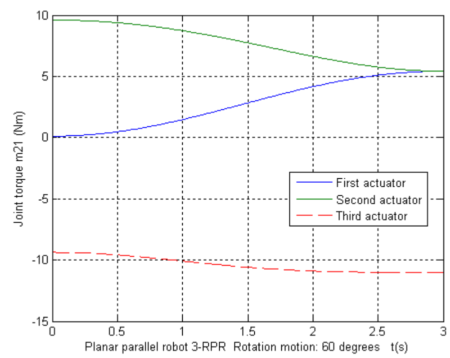

Fig. 6 Joint torques m_{21}^{i} from three legs

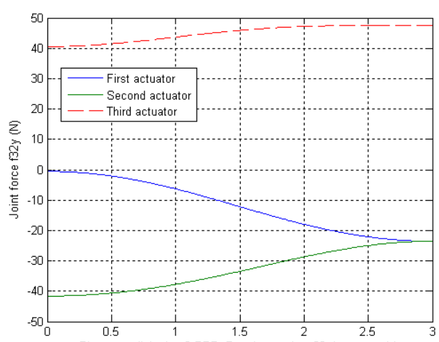

Fig. 7 Joint forces f_{32y}^{i} from three legs

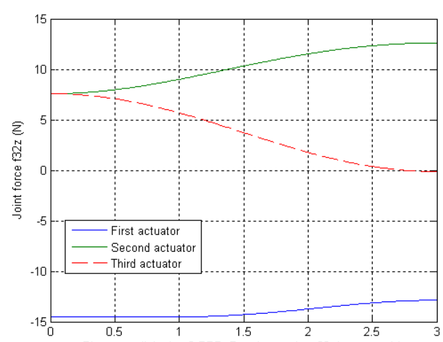

Fig. 8 Joint forces f_{32z}^{i} from three legs

If the platform's centre G moves along a *rectilinear planar trajectory* without rotation of the platform, the joint forces are calculated by the program and plotted versus time as follows: Fig. 9, Fig. 10, Fig. 11, Fig. 12, Fig. 13 and Fig. 14.

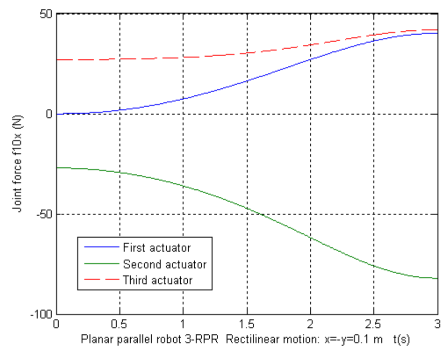

Fig. 9 Joint forces f_{10x}^{i} from three legs

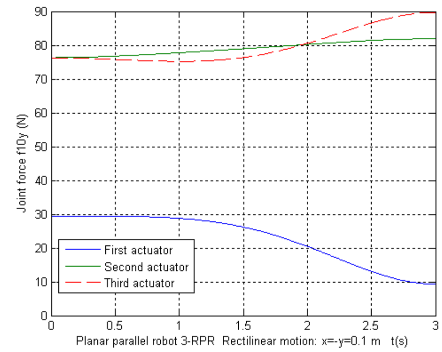

Fig. 10 Joint forces f_{10y}^{i} from three legs

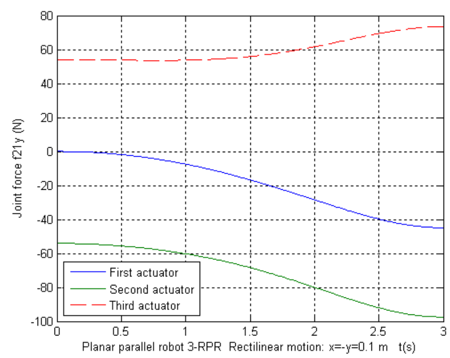

Fig. 11 Joint forces f_{21y}^{i} from three legs

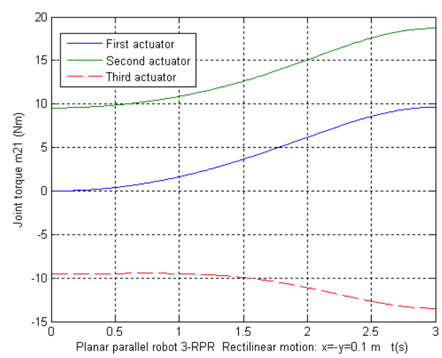

Fig. 12 Joint torques m_{21}^{i} from three legs

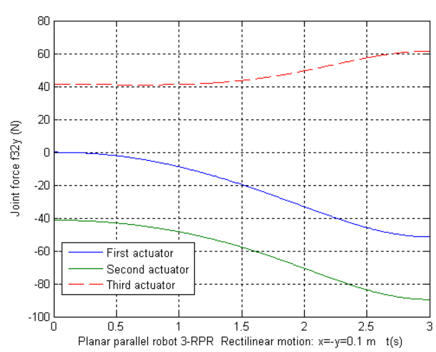

Fig. 13 Joint forces f_{32y}^{i} from three legs

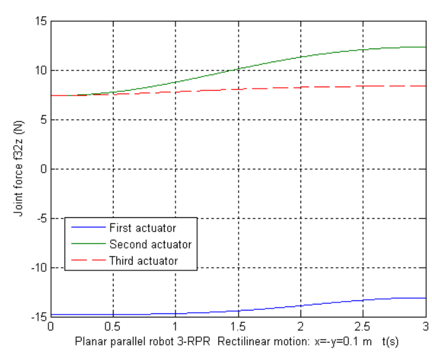

Fig. 14 Joint forces f_{32z}^{i} from three legs

4. Conclusions

The present dynamics model takes into consideration the mass, the tensor of inertia and the action of weight and inertia force introduced by all compounding elements of the parallel robot. Based on the principle of virtual work, this approach establishes a direct determination of the time-history evolution for the internal forces or torques in joints.

Choosing appropriate serial kinematical circuits connecting many moving platforms, the present method can easily be applied in forward and inverse mechanics of various types of parallel mechanisms, complex manipulators of higher degrees of freedom and particularly *hybrid structures*, when the number of components of the mechanisms is increased.

R E F E R E N C E S

[1] *Tsai, L-W.*, Robot analysis: the mechanics of serial and parallel manipulator, Wiley, 1999
[2] *Stewart, D.*, A Platform with Six Degrees of Freedom, Proc. Inst. Mech. Eng., 1, 15, 180, pp. 371-378, 1965
[3] *Merlet, J-P.*, Parallel robots, Kluwer Academic, 2000
[4] *Clavel, R.*, Delta: a fast robot with parallel geometry, Proceedings of 18th International Symposium on Industrial Robots, Lausanne, pp. 91-100, 1988
[5] *Tsai, L-W., Stamper, R.,* A parallel manipulator with only translational degrees of freedom, ASME Design Engineering Technical Conferences, Irvine, CA, 1996
[6] *Hervé, J-M., Sparacino, F.*, Star. A New Concept in Robotics, Proceedings of the Third International Workshop on Advances in Robot Kinematics, Ferrara, pp.176-183, 1992
[7] *Bonev, I., Zlatanov, D.,Gosselin, C.*, Singularity analysis of 3-DOF planar parallel mechanisms via screw theory, Journal of Mechanical Design, 25, 3, pp. 573-581, 2003
[8] *Pennock, G.R., Kassner, D.J.*, Kinematic Analysis of a Planar Eight-Bar Linkage: Application to a Platform-type Robot, ASME Mechanisms Conference, Paper DE - 25, pp. 37-43, 1990
[9] *Staicu, S.,* Power requirement comparison in the 3-RPR planar parallel robot dynamics, Mechanism and Machine Theory, Elsevier, 44, 5, pp. 1045-1057, 2009
[10] *Staicu, S.*, Modèle dynamique en robotique, UPB Scientific Bulletin, Series D: Mechanical Engineering, 61, 3-4, pp. 5-19, 1999
[11] *Staicu, S., Zhang, D.*, A novel dynamic modelling approach for parallel mechanisms analysis, Robotics and Computer-Integrated Manufacturing, Elsevier, 24, 1, pp. 167-172, 2008
[12] *Staicu, S.,* Dynamics analysis of the Star parallel manipulator, Robotics and Autonomous Systems, Elsevier, 57, 11, pp. 1057-1064, 2009
[13] *Dasgupta, B., Mruthyunjaya, T.S.,* A Newton-Euler formulation for the inverse dynamics of the Stewart platform manipulator, Mechanism and Machine Theory, Elsevier, 34, pp. 711-725, 1998
[14] *Geng, Z., Haynes, L.S., Lee, J.D., Carroll, R.L.,* On the dynamic model and kinematic analysis of a class of Stewart platforms, Robotics and Autonomous Systems, Elsevier, 40, 9, pp. 237-254, 1992
[15] *Zhang, C-D., Song, S-M.,* An Efficient Method for Inverse Dynamics of Manipulators Based on Virtual Work Principle, Journal of Robotic Systems, 10, 5, pp. 605-627, 1993
[16] *Staicu, S., Liu, X-J., Li, J.,* Explicit dynamics equations of the constrained robotic systems, Nonlinear Dynamics, Springer, 58, 1-2, pp. 217-235, 2009
[17] *Staicu, S.,* Dynamics of the 6-6 Stewart parallel manipulator, Robotics and Computer-Integrated Manufacturing, Elsevier, 27, 1, pp. 212-220, 2011