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DYNAMIC MODELING OF A 6-DOF PARALLEL MANIPULATOR

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The purpose of this paper is the kinematic and dynamic modeling of a mobile platform with six degrees of freedom. In this sense, both the direct kinematics and the indirect kinematics of the Stewart platform PS-6TL-1500 are presented. The problem of dynamic modeling for the parallel manipulator is approached considering the entire Stewart platform as a complete sistem, the methodology employed involves utilizing the Lagrange formalism.

This paper considers the validation of the dynamic model through two simulation cases that consider the attitude angles and the position of the Stewart platform. Based on the obtained results, the impact of certain parameters on the system can be observed, indicating which configurations can be considered for specific operational orientations.

Keywords: mobile platform, inverse kinematics, Stewart platform, dynamic modeling.

1. Introduction

A general Gough-Stewart platform is a parallel manipulator with six prismatic actuators, typically hydraulic jacks or linear electric actuators, which are attached in pairs of three positions on the platform base plate over three points mounted on a superior plate. The direct kinematics problem involves determining the position and orientation of the moving platform relative to the base, given the length of the legs and the coordinates of the attachment points in its local reference frame [1].

The Stewart platform mechanism, mainly called a hexapod, is a parallel kinematic structure that can be used as a basis for various controlled movements, such as manufacturing processes, or for testing various biological theories related to insect locomotion or neurobiology. Also, the desired position and orientation of the mobile platform is determined by combining the lengths of the six actuators, transforming the six transitional degrees of freedom into three translational and three rotational degrees of freedom [2].

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The main objectives of this paper include developing a kinematics model of a Stewart platform PS-6TL-1500 with 6-DOF, which was described by analyzing each point of the upper and lower plate. The dynamic modeling of the Stewart platform as a complete system is highlighted using the Lagrange formalism taking into consideration the angular velocity of the mobile platform relative to the base reference frame.

This paper aims to contribute to the field of understanding and controlling the Stewart platform PS-6TL-1500 by presenting a comprehensive model, that provides a more accurate and insightful perception of this platform, thereby enhancing its control and performance. Its originality lies in its ability to provide a holistic framework that integrates fundamental components of platform dynamics, leading to the establishment of stability and performance configurations for the Stewart platform.

2. Kinematics analysis of the Stewart platform

The Stewart platform contains parallel actuators with six degrees of freedom, linear motions consist of longitudinal, lateral, and vertical motion, and angular motions are expressed as Eulerian angular rotations known as roll, pitch, and yaw. Inverse kinematics determines the lengths of the actuators based on the position and orientation of the Stewart platform. The inverse kinematics model is developed based on simplified models, as found in works [3], [4], [5], [6]. Also, inverse kinematics deals with the mathematical problem of describing the position and orientation of the platform in terms of actuator variables. As seen in the figures below, two coordinate frames $\{P\}$ and $\{B\}$ are defined, assigned to the payload and base platforms respectively.

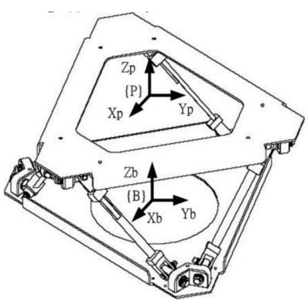

Fig. 1 Stewart platform coordinate frames [3]

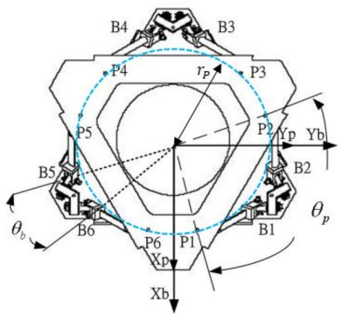

Fig. 2 Positioning points on the Stewart platform [3]

The origin of the frame $\{P\}$ represents the centroid of the upper platform, the y_P axis is directed outward, the x_P axis is perpendicular to the line connecting the two attachment points P_1 and P_6, and the angle between the actuators P_1 and

P_2 is denoted by θ_P. Regarding the base platform, the origin of frame $\{B\}$ represents the centroid of the lower platform, the angle between actuators B_5 and B_6 is denoted by θ_B. The Cartesian variables represent the position and relative orientation of the frame $\{P\}$ in relation to the frame $\{B\}$, assuming a vector describing the position of each reference point P_i in relation to the coordinate system $\{P\}$, it is expressed

$$\{P_i\} = \begin{Bmatrix} P_{i_x} \\ P_{i_y} \\ P_{i_z} \end{Bmatrix} = \begin{Bmatrix} r_P \cos\lambda_i \\ r_P \sin\lambda_i \\ 0 \end{Bmatrix}, i = \overline{1,6} \tag{1}$$

where r_P represents the radius of the top plate.

For P_1, P_3, P_5 points, the value of the angle $\lambda_i = i\dfrac{\pi}{3} - \dfrac{\theta_P}{2}$, and for P_2, P_4, P_6 points, $\lambda_i = \lambda_{i-1} + \theta_P$, where $\theta_P = \pi/2$. Assuming another vector B_i defined in Fig.2, it describes the position of an attachment point relative to the frame $\{B\}$, where B_i can be defined

$$\{B_i\} = \begin{Bmatrix} B_{i_x} \\ B_{i_y} \\ B_{i_z} \end{Bmatrix} = \begin{Bmatrix} r_B \cos\Lambda_i \\ r_B \sin\Lambda_i \\ 0 \end{Bmatrix}, i = \overline{1,6} \tag{2}$$

where r_B represents the radius of the bottom plate.

For B_1, B_3, B_5 points, the value of the angle $\Lambda_i = i\dfrac{\pi}{3} - \dfrac{\theta_B}{2}$ and for B_2, B_4, B_6 points, $\Lambda_i = \Lambda_{i-1} + \theta_B$, where $\theta_B = \pi/6$.

The rotation matrix $[R]$ represents the orientation of the frame $\{P\}$ relative to the frame $\{B\}$ [7]

$$[R]_P^B = \begin{bmatrix} \cos\psi_P \cos\theta_P & \cos\psi_P \sin\theta_P \sin\varphi_P - \sin\psi_P \cos\varphi_P & \cos\psi_P \sin\theta_P \cos\varphi_P + \sin\psi_P \sin\varphi_P \\ \sin\psi_P \cos\theta_P & \sin\psi_P \sin\theta_P \sin\varphi_P + \cos\psi_P \cos\varphi_P & \sin\psi_P \sin\theta_P \cos\varphi_P - \cos\psi_P \sin\varphi_P \\ -\sin\theta_P & \cos\theta_P \sin\varphi_P & \cos\theta_P \cos\varphi_P \end{bmatrix} \tag{3}$$

where φ_P, θ_P and ψ_P represent the rotation angles of the upper platform, corresponding to roll, pitch and yaw.

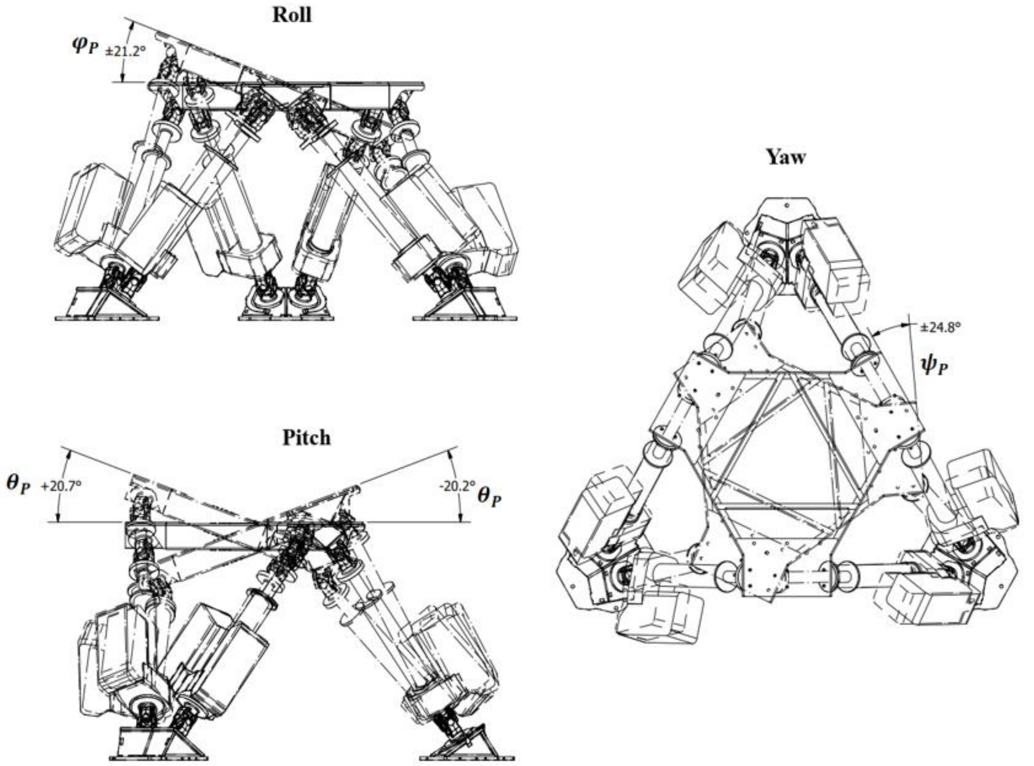

Fig. 3 Rotation angles of the upper platform of PS-6TL-1500

The rotations depicted in Fig.3 occur consecutively around the displacement coordinate axes, resulting in the final rotation matrix given by relation (3). For this analysis, the yaw-pitch-roll rotation matrix was considered, with the first rotation of ψ_P around z-axis, followed by the rotation of θ_P around y-axis and finally the rotation of φ_P around x-axis. This sequence of rotations can be represented as the product of three established rotation matrices, respectively $\left[R\right]_P^B = \left[R_{z_{\psi P}}\right]\left[R_{y_{\theta P}}\right]\left[R_{x_{\varphi P}}\right]$, where

$$\left[R_{z_{\psi P}}\right] = \begin{bmatrix} \cos\psi_P & -\sin\psi_P & 0 \\ \sin\psi_P & \cos\psi_P & 0 \\ 0 & 0 & 1 \end{bmatrix}; \left[R_{y_{\theta P}}\right] = \begin{bmatrix} \cos\theta_P & 0 & \sin\theta_P \\ 0 & 1 & 0 \\ -\sin\theta_P & 0 & \cos\theta_P \end{bmatrix}; \left[R_{x_{\varphi P}}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_P & -\sin\varphi_P \\ 0 & \sin\varphi_P & \cos\varphi_P \end{bmatrix}$$

As seen in Fig.3, each angle is constrained according to the physical limitations, therefore $\varphi_P \in \left[-21.2^o; 21.2^o\right]$, $\theta_P \in \left[-20.2^o; 20.7^o\right]$ and $\psi_P \in \left[-24.8^o; 24.8^o\right]$.

By adopting appropriate coordinate transformations, the actuation vector L_i corresponding to each actuator can be derived

$$\left\{L_i\right\} = \left[R\right]_P^B \left\{P_i\right\} + \left\{P\right\} - \left\{B_i\right\}, i = \overline{1,6} \tag{4}$$

where the vector $\left\{P\right\} = \left\{x \quad y \quad z\right\}^T$ represents the position of the coordinate system $\left\{P\right\}$, as observed in Fig.1.

Given that the length of the actuator is $l_i = \left|L_i\right|$, an inverse kinematics solution is obtained

$$l_i = \left|L_i\right| = \sqrt{(r_{11}P_{i_x} + r_{12}P_{i_y} + x - B_{i_x})^2 + (r_{21}P_{i_x} + r_{22}P_{i_y} + y - B_{i_y})^2 + (r_{31}P_{i_x} + r_{32}P_{i_y} + z)^2} \tag{5}$$

By performing mathematical calculations, the simplified equivalent form of the inverse kinematics solution of equation (5) has been obtained. This represents the link between the desired final state coordinates and the specific kinematic parameters of the parallel manipulator, obtaining

$$\begin{gathered} l_i^2 = x^2 + y^2 + z^2 + r_P^2 + r_B^2 + 2\left(r_{11}P_{i_x} + r_{12}P_{i_y}\right)\left(x - B_{i_x}\right) + 2\left(r_{21}P_{i_x} + r_{22}P_{i_y}\right)\left(y - B_{i_y}\right) \\ +2\left(r_{31}P_{i_x} + r_{32}P_{i_y}\right)z - 2\left(xB_{i_x} + yB_{i_y}\right) \end{gathered} \tag{6}$$

The direct kinematics of the upper platform of the six-degree-of-freedom parallel manipulator plays an important role in controlling or visualizing the motion of the platform, but it is difficult to define due to the nonlinearity and complexity of the platform. A popular method for solving the derivative problem is the Newton Raphson method, but it suffers from repetitive steps before the solution converges and therefore cannot become a real-time solution. Also, by imposing wrong values of the initial conditions this method can lead to an infinite loop in the solution.

3. Dynamic analysis of the Stewart platform

The parallel mechanism is composed of an upper (movable) platform and a lower (fixed) platform and six identical branches, each actuator in turn consisting of a cylinder and piston.

The dynamic analysis of the parallel manipulator is much more difficult compared to that of the serial manipulator due to the existence of multiple kinematic chains, all connected by the mobile platform [8], [9], [10]. In this chapter the Lagrange formulation will be used, because it offers a much better structure to describe the dynamics of the manipulator. Regarding the derivation of the dynamic equations for the Stewart platform, the approach considers the entire mobile platform. It involves calculating both kinetic and potential energies, and subsequently deriving the dynamic equations based on these energy considerations.

The figure below shows the spatial direction of the Stewart platform, with the lengths of each actuator being variable

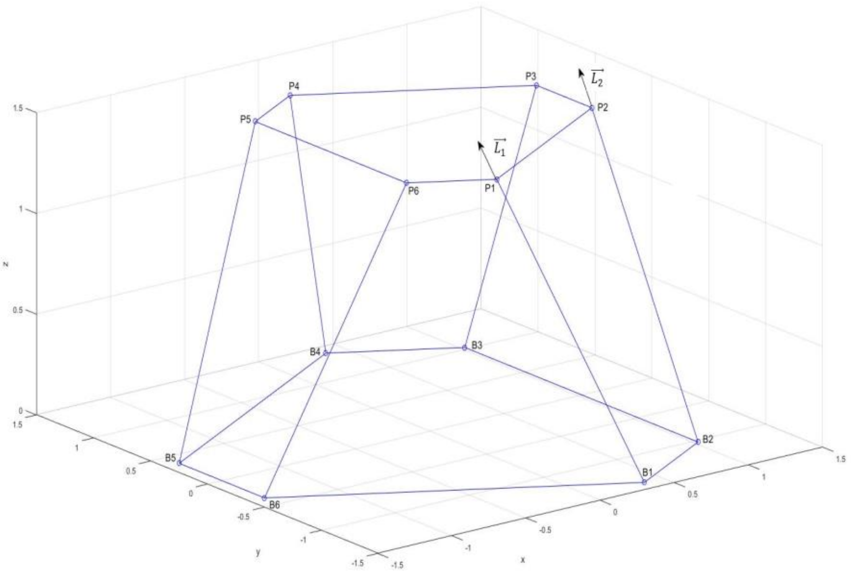

Fig. 4 The structure and geometry of the Stewart platform

The dynamic analysis of the Stewart platform can be performed by applying the Lagrange equations [5], [11], [12]

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right)-\frac{\partial E}{\partial q}+\frac{\partial U}{\partial q}=Q \tag{7}$$

In this case q represent the generalized coordinates, respectively $q=\left[x \quad y \quad z \quad \varphi \quad \theta \quad \psi\right]^T$, x, y and z are positional displacement of the upper platform, φ, θ and ψ are the angular displacement of the upper platform , Q are the generalized forces, E and U represent the kinetic energy and the potential energy of the upper platform, respectively.

Knowing that the kinetic energy of the upper platform is composed of translational and rotational kinetic energy, the two formulations will be explained below. The translational kinetic energy arising due to the translational movement of the center of mass is defined

$$E_t = \frac{1}{2} m_p \left(\dot{P}_x^2 + \dot{P}_y^2 + \dot{P}_z^2 \right) \tag{8}$$

where m_p is the mass of the upper platform, $\dot{P}_x^2, \dot{P}_y^2, \dot{P}_z^2$ are the velocities on the three axes of the center of mass.

In terms of the rotational motion of the mobile platform about its center of mass, the rotational kinetic energy can be written:

$$E_r = \frac{1}{2} \omega_p^T I_p \omega_p \tag{9}$$

where I_p and ω_p are the rotational inertial mass and the angular velocity of the mobile platform.

The parameters I_p and ω_p are described as follows

$$I_p = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \tag{10}$$

$$\left\{\omega_p\right\} = \left[R_{z,\psi}\right]^T \left[R_{x,\varphi}\right]^T \left[R_{y,\theta}\right]^T \left\{\omega_{p_1}\right\} \tag{11}$$

where ω_{p_1} represents the angular velocity of the mobile platform relative to the base reference frame.

Given the definition of Euler angles, ω_{p_1} can be written

$$\omega_{p_1} = \dot{\varphi}\left[R_{y,\theta}\right] X + \dot{\theta} Y + \dot{\psi}\left[R_{x,\varphi}\right]\left[R_{z,\psi}\right] Z \tag{12}$$

$$\{\omega_{p_1}\} = \left(\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{Bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} \tag{13}$$

$$\{\omega_{p_1}\} = \begin{bmatrix} \cos\theta & 0 & 0 \\ 0 & 1 & -\sin\varphi \\ -\sin\theta & 0 & \cos\varphi \end{bmatrix} \begin{bmatrix} \dot{\varphi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T \tag{14}$$

Substituting the relation thus obtained into (12)

$$\{\omega_p\} = \begin{bmatrix} c\psi & c\varphi s\psi & -c\varphi c\psi s\theta - c\varphi s\varphi s\psi + c\varphi c\theta s\varphi s\psi \\ -s\psi & c\varphi c\psi & -c\varphi c\psi s\varphi + c\varphi s\theta s\psi + c\varphi c\theta s\varphi c\psi \\ 0 & -s\varphi & s^2\varphi + c^2\varphi c\theta \end{bmatrix} \begin{bmatrix} \dot{\varphi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T \tag{15}$$

where the notations were made $s(\bullet) = \sin(\bullet), c(\bullet) = \cos(\bullet)$.

The total kinetic energy of the upper platform written in compact form is

$$E = E_t + E_r = \frac{1}{2} \begin{bmatrix} \dot{P}_x & \dot{P}_y & \dot{P}_z & \dot{\varphi} & \dot{\theta} & \dot{\psi} \end{bmatrix} [M] \begin{bmatrix} \dot{P}_x & \dot{P}_y & \dot{P}_z & \dot{\varphi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T \tag{16}$$

where M is a 6x6 diagonal matrix of the upper platform.

The mass matrix can be written

$$[M] = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{44} & M_{45} & M_{46} \\ 0 & 0 & 0 & M_{54} & M_{55} & M_{56} \\ 0 & 0 & 0 & M_{64} & M_{65} & M_{66} \end{bmatrix} \tag{17}$$

where

$$M_{44} = I_x \cos^2\beta \cos^2\gamma + I_y \cos^2\beta \sin^2\gamma + I_z \sin^2\beta$$

$$M_{45} = M_{54} = \left(I_x - I_y\right) \cos\beta \cos\gamma \sin\gamma$$

$$M_{46} = M_{64} = I_z \sin\beta$$

$$M_{55} = I_x \sin^2\gamma + I_y \cos^2\gamma$$

$$M_{66} = I_z$$

Also, the potential energy of the upper platform is

$$U = \begin{bmatrix} 0 & 0 & m_p g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_x & P_y & P_z & \varphi & \theta & \psi \end{bmatrix}^T = m_p g P_z \qquad (18)$$

and g represents the gravitational acceleration.

Formulating the equation using redundant coordinates (the equations use more coordinates than the degrees of freedom of the underlying system) of the mechanism's kinematics is

$$M(X)\ddot{X} + C\left(X, \dot{X}\right)\dot{X} + G(X) = J^T(X)\tau \qquad (19)$$

where $M(X)$ is the mass matrix, $C\left(X, \dot{X}\right)$ is the Coriolis and centrifugal force term, $G(X)$ is the gravitational force, J is the Jacobian matrix, J^T is the transposed Jacobian matrix, τ represents torque.

The dynamic model of the six-degree-of-freedom parallel manipulator can be described using the inverse transposed Jacobian matrix, J^{-T}, thus

$$\dot{X} = J^{-T}\dot{q} \qquad (20)$$

Given that q represent the generalized coordinates, respectively $[x \quad y \quad z \quad \varphi \quad \theta \quad \psi]^T$ and $\dot{q}$ represents their derivative, $\begin{bmatrix} \dot{x} & \dot{y} & \dot{z} & \dot{\varphi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T$, the Lagrange equation can be expressed

$$M(q)\ddot{q} + C\left(q, \dot{q}\right)\dot{q} + G(q) = \tau \qquad (21)$$

where

$M(q) = J^T M(X) J$ is symmetric and positive definite for any $q \in \mathbb{R}^n$

$C\left(q, \dot{q}\right) = J^T\left[C(X) - M(X) J \dot{J}\right] J$

$G(q) = J^T G(X)$

Given that $\dot{M} - 2C$ can represents a skew symmetric matrix, it will result that $x^T\left(\dot{M} - 2C\right)x = 0, \forall x \in \mathbb{R}^n$, this relation being fundamental in the use of adaptive or robust control for classical robots with serial and parallel linkage.

The matrix of Coriolis and centrifugal forces can be written

$$C = \begin{bmatrix} 0 & 0 \\ 0 & C_{22} \end{bmatrix} \tag{22}$$

where

$$C_{22} = \begin{bmatrix} -C_1\dot{\beta} - C_2\dot{\gamma} & -C_1\dot{\alpha} - C_3\dot{\beta} + C_4\dot{\gamma} & -C_2\dot{\alpha} + C_4\dot{\beta} \\ C_1\dot{\alpha} + C_4\dot{\gamma} & C_5\dot{\gamma} & C_4\dot{\alpha} + C_5\dot{\beta} \\ C_2\dot{\alpha} - C_4\dot{\beta} & -C_4\dot{\alpha} - C_5\dot{\beta} & 0 \end{bmatrix} \tag{23}$$

$$\begin{aligned} C_1 &= \cos\beta\sin\beta(\cos^2\gamma I_x + \sin^2\gamma I_y - I_z) \\ C_2 &= \cos^2\beta\cos\gamma\sin\gamma(I_x - I_y) \\ C_3 &= \cos\gamma\sin^2\gamma(I_x - I_y) \\ C_4 &= \frac{1}{2}\cos\beta(\cos^2\gamma - \sin^2\gamma)(I_x - I_y) \\ C_5 &= \cos\gamma\sin\gamma(I_x - I_y) \end{aligned} \tag{24}$$

The Jacobian matrix is defined according to [9]

$$\begin{aligned} J(i,1) &= \left(x + P_{i_x}r_{11} + P_{i_y}r_{12} - B_{i_x}\right)/l_i \\ J(i,2) &= \left(y + P_{i_x}r_{21} + P_{i_y}r_{22} - B_{iy}\right)/l_i \\ J(i,3) &= \left(z + P_{i_x}r_{31} + P_{i_y}r_{32}\right)/l_i \\ J(i,5) &= \left(\left(-P_{i_x}s\theta c\psi + P_{i_y}r_{32}c\psi\right)\left(x - B_{i_x}\right) + \left(-P_{i_x}s\theta s\psi + P_{i_y}r_{23}s\psi\right)\left(y - B_{i_y}\right) + \left(-P_{i_x}c\theta - P_{i_y}s\varphi s\theta\right)z\right)/l_i \\ J(i,6) &= \left(-\left(P_{i_x}r_{21} + P_{i_y}r_{22}\right)\left(x - B_{i_x}\right) + \left(P_{i_x}r_{11} + P_{i_y}r_{12}\right)\left(y - B_{i_y}\right)\right)/l_i \end{aligned} \tag{25}$$

where $i = \overline{1,6}, r_{11}, r_{12}, r_{21}, r_{22}, r_{31}, r_{32}, r_{13}, r_{23}$ are the coefficients of the rotation matrix and l_i represents the length of the actuator. Using the obtained relations (25) the force of the actuators can be determined by replacing them in the relation (21).

4. Results

In this section, several dynamic simulations were carried out to highlight the inertial effect of the actuators and their component parts on the dynamics of the entire system. The simulation parameters of the PS-6TL-1500 platform are as follows: $r_P = 0.94m; r_B = 1.449m; \theta_P = 100^o; \theta_B = 31^o; m_p = 3kg; \omega = 3rad/s$

$$I_p = \begin{bmatrix} 0.085 & 0 & 0 \\ 0 & 0.085 & 0 \\ 0 & 0 & 0.085 \end{bmatrix} kgm^2;\ I_b = \begin{bmatrix} 0.006 & 0 & 0 \\ 0 & 0.006 & 0 \\ 0 & 0 & 0.006 \end{bmatrix} kgm^2;\ I_t = \begin{bmatrix} 0.006 & 0 & 0 \\ 0 & 0.006 & 0 \\ 0 & 0 & 0 \end{bmatrix} kgm^2$$

where m_p is the mass of platform including the payload, I_p is inertia matrix with respect to the control point of platform including the payload, I_b is inertia matrix with respect to the center of gravity of the lower rotating part of the ***i***th actuator and I_t is inertia matrix with respect to the center of gravity of the upper moving part of the ***i***th actuator. The positioning of the points associated with both the mobile platform and the base are as follows

$$P_1 = [0.9257 \quad 0.1632 \quad 1.6000]m \qquad B_1 = [1.0335 \quad 1.0156 \quad 0]m$$
$$P_2 = [-0.3215 \quad 0.8833 \quad 1.6000]m \qquad B_2 = [0.3628 \quad 1.4028 \quad 0]m$$
$$P_3 = [-0.6042 \quad 0.7201 \quad 1.6000]m \qquad B_3 = [-1.3963 \quad 0.3872 \quad 0]m$$
$$P_4 = [-0.6042 \quad -0.7201 \quad 1.6000]m \qquad B_4 = [-1.3963 \quad -0.3872 \quad 0]m$$
$$P_5 = [-0.3215 \quad -0.8833 \quad 1.6000]m \qquad B_5 = [0.3628 \quad -1.4028 \quad 0]m$$
$$P_6 = [0.9257 \quad 0.1632 \quad 1.6000]m \qquad B_6 = [1.0335 \quad -1.0156 \quad 0]m$$

First simulation case represents a sinusoidal trajectory imposed along the x-axis, while the orientation of the moving platform is kept constant.

$$[\varphi \quad \theta \quad \psi]^T = [0 \quad 0 \quad 0]^T;\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.3\sin(\omega t) \\ 0.2 \\ 0.3 \end{bmatrix}$$

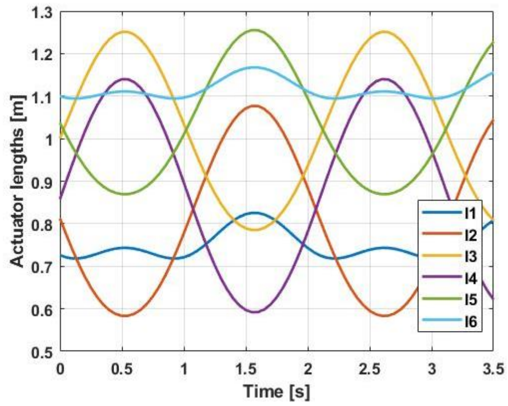

Fig. 5 Actuator lengths as a function of time case 1

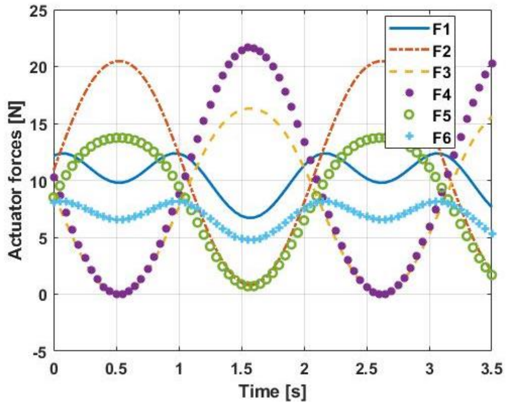

Fig. 6 Actuator forces as a function of time case 1

The second simulation case represents a sinusoidal trajectory, while the orientation of the moving platform is variable only as a function of the angle ψ.

$$[\varphi \quad \theta \quad \psi]^T = [0 \quad 0 \quad 20]^T; \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -0.3 + 0.25\sin(\omega t) \\ -0.4\sin(\omega t) \\ 0.7 + 0.5\sin(\omega t) \end{bmatrix}$$

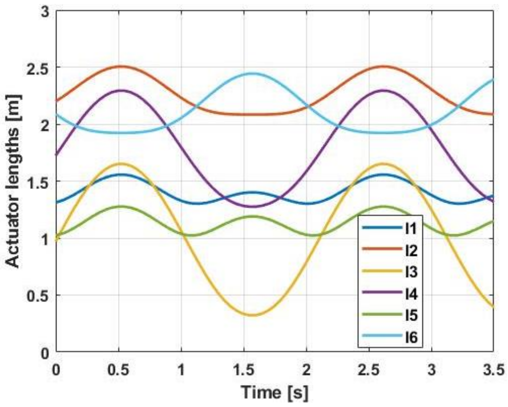

Fig. 7 Actuator lengths as a function of time case 2

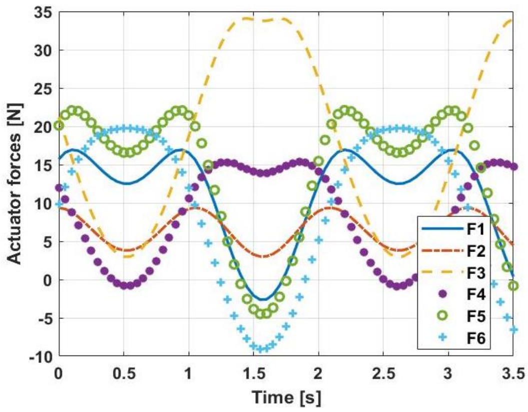

Fig. 8 Actuator forces as a function of time case 2

Sensitivity analysis is a methodology employed to examine how alterations in input parameters or variables influence the outcome or effectiveness of a system. The sensitivity analysis procedure involves identifying the essential parameters for the performance of the Stewart platform, these being the parameters expected to exert the most significant influence on the system. Subsequently, a systematic modification of certain parameters is considered, while keeping others constant, in this case, three simulations were considered.

In the first phase, the input parameters of the Stewart platform were kept the same as in the second case, followed by a modification of the θ angle and in the last simulation, the φ angle was modified. In the figures below, variations in forces have been identified based on the changes in input parameters.

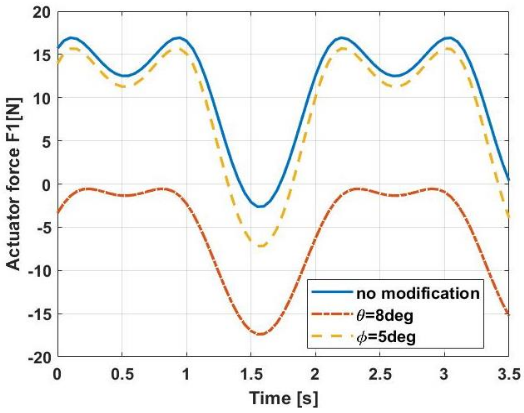

Fig. 9 Sensitivity analysis for F1

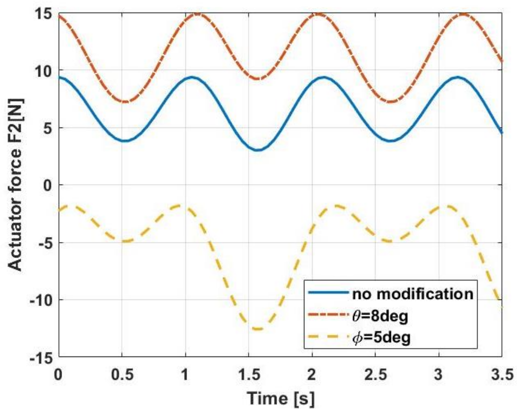

Fig. 10 Sensitivity analysis for F2

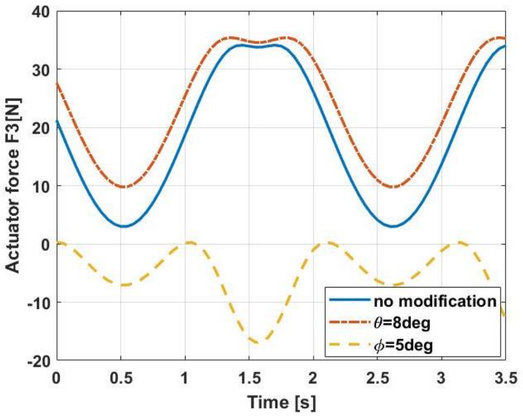

Fig. 11 Sensitivity analysis for F3

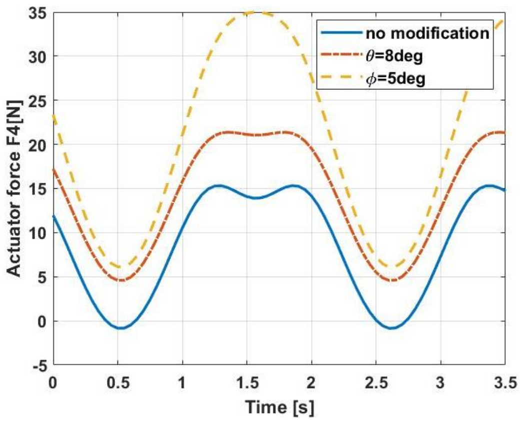

Fig. 12 Sensitivity analysis for F4

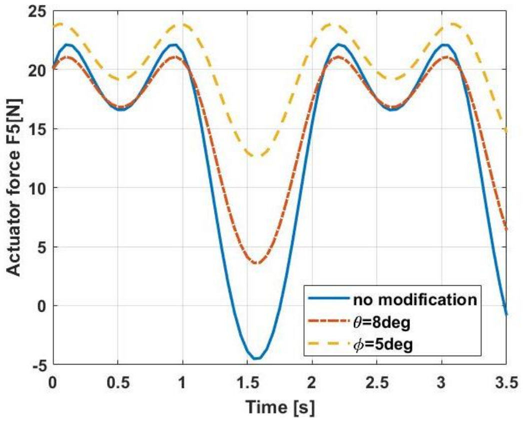

Fig. 13 Sensitivity analysis for F5

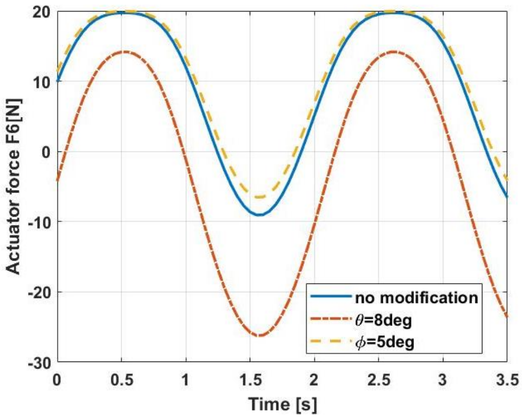

Fig. 14 Sensitivity analysis for F6

As observed in Fig.9 and Fig.14, the change in the pitch angle has a more pronounced impact on the two actuators, resulting in a force variation of 10N and 15N, respectively, compared to the other two simulation cases.

Alterations in the roll angle result in reduced actuator forces, as evidenced in Fig.10 and Fig.11, reaching values of -15N. This adjustment helps prevent configurations that impose an undue burden on the actuators.

As observed in Fig.12 and Fig.13, the actuator forces follow a consistent pattern of variation over the simulation time, reaching either maximum values (Fig.12) or minimum values (Fig.13) after 1.5 seconds, regardless of the initial parameter modifications. The purpose of the actuator force sensitivity analysis is to highlight the dynamic capabilities of the PS-6TL-1500 Stewart platform and to evaluate the stability and performance of the platform. This is achieved by identifying some specific combinations of pitch, roll, and yaw angles that can result in more stable platform configurations or enhanced performance in terms of load capacity, accuracy, or speed.

5. Conclusions

In the present paper, the system of closed-form dynamic equations of a parallel manipulator using the Lagrangian formalism has been presented, this approach used a configuration of Stewart platform with six degrees of freedom, as can be observed in others specialized papers [13], [14], [15], [16].

The algorithm was implemented using the Matlab simulation environment, and the numerical results were studied to validate the dynamic formulation presented in the chapter.

The results of the simulations demonstrate that the derivation of the explicit dynamic equations in the load space is available for a manipulator with six degrees of freedom, thus obtaining the actuation forces on the mobile platform actuators. The previously presented case studies define an imposed movement of the mobile platform, on a single axis, x, y, z, or according to the attitude angles, φ, θ, ψ, following the variation of the actuation forces on the actuators and their displacement during the simulation. Also, in the simulations, the stiffness of the actuators caused by the movement of the platform was neglected, their joints being considered ideal, having no frictional forces. The method described in the dynamic analysis is used in both direct and inverse mechanics of serial and parallel mechanisms in the context of autonomous control, which can be used in a robust model for the computer control part of the Stewart manipulator.

As future development, the dynamic model obtained in this paper will be verified and validated during experimental tests using the Stewart platform PS-6TL-1500 and a sensor system of GPS (Global Positioning System) and IMU (Inertial Measurement Unit). This experimental analysis will be carried out in the

SpaceSysLab Maneciu Laboratory of the National Institute for Aerospace Research “Elie Carafoli”.

Fig. 15 INCAS – Stewart Platform PS-6TL-1500

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