THE ANALYSIS OF A REACTIVE HYDROMAGNETIC FLUID FLOW IN A CHANNEL THROUGH A POROUS MEDIUM WITH CONVECTIVE COOLING

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This paper investigates the analysis of a reactive hydromagnetic fluid flowing between two parallel plates through a porous medium with convective boundary conditions. Neglecting the consumption of the material which is exothermic under Arrhenius kinetics; it is assumed that the flow system exchanges heat with the ambient following Newton's law of cooling. Approximate solutions of the nonlinear dimensionless equations governing the fluid flow are obtained using the traditional perturbation method and Adomian decomposition method (ADM). Also, the diagonal Pade approximation technique is used to determine the thermal criticality values as well as bifurcation conditions. The entropy generation analysis and effects of all – important flow properties on the fluid flow are also presented and discussed.

Keywords: Reactive fluids, porous medium, thermal criticality, entropy generation, convective cooling, Adomian decomposition method (ADM), Pade approximation technique and Arrhenius kinetics.

1. Introduction

Over the past few decades, studies relating to analysis of a reactive hydromagnetic fluid flow are on the increase due to its immense applications in many engineering and industrial processes as described in [1] – [6] such as, petroleum industries, chemical engineering, etc. In a reacting material undergoing an exothermic reaction in which reactant consumption is neglected, heat is being produced in accordance with Arrhenius rate law and Newtonian cooling where convection forms an integral part of heat transfer due to differences in ambient temperatures. The process of convection not only affects heat transfer, but also helps maintain comfort conditions. In addition to that, [7] mentioned that thermal explosions occur when the reactions produce heat too rapidly for a stable balance between heat production and heat loss to be preserved.

Moreover, studies involving the fluid properties in a channel through a porous medium have been investigated in [8] – [11], just to mention few. Also, studies in [12] – [16] examined fluid flowing between walls with convective cooling effects because of its importance in technological applications, for

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example, the cooling processes of nuclear reactors and refrigerators where investigations were done on convective boundary conditions of the flow system.

However, as discussed in [11], it is needed to find out the property of porous medium which measures the capacity and ability of the formation to transmit fluids. Hence, in the present study, the analysis of [6] and [17] are extended to include and investigate the effects of fluid flow through a porous medium and symmetrical convective cooling on the overall flow structure in a reactive hydromagnetic fluid between two parallel porous plates which was not accounted for in the previously obtained results.

This present study has significant benefits in engineering and industrial processes where there is an inherent simplicity for the applications just requiring some provision for natural heat flow to the ambient which is often achieve by adequate venting on the system of flow rather than forced convection. In order to obtain approximate solutions for the nonlinear dimensionless equations governing the fluid flow, traditional perturbation method shall be used to determine the temperature profile. Also, entropy generation analysis shall be investigated while Adomian decomposition method (ADM) together with the diagonal Pade approximation technique shall be used to determine the thermal criticality values as well as bifurcation conditions of the fluid flow system.

In the rest of this paper, the problem is formulated in section 2. The governing equations are solved using traditional perturbation method in section 3.The entropy generation analysis were derived and the thermal criticality conditions were determined using ADM and diagonal Pade approximation technique in section 4. Presentations of analytical results of the problem are shown in tables and graphs in section 5; while section 6 gives the concluding remarks.

2. Mathematical Formulation

Let us consider the steady flow of an incompressible reactive fluid through a channel made up of two parallel porous plates distant 2a apart and the fluid is subjected to convective cooling at the boundaries. The fluid is electrically conducted under the influence of a transversely applied magnetic field, B_0. The *x*- and *y*-axes are chosen parallel and perpendicular to the plates respectively as shown in Fig. 1.

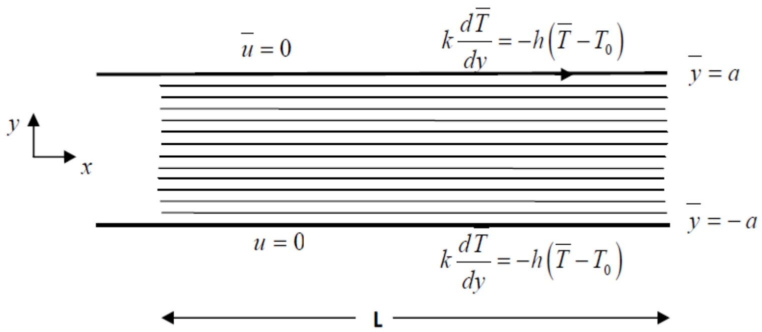

Figure 1: Geometry of the problem

Neglecting the consumption of the reactant, the differential equations governing the fluid flow in non – dimensionless form as in [6] and [17] may be written as:

$$\frac{dP}{d\bar{x}}=\mu\frac{d^2\bar{u}}{d\bar{y}^2}-\sigma_0B_0{}^2\bar{u}-\frac{\mu}{K}\bar{u} \tag{1}$$

$$k\frac{d^2\bar{T}}{d\bar{y}^2}+\mu\left(\frac{d\bar{u}}{d\bar{y}}\right)^2+QC_0Ae^{-\frac{E}{R\bar{T}}}+\sigma_0B_0{}^2\bar{u}^2+\frac{\mu}{K}\bar{u}^2=0 \tag{2}$$

The flow is symmetric about the vertical x – axis. Hence the corresponding boundary conditions along the channel centreline is given as

$$\frac{d\bar{u}}{d\bar{y}}=\frac{d\bar{T}}{d\bar{y}}=0 \text{ on } \bar{y}=0 \qquad \text{and}$$

$$\bar{u}=0, k\frac{d\bar{T}}{d\bar{y}}=-h\left(\bar{T}-T_0\right) \text{ on } \bar{y}=\pm a. \tag{3}$$

In equations (1) – (3), u is the axial velocity, T is the absolute Temperature, P is the modified pressure, μ is the fluid viscosity, σ_0 is the electrical conductivity, B_0 is the magnetic field, K is the porous permeability of the medium, k is the thermal conductivity, Q is the heat of reaction term, C_o is the reactant species initial concentration, A is the reaction rate constant, E is the activation energy, R is the universal gas constant, h is the heat transfer coefficient, a is the channel half width and $\bar{x}, \bar{y}$ is the coordinate system measured in the axial and normal directions respectively. It should be noted that the last term in equations (1) and (2) are due to the influence of porosity as in [8 – 11]. Also, the first term in equation (4) is the rate of heat transfer while other terms account for viscous dissipations and magnetic effect.

Introducing the following dimensionless parameters and variables:

$$y=\frac{\bar{y}}{a},x=\frac{\bar{x}}{a},u=\frac{\bar{u}}{U},T=\frac{E\left(\bar{T}-T_0\right)}{RT_0^2},G=-\frac{a^2}{\mu U}\frac{dP}{dx},\ Br=\frac{E\mu U^2}{kRT_0^2},\delta=\frac{RT_0}{E},Bi=\frac{ah}{k},$$
$$\gamma=\frac{\mu U^2}{QAa^2C_0}e^{\frac{E}{RT_0}},\ H^2=\frac{\sigma B_0^2a^2}{\mu},\ \lambda=\frac{QEAa^2C_0}{kRT_0^2}e^{-\frac{E}{RT_0}},\ \alpha=\frac{a^2}{K}\text{ and }\Omega=\frac{RT_0}{E} \tag{4}$$

The governing boundary value problem equations (1) – (3) become the following in dimensionless form:

$$\frac{d^2u}{dy^2}+G-\left(H^2+\alpha\right)u=0. \tag{5}$$

$$\frac{d^2T}{dy^2}+\lambda\left[e^{\frac{T}{1+\delta T}}+\gamma\left(\left(\frac{du}{dy}\right)^2+\left(H^2+\alpha\right)u^2\right)\right]=0 \tag{6}$$

together with the boundary conditions

$$\frac{dT}{dy}=\frac{du}{dy}=0\text{ on }y=0\text{ and }u=0,\ \frac{dT}{dy}=-BiT\text{ on }y=1 \tag{7}$$

In equations (1) – (7), other variables and parameters like T_0 is the wall temperature, G is the pressure gradient, U is the fluid characteristic velocity, δ is the activation energy parameter, γ is the viscous heating parameter, α is the porous medium permeability parameter, Br is the Brinkman number, H is the Hartmann number, Bi is the Biot Number, λ is the Frank – Kamenettski parameter, Ω is the wall temperature parameter and Da is the Darcy number.

3. Perturbation Method

The fluid velocity equation (5) is a linear second order non-homogeneous differential equation that has exact solution with the appropriate boundary conditions as

$$u(y)=\frac{1}{H^2+\alpha}\left[\begin{array}{l}G-GCosh[y\sqrt{H^2+\alpha}]\\+\dfrac{H^2(-G+GCosh[\sqrt{H^2+\alpha}])Cosh[y\sqrt{H^2+\alpha}]Sech[\sqrt{H^2+\alpha}]}{H^2+\alpha}\\+\dfrac{\alpha(-G+GCosh[\sqrt{H^2+\alpha}])Cosh[y\sqrt{H^2+\alpha}]Sech[\sqrt{H^2+\alpha}]}{H^2+\alpha}\end{array}\right]. \tag{8}$$

Substituting (8) in (6), it will be convenient to assume a series solution in the Frank Kamenettski parameter due to the non-linear nature of (6) in this form following [18]:

$$T(y)=\sum_{n=0}^{\infty}\lambda^n T_n(y) \tag{9}$$

Where 0 < λ << 1, clearly, $e^{\frac{T}{1+\delta T}}$ can be Taylor's series expanded, using the solution series (9) in (6) and equating the orders of λ, we obtain and solve the following:

$$O(\lambda^0) \qquad \frac{d^2T_0}{dy^2}=0, \tag{10}$$

such that $T_0\,'(0)=0$, $T_0\,'(1)=-BiT_0(1)$,

$$O(\lambda^1) \qquad \frac{d^2T_1}{dy^2}+e^{\frac{T_0}{1+\delta T_0}}+\gamma\left[\left(\frac{du}{dy}\right)^2+\left(H^2+\alpha\right)u^2\right]=0, \tag{11}$$

such that $T_1\,'(0)=0, T_1\,'(1)=-BiT_1(1)$

$$O(\lambda^2) \qquad \frac{d^2T_2}{dy^2}+\frac{e^{\frac{T_0}{1+\delta T_0}}T_1}{(1+\delta T_0)^2}=0 \tag{12}$$

such that $T_2\,'(0)=0, T_2\,'(1)=-BiT_2(1)$

$$O(\lambda^3) \qquad \frac{d^2T_3}{dy^2}+\frac{e^{\frac{T_0}{1+\delta T_0}}(T_1^2-2\delta T_1^2-2\delta^2T_0T_1^2+2T_2+4\delta T_0T_2+2\delta^2T_0^2T_2)}{2(1+\delta T_0)^4}=0 \tag{13}$$

such that $T_3\,'(0)=0, T_3\,'(1)=-BiT_3(1)$ and so on.

Solving equations (10) – (13) give us the fluid temperature profile and the effects of physical aspects of the flow properties are discussed in section 5.

4.1. Entropy Generation Analysis

The total entropy change observed in a closed system is the sum of the entropy change which can be attributed to reversible heat transfer and the entropy change attributable to irreversibility. Although, it is difficult to directly measure the magnitude of irreversibility in a closed system, but can be calculated from the entropy generation equation. The entropy production is due to heat transfer and the combined effects of fluid friction and Joules dissipation. Following [3, 5, 6 and 20], the general equation for the entropy generation per unit volume in the presence of a magnetic field and porous medium is given by:

$$S^m=\frac{k}{T_0^2}\left(\frac{d\bar{T}}{d\bar{y}}\right)^2+\frac{\mu}{T_0}\left(\frac{d\bar{u}}{d\bar{y}}\right)^2+\frac{\sigma_0B_0^2\bar{u}^2}{T_0}+\frac{\mu\bar{u}^2}{KT_0} \tag{14}$$

The first term in (14) is the irreversibility due to heat transfer; the second term is the entropy generation due to viscous dissipation and the last two are the local entropy generation due to the effects of magnetic field and porosity respectively.
We express the entropy generation number in dimensionless form using the existing dimensionless variables and parameter in (4) as:

$$N^s = \frac{S^m a^2 E^2}{kR^2 T_0^2} = \left(\frac{dT}{dy}\right)^2 + \frac{Br}{\Omega}\left[\left(\frac{du}{dy}\right)^2 + (H^2+\alpha)u^2\right] \tag{15}$$

The first term, $\left(\frac{dT}{dy}\right)^2$ is assigned N_1 which is the irreversibility due to heat transfer and the second term, $\frac{Br}{\Omega}\left[\left(\frac{du}{dy}\right)^2 + (H^2+\alpha)u^2\right]$ referred to as N_2 is the entropy generation due to the combined effects of viscous dissipation, magnetic field and porosity of the flow regime where $\Omega = \frac{RT_0}{E}$ is the wall temperature parameter. We defined

$$\phi = \frac{N_2}{N_1} \tag{16}$$

as the irreversibility distribution ratio. Relation (16) shows that heat transfer dominates when $0 \le \phi < 1$ and fluid friction dominates when $\phi > 1$. This is used to determine the contribution of heat transfer in many engineering designs. As an alternative to irreversibility parameter, the Bejan number (Be) is defined as

$$Be = \frac{N_1}{N_s} = \frac{1}{1+\phi} \quad where \quad 0 \le Be \le 1. \tag{17}$$

4.2. Thermal Criticality

The analysis of the thermal criticality for the fluid flow through a porous medium with convective cooling is done by using Adomian Decomposition Method (ADM) and Pade approximation to obtain the solution of the non – linear boundary value problem equations governing the fluid flow.
Using ADM, the solution of the temperature profile is given as

$$T(y) = a_0 - \lambda \int_0^y \int_0^y \left[e^{\frac{T}{1+\delta T}} + \gamma\left(\left(\frac{du}{dy}\right)^2 + \left(H^2+\alpha\right)u^2\right)\right] dydy \tag{18}$$

where $a_0 = T(0)$ is to be determined by using the boundary conditions.
The ADM requires that the approximate solution is the partial sum

$$T(y) = \sum_{n=0}^{k} T_n(y) \tag{19a}$$

of the following series

$$T(y) = \sum_{n=0}^{\infty} T_n(y) \tag{19b}$$

where the components $T_0, T_1, T_2, ..., T_k$ are to be determined. Writing the non – linear term in (18) as a series of Adomian polynomials, we have

$$\sum_{n=0}^{\infty} A_n(y) = e^{\frac{\sum_{n=0}^{\infty} T_n(y)}{1+\delta \sum_{n=0}^{\infty} T_n(y)}} \tag{20}$$

such that (18) becomes

$$T(y) = a_0 - \lambda \int_0^y \int_0^y \left[\sum_{n=0}^{\infty} A_n(y) + \gamma \left(\left(\frac{du}{dy} \right)^2 + \left(H^2 + \alpha \right) u^2 \right) \right] dydy. \tag{21}$$

and some of the Adomian polynomials obtained from (20) are

$$A_0 = e^{\frac{T_0(y)}{1+\delta T_0(y)}}, \tag{22a}$$

$$A_1 = \frac{e^{\frac{T_0(y)}{1+\delta T_0(y)}} T_1(y)}{\left[1+\delta T_0(y)\right]^2}, \tag{22b}$$

$$A_2 = \frac{e^{\frac{T_0(y)}{1+\delta T_0(y)}} \left[\left(1 - 2\delta - 2\delta^2 T_0(y)\right) T_1(y)^2 + 2(1+\delta T_0(y)^2 T_2(y)) \right]}{2\left[1+\delta T_0(y)\right]^4} \tag{22c}$$

Following [3, 4, 17 and 20] and taking the zeroth components of (21), we have

$$T_0(y) = a_0 \tag{23}$$

$$T_1(y) = -\lambda \int_0^y \int_0^y \left[A_0(y) + \gamma \left(\left(\frac{du}{dy} \right)^2 + \left(H^2 + \alpha \right) u^2 \right) \right] dydy \tag{24}$$

$$T_{n+1}(y) = -\lambda \int_0^y \int_0^y \left[A_n(y) \right] dydy, \; n \geq 1 \tag{25}$$

To this end, the diagonal form of the series solutions (19a) is evaluated using the built – in Pade approximant procedure in MATHEMATICA and the boundary conditions in (7) given as:

$$T'(1) = -BiT(1) \tag{26}$$

Taking the diagonal Pade approximant of (19a) at various values leads to an eigenvalue problem. To show that the series converge, the unknown constant a_0 is evaluated using values for the known parameters. The critical values of the Frank – Kamenettski parameter (λ_c) for the non – existence of solution or thermal runaway for the fluid flow are presented and discussed in the next section.

5. Discussion of Results

In this section, we discuss the solutions of velocity and temperature profiles, solution branches, entropy generation and thermal criticality for hydromagnetic fluid flow through a porous medium with convective cooling.

The rapid convergence of the series solutions of the temperature profile which clearly shows the efficiency and reliability in the approximation is shown in Table 1 while Table 2 displays the computation of the entropy generation analysis which indicates that the entropy generation rate is maximum at the plate surfaces and minimum around the core region of the channel. Also, the irreversibility distribution ratio (ϕ) shows that heat transfer dominates at upper and lower plate surfaces because $0 \le \phi < 1$ and fluid friction dominates at the centerline of the region because $\phi > 1$.

Table 1

Rapid convergence of the series solutions of the Temperature Profiles

$y = \gamma = H = G = \delta = \alpha = 1, \quad Bi = 10, \quad \lambda = 0.5,$		
n	T_n	$\sum_{n=0}^{n=k} \lambda^n T_n$
0	0	0
1	0.109963	0.0549816
2	0.0476533	0.0668949
3	0.0103951	0.0681943
4	− 0.0052743	0.0678646
5	− 0.0045403	0.0677161
6	− 0.0004259	0.0677161

Table 2

Computation of the Entropy Generation Analysis

$\gamma = H = G = \delta = \alpha = 1, \quad Bi = 10, \quad \lambda = 0.5, \quad Br\Omega^{-1} = 0.1$					
y	N_1	N_2	N_s	ϕ	$Be = \frac{1}{1+\Phi}$
-1	0.535876	0.0394614	0.575338	0.0736391	0.931412
-0.75	0.306524	0.020341	0.326865	0.0663601	0.93777
-0.5	0.140285	0.0150814	0.155367	0.107505	0.902931
-0.25	0.0358832	0.0144756	0.0503588	0.40341	0.71255
0	1.73334×10^{-31}	0.00146287	0.0146287	8.43964×10^{28}	1.18488×10^{-29}
0.25	0.0358832	0.0144756	0.0503588	0.40341	0.71255
0.5	0.140285	0.0150814	0.155367	0.107505	0.902931
0.75	0.306524	0.020341	0.326865	0.0663601	0.93777
1	0.535876	0.0394614	0.575338	0.0736391	0.931412

Table 3

Effect of different parameters on the development of thermal runaway

Pade	H	γ	δ	G	α	Bi	λ_c
2/2	**1**	**0.1**	**0.1**	**1**	**0.1**	**10**	**0.8939930073508189**
2/2	1	0.1	0.1	1	0.5	10	0.8943907315538621
2/2	1	0.1	0.1	1	1.0	10	0.8948222921987654
2/2	**1**	**0.1**	**0.1**	**1**	**0.1**	**10**	**0.8939930073508189**
2/2	1	0.1	0.1	1	0.1	25	1.0072845560814208
2/2	1	0.1	0.1	1	0.1	50	1.0488443355711630
2/2	**1**	**0.1**	**0.1**	**1**	**0.1**	**10**	**0.8939930073508189**
2/2	2	0.1	0.1	1	0.1	10	0.8960742476940970
2/2	3	0.1	0.1	1	0.1	10	0.8974303789687192

Meanwhile, Table 3 shows the effects of different parameters on the development of thermal runaway. It shows that the magnitude of thermal criticality increases with increasing values of porous medium term (α), convective cooling term (Bi) and magnetic field intensity (H) which stabilizes the fluid flow.

The velocity profiles with variations in porous medium term and magnetic field are respectively shown in Figs. 2 and 3. It is shown that the fluid velocity reduces with increasing values of porous medium term (α) and magnetic field intensity (H) which is due to the retarding effect of the porosity and magnetic force present in the channel.

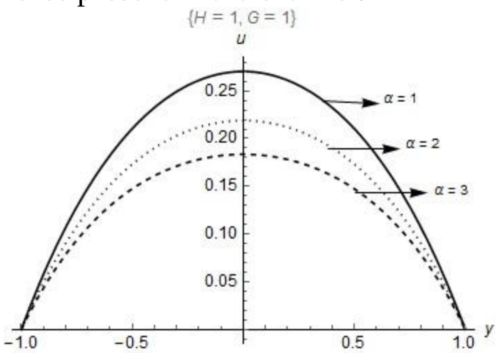

Fig. 2: Fluid velocity profile with variations in porous medium term

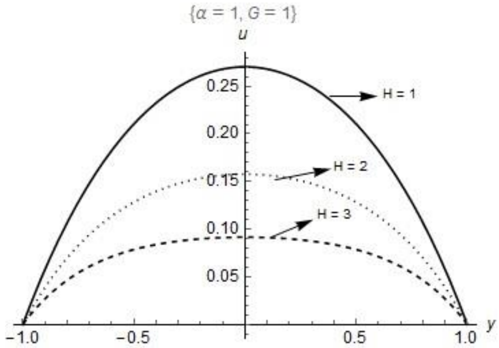

Fig. 3: Fluid velocity profile with variations in magnetic field intensity

The temperature profiles are shown in Figs. 4 – 8. In fig. 4, the fluid temperature increases as the viscous heating parameter increases, this is caused by the conversion of kinetic energy in the moving fluid to internal energy. The maximum fluid temperature is obtained at the minimum values of magnetic field intensity parameter (H) as shown in fig. 5. Also, in Fig. 6, the fluid temperature reduces as the porous medium term increases; this is due to the reduction in fluid flow and the time taken for fluid to flow within the porous medium thereby reduces the temperature.

The fluid temperature profile with variations in convective cooling term (*Bi*) is shown in figure 7; it is observed that the minimum value of temperature is obtained at the maximum value of Biot number due to the influence of thermal conductivity on the fluid temperature. Also, the fluid temperature increases as Frank – Kamenettski parameter (λ) increases as shown in figure 8; this is due to an increase in the heat generated within the flow channel.

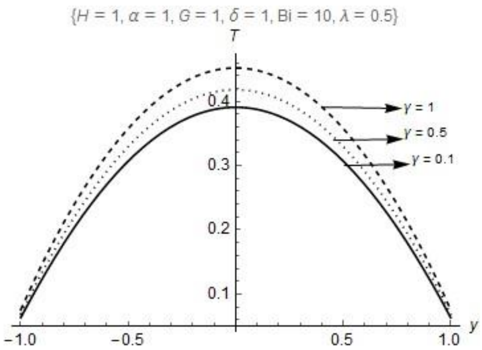

Fig. 4: Fluid temperature profile with variations in viscous heating parameter

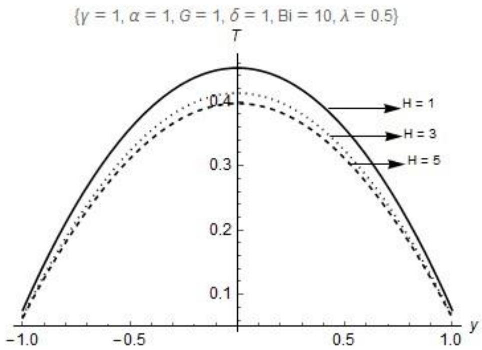

Fig. 5: Fluid temperature profile with variations in magnetic field intensity

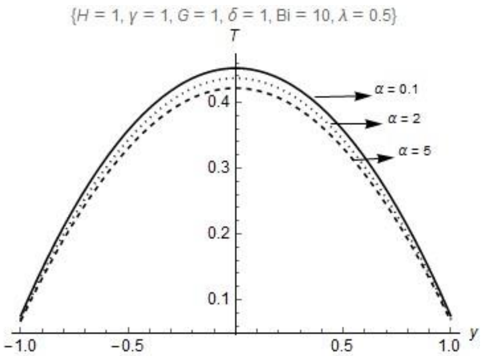

Fig. 6: Fluid temperature profile with variations in porous medium parameter

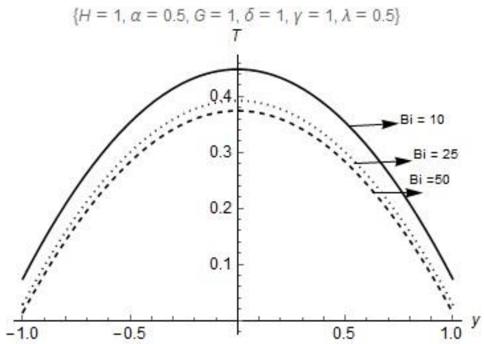

Fig. 7: Fluid temperature profile with variations in convective cooling term

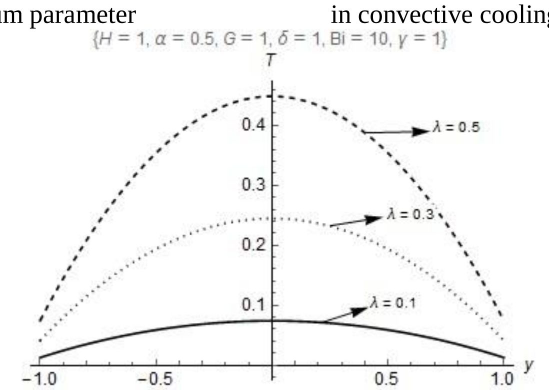

Fig. 8: Fluid temperature profile with variations in Frank – Kamenettski parameter

Figs. 9 to 12 display the variation of parameters on entropy generation rate. Generally, it is noticed that the entropy generation rate is at maximum at the surfaces and at minimum around the core region of the channel of fluid flow. In figure 9, the influence of porous medium parameter (α) is clearly noticed as it yields an interesting result with respect to the entropy generation rate with increasing value of α over moving surfaces. On the other hands, Figs. 10 and 11 showed that the entropy generation rate increases respectively with increasing values of Frank – Kamenettski parameter (λ) and wall temperature parameter ($Br\Omega^{-1}$) in the thermodynamic performance of the flow system. In Fig. 12, the rate of disorder is reduced with an increase in magnetic field intensity (H)

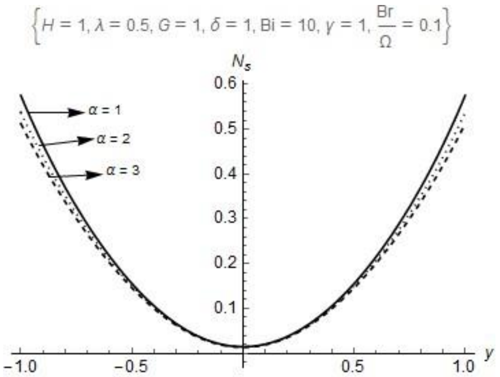

Fig. 9: Entropy generation rate for various values of porous medium parameter

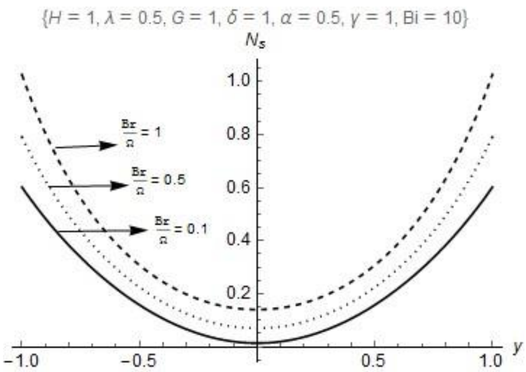

Fig. 10: Entropy generation rate for various values of wall temperature parameter

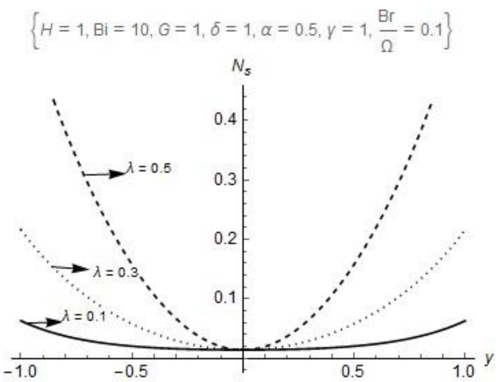

Fig. 11: Entropy generation rate for various values of Frank – Kamenettski parameter

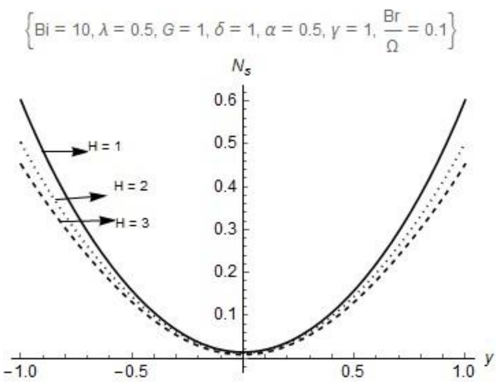

Fig. 12: Entropy generation rate for various values of magnetic field intensity

However, Figs. 13 – 15 show the Bejan number (Be) for various parametric values in the channel width. The general observation is that the fluid friction over irreversibility dominates at the channel core region while heat transfer rate over irreversibility dominates at both upper and lower wall surfaces. It is clearly noticed that, the dominant influence of heat irreversibility of the plate increases with increasing values of porous medium parameter (α) and Frank –

Kamenettski parameter (λ) in Figs. 13 and 14, while the reverse is the case in Fig. 15 where heat irreversibility of the plate decreases with an increasing value of the wall temperature parameter ($Br\Omega^{-1}$)

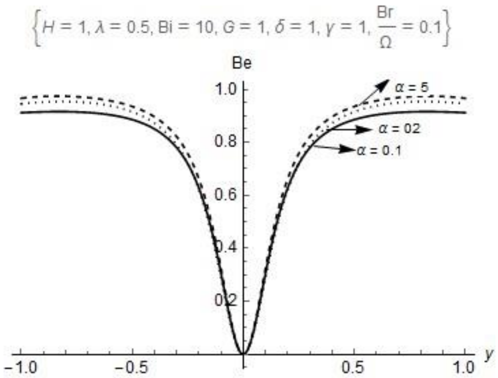

Fig. 13: Bejan number for various values of porous medium parameter

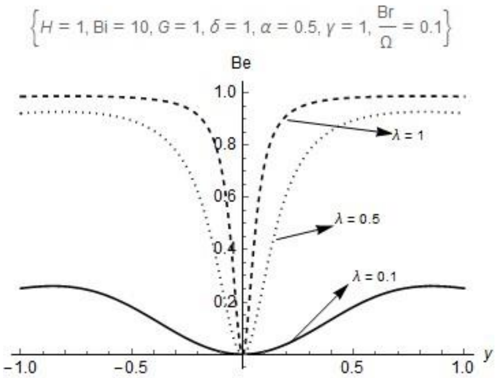

Fig. 14: Bejan number for various values of Frank – Kamenettski parameter

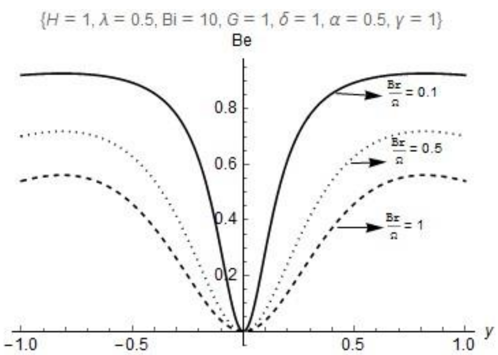

Fig. 15: Bejan number for various values of wall temperature parameter

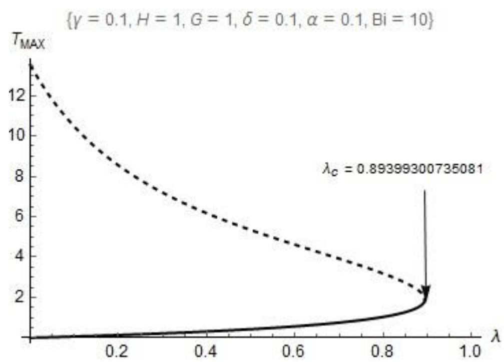

Fig. 16: A slice of approximate bifurcation diagram

Finally, another interesting aspect of the problem is the critical point shown in figure 16, a slice of approximate bifurcation diagram, it is noticed that the problem has upper and lower solutions at $\lambda < \lambda_c$, a single solution at $\lambda = \lambda_c$ and no solution at $\lambda > \lambda_c$.

6. Conclusion

The analysis of a reactive hydromagnetic fluid flow between two parallel plates through a porous medium with convective boundary conditions is investigated using the traditional perturbation method together with Adomian Decomposition Method (ADM) and diagonal Pade Approximant to determine the thermal criticality values as well as bifurcation conditions. It is observed that the fluid velocity reduces with increasing values of porous medium and magnetic intensity parameters. The fluid temperature decreases with increasing values of activation energy, porous medium, magnetic intensity and convective cooling terms. Also, an increase in the convective cooling, porous medium and magnetic intensity fields on the fluid flow will improve stability and this will help to bring about a delay in the appearance of thermal runaway.

R E F E R E N C E S

[1]. *J. A. Gbadeyan and A.R. Hassan*, " Multiplicity of solutions for a reactive variable viscous Couette flow under Arrhenius kinetics", in Mathematical Theory and Modelling **Vol. 2**, no.9, 2012, pp. 39 – 49.

[2]. *A. R. Hassan and J. A. Gbadeyan*, "The effect of heat absorption on a variable viscosity reactive Couette flow under Arrhenius kinetics", in Theoretical Mathematics & Applications. **Vol. 3**, no.1, 2013, pp. 145 – 159

[3]. *A. R. Hassan and J. A. Gbadeyan*, "Entropy generation analysis of a reactive hydromagnetic fluid flow through a channel", in U. P. B. , Series A, **Vol. 77**, Iss. 2, 2015, pp 287 – 296.

[4]. *A. R. Hassan and J. A. Gbadeyan*, "Thermal stability of a reactive hydromagnetic Poiseuille fluid flow through a channel", in American Journal of Applied Mathematics, **Vol. 2** , no.1, 2014, pp 14 – 20.

[5]. *A. R. Hassan and J. A. Gbadeyan*, "A reactive hydromagnetic internal heat generating fluid flow through a channel", in International Journal of Heat and Technology, **Vol. 33**, no 3, 2015, pp 43 – 50.

[6]. *O. D. Makinde and O. Anwar Beg,*"On inherent irreversibility in a reactive hydromagnetic Channel Flow", in Journal of Thermal Science, **Vol.** 19, no 1, 2010, pp. 72 -79.

[7]. *K Taira*, "A mathematical analysis of thermal explosions", in IJMMS, **Vol. 28**, no 10, 2001, pp. 581 – 607.

[8]. *P. K. Singh*, "Effects of variable fluid properties and viscous dissipation on mixed convection fluid flow past a vertical plate in porous medium", in International Journal of Scientific and Engineering Research, **Vol. 3**, no 1, 2012, pp. 1 – 5.

[9]. *O. D. Makinde*, "Thermal stability of a reactive viscous flow through a porous – saturated channel with convective boundary conditions", in International Journal of Numerical Methods for Heat & Fluid Flow, **Vol. 17**, no. 8, 2007, pp. 836 – 844.

[10]. *O. D. Makinde*, "Thermal stability of a reactive viscous flow through a porous – saturated channel with convective boundary conditions", in Applied Thermal Engineering, **Vol. 29**, 2009, pp. 1773 - 1777.

[11]. *A. R. Hassan and R. Maritz*, "The analysis of a variable – viscosity fluid flow between parallel porous plates with non – uniform wall temperature", In Italian Journal of Pure and Applied Mathematics, **Vol. 36**, no 1, 2016, pp 1 – 12.

[12]. *O. D. Makinde and M. Maserumule* , "Inherent irreversibility and thermal stability for steady flow of variable viscosity liquid film in a cylindrical pipe with convective cooling at the surface", In International Journal of Numerical Methods for Heat & Fluid Flow, **Vol. 20**, no. 1, 2010, pp. 5 – 16.

[13]. *O. D. Makinde*, "On Thermal Stability of a reactive third - grade fluid in a channel with convective cooling the walls", In Applied Mathematics and Computation, **Vol. 213**, 2009, pp. 170 – 176.

[14]. *P. O. Olanrewaju, J. A. Gbadeyan , T. Hayat. and A. A. Hendi*, "Effects of internal heat generation thermal radiation and buoyancy force on a boundary layer over a vertical plate with a convective surface boundary condition", in South African Journal of Science, **Vol. 107**, no. (9/10), 2011, pp. 476 – 482.

[15] *T. Chinyoka and O. D. Makinde*, “Analysis of transient generalized couette flow of a reactive variable viscosity third - grade liquid with asymmetric convective cooling”, in Mathematical and Computer Modelling, **Vol. 54**, 2011, pp. 160 – 174.
[16]. *O. D. Makinde and T. Chinyoka*, “Numerical study of unsteady hydromagnetic generalized Couette flow of a reactive third – grade fluid with asymmetric convective cooling”, in Journal of Computers and Mathematics with Applications, **Vol. 61**, 2011, pp. 1167 – 1179.
[17]. *A. R. Hassan and R. Maritz*, “The analysis of a reactive hydromagnetic internal heat generating Poiseuille fluid flow through a channel”, in SpringerPlus, **Vol. 5**, no 1, 2016 pp. 1332
[18]. *A. H. Nayfeh* , Perturbation methods, A Wiley – Interscience Publication, John Wiley & Sons, New York, 2008.
[19]. *L. C. Wood*, The thermodynamics of fluid system, Oxford University Press, Oxford, 1975.
[20]. *A. M. Wazwaz and El-Sayed*, “A new modification of the Adomian decomposition method for linear and nonlinear operators”, in Journal of Applied Maths. Computation, **Vol. 122**, 2001**,** pp. 393 – 405.