NEW RESULTS ON TOPOLOGICAL INDICES FOR BENES AND BUTTERFLY NETWORKS

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Networks play vital role in the advanced technology era. From simple to complex networks are constructed for efficient and accurate information exchange. Before using different networks their topological properties are important to keep in view, thats why various kind of topological indices were investigated and researcher studying topological indices for new networks. The topological indices are main tool in the study of Quantitative structure activity (QSAR) and structured property relationships (QSPR) to examine the architecture of networks. Topological index is obtained by converting a graphical network structure into a numerical value. In the present paper, we study basic butterfly and benes networks. We derive analytical result for general Randic connectivity index (R_α), second Zegreb (M_α) index, general sum-connectivity (χ_α) index, fourth atom-bond connectivity index ABC_4, and fifth geometric-arithematic index GA_5, multiple Zegreb indices and Zegrab polynomial indices of the butterfly and benes networks.

Keywords: Topological indices, Zegreb polynomials, Butterfly network, Benes network.
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1. Introduction and preliminaries

Graphs are used to design interconnected networks in a very natural way, in which the processors or components represent vertices and edges represent the communication links e.g. fiber optic cables. The way in which all these components work will be carried out by incidence functions. Graphs show the topological properties of the networks, therefore the graph and networks are basically same in a sense that when we are considering a networks, components and links we actually speak of graph, vertices and edges.

In interconnection networks, the processing nodes are the multiprocessor used to build a network of homogeneously same processor memory pairs. Programs are compiled and executed through message sending. Considerable importance to the architecture and utilization of multiprocessor interconnection network is due to low cost , more efficient microprocessors and chips [1]. Interconnection networks resembled the communication pattern of the natural scenario, which make it more valuable and important. Most of the networks are interconnected and dependent on each other which need to be reviewed for the future work. In particularly, the cooperation which dependent on such a networks face many failure need to be study for better solution to their problems and improvements [2, 3].

In the present paper all graphs are simple, undirected and finite. Formally we represent a graph by $H(V, E)$ with the set V as vertices and set E as edges. An array of different

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vertices and edges in which vertices are connected by edges and no vertex or edge repeat such that the initial and final vertices are different is called a path. A connected graph are those graphs in which there is a path between any two vertices. A network is basically a connected graph in which vertices represents processors and edges represents links.

The notions used in the present paper are taken from the book [5, 7] and [8]. The two vertices connected by an edge are called adjacent vertices. The number of all adjacent vertices to a vertex x is called degree of x denoted by d_x, where the vertices adjacent to x are called neighbors of x [6]. The set of all neighbors of x denoted by $N(x)$ which is called neighborhood of x i.e $N(x) = \{y \in V(H) : \quad xy \in E(H)\}$. The sum of vertices degree from the set $N(x)$ is denoted by S_x i.e $S_x = \sum_{y \in N(x)} d_y$.

The notion of topological index [4] were introduced by Herold Wiener in 1947. After Herold Wiener in 1975 Milan Randic extend the study of topological index and nowadays different researchers are working on different indices. In the present paper we extend the results discussed in the paper [6] by M. Imran et al. To derived our results we defined some degree based topological indices.

Definition 1.1. [9]
In 1998, the Randic index discovery leads to a General Randic index given by Bollobás and Erdös [9] *denoted by* $R_\alpha(H)$ *given as:*

$$R_\alpha(H) = \sum_{xy \in E(H)} (d_x d_y)^\alpha. \tag{1}$$

For $\alpha \in R$. *The Randic index is a particular case of general Randic index when* $\alpha = -\frac{1}{2}$.

Definition 1.2. [10]
Trinajstic and Zhou change the idea of Randic index and modified it into general sum connectivity index χ_α *as follows*

$$\chi_\alpha(H) = \sum_{xy \in E(G)} (d_x + d_y)^\alpha \tag{2}$$

where $\alpha \in R$.

Definition 1.3. [11] *In* [11] *Shirdel et al. introduced a new degree based Zegreb index named as " hyper-Zegreb index " which is defined as:*

$$M_\alpha(H) = \sum_{xy \in E(H)} (d_x)^\alpha. \tag{3}$$

Definition 1.4. [13] *Ghorbani and Hosseinzadeh* [13] *give idea of the fourth version of ABC index define as:*

$$ABC_4(H) = \sum_{xy \in E(H)} \sqrt{\frac{S_x + S_y - 2}{S_x S_y}}. \tag{4}$$

Where idea for the fifth version of GA given by Graovac et al. [14] *and defined as:*

$$GA_5(H) = \sum_{xy \in E(H)} \frac{2\sqrt{S_x S_y}}{S_x + S_y}. \tag{5}$$

Definition 1.5. [15]
The first multiple Zagreb index $PM_1(G)$ *and second multiple Zagreb index* PM_2 *defined by Ghorbani and Azimi* [15] *as:*

$$PM_1(H) = \prod_{xy \in E(H)} (d_x + d_y) \tag{6}$$

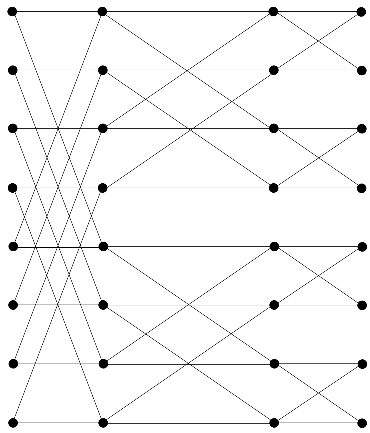

FIGURE 1. 3-dimentional butterfly network

$$PM_2(H) = \prod_{xy \in E(H)} (d_x d_y). \tag{7}$$

Definition 1.6. [16] *In 2013, Hyper Zagreb index was proposed by Shirdel et al.* [16]

$$HM(H) = \sum_{xy \in E(H)} (d_x + d_y)^2.$$

Definition 1.7. [15] *The first Zagreb polynomial* $M_1(H,t)$ *and second Zagreb polynomial* $M_2(H,t)$ *are defined as:*

$$M_1(H,t) = \sum_{xy \in E(H)} t^{(d_x + d_y)} \tag{8}$$

$$M_2(H,t) = \sum_{xy \in E(H)} t^{(d_x d_y)}. \tag{9}$$

Some results on degree base topological indices can be found in [17, 18, 19].

2. **Main Results**

In this paper, we constructed butterfly and benes networks and then study the topological indices and formulated general result for second and hyper zegreb index, ABC_4, GA_5, first and second multiple zegreb $PM_1(G)$, $PM_2(G)$ and polynomial Zegreb index $M_1(G,t)$ and $M_2(G,t)$

Results for Butterfly Network BF(r): The most important and commonly used degree based network is the butterfly network. It is made up of a butterfly patterns. The set V of vertices of an r-dimensional butterfly give pairs (i,j) where j is the level or stages of nodes $(0 \leq j \leq r)$ and i is an r-bit binary number that denotes the row of the node. Two node are connected by an edge using walk cycle such as node(i,j) is connected to node$(i,j+1)$ and node$(m,j+1)$ where m is obtained by flipping j^th bit which represent a butterfly. The networks has undirected edges. An r-dimensional butterfly network is denoted by $BF(r)$, the number of vertices in $BF(r)$ are $(r+1)2^r$ and number of edges are $r2^{r+1}$. In the Figure 1 a 3-dimentional butterfly network $BF(3)$ can be seen.

Table 1. Degree based vertex partition of $BF(r)$.

Vertex degree	Number of vertices
2	2^{r+1}
4	$2^r(r-1)$

Table 2. End vertices degree based edge partition of graph $(BF(r))$.

(d_x, d_y),where xy$\in E(BF(r))$	Number of edges
(2,4)	2^{r+2}
(4,4)	$2^{r+2}(r-2)$

General result of Randic, First and second Zagreb, ABC and GA indecies for $BF(r)$ are calculated in [6]. We will calculate more degree base indices for $BF(r)$.

Theorem 2.1. *For the butterfly network* $BF(r)$*, we have*

(1) $M_\alpha(BF(r)) = 2^{r+1+\alpha} + 2^{r+2\alpha}(r-1)$*;*
(2) $R_\alpha(BF(r)) = 2^{r+2+3\alpha} + 2^{r+1+4\alpha}(r-2)$*;*
(3) $\chi_\alpha(BF(r)) = 2^{r+2}(6)^\alpha + 2^{r+1+3\alpha}(r-2)$*;*

where α *is a real number.*

Proof. For the graph $BF(r)$ we have,

$$M_\alpha(BF(r)) = \sum_{x\in V(BF(r))} (d_x)^\alpha.$$

Since we have two types vertices in $BF(r)$ under degree base shown in Table 1. Using Table 1 and above definition of $M_\alpha(BF(r))$ we get,

$$M_\alpha(BF(r)) = V_2(2)^\alpha + V_4(4)^\alpha = (2)^{r+1+\alpha} + (2)^{r+2\alpha}.$$

The general Randic index for the graph $BF(r)$ define as:

$$R(BF(r)) = \sum_{xy\in E(BF(r))} (d_x d_y)^\alpha.$$

Using Table 2 and above mentioned definition we have,

$$R_\alpha(BF(r)) = e_{2,4}(2\times 4)^\alpha + e_{4,4}(4\times 4)^\alpha = 2^{r+2+3\alpha} + 2^{r+1+4\alpha}(r-2).$$

The formula of general sum-connectivity for $BF(r)$ is:

$$\chi_\alpha(BF(r)) = \sum_{xy\in E(BF(r))} (d_x + d_y)^\alpha.$$

Getting values from Table 3 and using above formula we get,

$$\chi_\alpha(BF(r)) = e_{2,4}(2+4)^\alpha + e_{4,4}(4+4)^\alpha = 2^{r+1+3\alpha}(r-2) + 2^{r+2}(6)^\alpha.$$

□

To calculate the fourth atom-bond connectivity index ABC_4 and the fifth geometric-arithmetic index GA_5 for $BF(r)$, we use degree based sum of neighbors vertices of each edge. The partition of edges of $BF(r)$ on the base of degree sum of neighbors vertices shown in below tables.

Table 3(a). The edge partition of graph BF(3)

$(S_x, S_y), wherexy \in E(BF(r))$	Number of edges
(8,12)	32
(12,12)	16

Table 3(b). The edge partition of graph BF(r) for $r \geq 4$.

$(S_x, S_y), wherexy \in E(BF(r))$	Number of edges
(8,12)	2^{r+2}
(12,16)	2^{r+2}
(16,16)	$2^{r+1}(r-4)$

Theorem 2.2. *The ABC_4 and GA_5 indices for the graph $BF(r)$ are,*

(1) $ABC_4(BF(r)) = \sqrt{3}(2^r) + \frac{1}{\sqrt{2}}(2^r) + \sqrt{30}(2^{r-3})(r-4), \quad for \quad r \geq 4;$

(2) $ABC_4(BF(3)) = 8\sqrt{3} + \frac{4\sqrt{22}}{3};$

(3) $GA_5(BF(r)) = \frac{\sqrt{6}}{5}(2^{r+3}) + \frac{\sqrt{6}}{5}(2^{r+4}) + 2^{r-1}(r-4), \quad for \quad r \geq 4;$

(4) $GA_5(BF(3)) = \frac{64\sqrt{6}}{5} + 16.$

Proof. For $i = S_x$ and $j = S_y$ we denotes the number of edges of butterfly network by $m_{i,j}$. The ABC_4 is defined as:

$$ABC_4(BF(r)) = \sum_{xy \in E(BF(r))} \sqrt{\frac{S_x + S_y - 2}{S_x \times S_y}}.$$

Using Table 3(a), Table 3(b) and above definition of ABC_4 we have,

$$\begin{aligned} ABC_4(BF(3)) &= m_{8,12}\sqrt{\frac{8+12-2}{8 \times 12}} + m_{12,12}\sqrt{\frac{12+12-2}{12 \times 12}} \\ &= 8\sqrt{3} + \frac{4\sqrt{22}}{3} \end{aligned}$$

and for $r \geq 4$

$$\begin{aligned} ABC_4(BF(r)) &= m_{8,12}\sqrt{\frac{8+12-2}{8 \times 12}} + m_{12,16}\sqrt{\frac{12+16-2}{12 \times 16}} + m_{16,16}\sqrt{\frac{16+16-2}{16 \times 16}} \\ &= \sqrt{3}2^r + \frac{1}{\sqrt{2}}2^r + \sqrt{30}2^{r-3}(r-4). \end{aligned}$$

The GA_5 defines as:

$$GA_5(BF(r)) = \sum_{xy \in E(BF(r))} \frac{2\sqrt{S_x \times S_y}}{S_x + S_y}.$$

Using Table 3(a), Table 3(b) and GA_5 definition we get,

$$GA_5(BF(3)) = m_{8,12}(2\frac{\sqrt{8 \times 12}}{8+12}) + m_{12,12}(2\frac{\sqrt{12 \times 12}}{12+12}) = \frac{64\sqrt{6}}{5} + 16.$$

For $r \geq 4$ we have,

$$\begin{aligned} GA_5(BF(r)) &= m_{8,12}(2\frac{\sqrt{8 \times 12}}{8+12}) + m_{12,16}(2\frac{\sqrt{12 \times 16}}{12+16}) + m_{16,16}(2\frac{\sqrt{16 \times 16}}{16+16}) \\ &= \frac{\sqrt{6}}{5}2^{r+3} + \frac{\sqrt{6}}{5}2^{r+4} + 2^{r-1}(r-4). \end{aligned}$$

□

Theorem 2.3. *Let $BF(r)$ be a butterfly network, then*

(1) $HM(BF(r)) = (9)2^{r+4} + 2^{r+7}(r-2);$

(2) $PM_1(BF(r)) = 2^{2^{r+1}(3r-4)} \times 3^{2^{r+2}};$

(3) $PM_2(BF(r)) = 2^{2^{r+2}(2r-1)};$

(4) $M_1(BF(r), t) = 2^{r+2}t^6 + 2^{r+1}(r-2)t^8;$

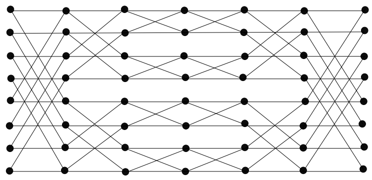

Figure 2. 3-dimentional Benes network

(5) $M_2(BF(r),t) = 2^{r+2}t^8 + 2^{r+1}(r-2)t^{16}$.

Proof. For the graph $BF(r)$, the $HM(BF(r))$, $PM_1(BF(r))$, $PM_2(BF(r))$, $M_1(BF(r),t)$ and $M_2(BF(r),t)$ are define as follows:

$$HM(BF(r)) = \sum_{xy\in E(BF(r))} (d_x+d_y)^2.$$
$$PM_1(BF(r)) = \prod_{xy\in E(BF(r))} (d_x+d_y)$$
$$PM_2(BF(r)) = \prod_{xy\in E(BF(r))} (d_x\times d_y)$$
$$M_1(BF(r),t) = \sum_{xy\in E(G)} t^{(d_x+d_y)}$$
$$M_2(BF(r),t) = \sum_{xy\in E(BF(r))} t^{(d_x\times d_y)}.$$

By using Table 2 and above definitions we get,

$$HM(BF(r))) = e_{2,4}(2+4)^2 + e_{4,4}(4+4)^2 = (9)2^{r+4} + 2^{r+7}(r-2),$$
$$PM_1(BF(r)) = (2+4)^{e_{2,4}} \times (4+4)^{e_{4,4}} = 2^{2^{r+1}(3r-4)} \times 3^{2^{r+2}},$$
$$PM_2(BF(r)) = (2\times 4)^{e_{2,4}} \times (4\times 4)^{e_{4,4}} = 2^{2^{r+2}(2r-1)},$$
$$M_1(BF(r),x) = e_{2,4}x^{2+4} + e_{4,4}x^{4+4} = 2^{r+2}x^6 + 2^{r+1}(r-2)x^8,$$
$$M_2(BF(r),x) = e_{2,4}x^{2\times 4} + e_{4,4}x^{4\times 4} = 2^{r+2}x^8 + 2^{r+1}(r-2)x^{16}.$$

□

Results for Benes Network $B(r)$: Benes is the most important and commonly used network. A butterfly network gives the Benes Network i-e first and last stages are same, second and second last stages are same and also refer as back to back butterflies. All the nodes (processors) are connected in the form of butterfly in the whole Benes network. Butterflies in the middle stage are shared in the Benes network[5]. $B(r)$ represents a Benes network with dimension r. In the Figure 2 a 3-dimentional Benes network $B(3)$ can be seen.

Table 1.The vertex partition of graph $B(r)$ based on degree of vertices

Degree of vertex	Number of vertices
2	2^{r+1}
4	$2^r(2r-1)$

Table 2. The edge partition of graph $B(r)$ based on degree of end vertices of each edge.

(d_x,d_y),where xy∈ E(G)	Number of edges
(2,4)	2^{r+2}
(4,4)	$2^{r+2}(r-1)$

General result of Randic,First Zagreb, ABC and GA indecies for $B(r)$ are calculated in [6]. We will calculate some further indices for $B(r)$.

Theorem 2.4. *Consider the benes network $B(r)$, then*
(1) $M_2(B(r)) = 2^{r+5} + 2^{r+6}(r-1)$;
(2) $M_\alpha(B(r)) = 2^{r+1+\alpha} + 2^{r+2\alpha}(2r-1)$;
(3) $R_\alpha(B(r)) = 2^{r+2+3\alpha} + 2^{r+2+4\alpha}(r-1)$;
(4) $\chi_\alpha(B(r)) = 2^{r+2+\alpha} \times 3^\alpha + 2^{r+2+3\alpha}(r-1)$;
(5) $HM(B(r)) = 2^{r+2}[64r-28]$;
where α is a real number.

Proof. Since the formula for $M_2(B(r))$ is given as:

$$M_2(B(r)) = \sum_{xy \in E(B(r))} (d_x \times d_y).$$

Using vales from Table 4, we get

$$M_2(B(r)) = e_{2,4}(2 \times 4) + e_{4,4}(4 \times 4) = 2^{r+5} + 2^{r+6}(r-1).$$

The general formula for $M_\alpha(B(r))$ define as:

$$M_\alpha(B(r)) = \sum_{x \in V(B(r))} (d_x)^\alpha.$$

Using Table 4 and Table 5 we have,

$$M_\alpha(B(r)) = V_2(2)^\alpha + V_4(4)^\alpha = 2^{r+1+\alpha} + 2^{r+2\alpha}(2r-1).$$

For the graph $B(r)$ general Randic index is:

$$R_\alpha(B(r)) = \sum_{xy \in E(B(r))} (d_x \times d_y)^\alpha.$$

This implies that

$$R_\alpha(B(r)) = e_{2,4}(2 \times 4)^\alpha + e_{4,4}(4 \times 4)^\alpha = 2^{r+2+3\alpha} + 2^{r+2+4\alpha}(r-1).$$

The formula of general sum-connectivity is:

$$\chi_\alpha(B(r)) = \sum_{xy \in E(B(r))} (d_x + d_y)^\alpha.$$

$$\chi_\alpha(B(r)) = e_{2,4}(2+4)^\alpha + e_{4,4}(4+4)^\alpha = 2^{r+2+\alpha} \times 3^\alpha + 2^{r+2+3\alpha}(r-1).$$

Hyper Zagreb index is given as:

$$HM(B(r)) = \sum_{xy \in E(B(r))} (d_x + d_y)^2.$$

$$HM(B(r)) = e_{2,4}(2+4)^2 + e_{4,4}(4+4)^2 = 2^{r+2}[64r-28].$$

□

For calculation of fourth atom-bond connectivity index ABC_4 and the fifth geometric-arithmetic index GA_5 for $B(r)$, we use edges partation on the bases of degree sum of neighbors vertices of each edge in the benes network shown in below table.

Table 6. Edge partition of graph $B(r)$ for $r \geq 3$.

$(S_x, S_y), where xy \in E(BF(r))$	Number of edges
(8,12)	2^{r+2}
(12,16)	2^{r+2}
(16,16)	$2^{r+2}(r-2)$

Theorem 2.5. *The ABC_4 and GA_5 of $B(r)$ are given by*

(1) $ABC_4(B(r)) = 2^r[\sqrt{3} + \sqrt{\frac{13}{6}} + (r-2)\frac{\sqrt{30}}{4}]$;
(2) $GA_5(B(r)) = 2^{r+2}[\frac{2\sqrt{6}}{5} + \frac{4\sqrt{3}}{7} + (r-2)]$.

Proof. From the symbol $m_{i,j}$ we denotes the number of edges of $B(r)$ with $i = S_x$ and $j = S_y$. The ABC_4 for $B(r)$ is defined as:

$$ABC_4(B(r)) = \sum_{xy \in E(G)} \sqrt{\frac{S_x + S_y - 2}{S_x \times S_y}}.$$

By using Table 6 and definition of ABC_4 we have,

$$\begin{aligned} ABC_4(B(r)) &= m_{8,12}\sqrt{\frac{8+12-2}{8\times 12}} + m_{12,16}\sqrt{\frac{12+16-2}{12\times 16}} + m_{16,16}\sqrt{\frac{16+16-2}{16\times 16}} \\ &= 2^r[\sqrt{3} + \sqrt{\frac{13}{6}} + (r-2)\frac{\sqrt{30}}{4}]. \end{aligned}$$

The fifth geometric -arithmetic index GA_5 is defined as:

$$GA_5(B(r)) = \sum_{xy \in E(B(r))} \frac{2\sqrt{S_x \times S_y}}{S_x + S_y}.$$

This implies that

$$\begin{aligned} GA_5(B(r)) &= m_{8,12}(2\frac{\sqrt{8\times 12}}{8+12}) + m_{12,16}(2\frac{\sqrt{12\times 16}}{12+16}) + m_{16,16}(2\frac{\sqrt{16\times 16}}{16+16}) \\ &= 2^{r+2}[\frac{2\sqrt{6}}{5} + \frac{4\sqrt{3}}{7} + (r-2)]. \end{aligned}$$

□

Theorem 2.6. *Let $B(r)$ be a benes network, then*

(1) $PM_1(B(r)) = 3 \times 2^{2r+8}(r-1)$;
(2) $PM_2(B(r)) = 2^{2r+11}(r-1)$;
(3) $M_1(B(r), t) = 2^{r+2}t^6 + 2^{r+2}(r-1)t^8$;
(4) $M_2(B(r), t) = 2^{r+2}t^8 + 2^{r+2}(r-1)t^{16}$.

Proof. : For the benes network $B(r)$, the number $PM_1(B(r))$, $PM_2(B(r))$, $M_1(B(r), t)$ and $M_2(B(r), t)$ are define as:

$$PM_1(B(r)) = \prod_{xy \in E(B(r))} (d_x + d_y)$$

$$PM_2(B(r)) = \prod_{xy \in E(B(r))} (d_x \times d_y)$$

$$M_1(B(r), t) = \sum_{xy \in E(B(r))} t^{(d_x + d_y)}$$

$$M_2(B(r),t) = \sum_{xy \in E(B(r))} t^{(d_x \times d_y)}.$$

Using these definitions and Table 5 and Table 6, we get:

$$PM_1(B(r)) = (2+4)e_{2,4} \times (4+4)e_{4,4} = 3 \times 2^{2r+8}(r-1).$$

$$PM_2(B(r)) = (2 \times 4)e_{2,4} \times (4 \times 4)e_{4,4} = 2^{2r+11}(r-1).$$

$$M_1(B(r),t) = e_{2,4}t^{2+4} + e_{4,4}t^{4+4} = 2^{r+2}t^6 + 2^{r+2}(r-1)t^8.$$

$$M_2(B(r),t) = e_{2,4}t^{2\times 4} + e_{4,4}t^{4\times 4} = 2^{r+2}t^8 + 2^{r+2}(r-1)t^{16}.$$

Which completes the proof. □

3. Conclusion and general remarks

Designing new network structure always attract and open ways for the researchers in the networking and other structural sciences . In this paper we study butterfly and benes network and then generalized some of the degree based topological indices such as Randic connectivity index (R_α), Zegreb (M_α) index, general sum-connectivity (χ_α) index, fourth atom-bond connectivity ABC_4 index, and fifth geometric-arithematic GA_5 index, multiple Zegreb indices and Zegrab polynomial indices for the above mentioned network. In future, it will help to those who are interested in problems related to interconnected networks and will be able to deal with the complex networks and their topological properties.

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