KINEMATIC ANALYSIS OF THE *RTaRT* MOTORS GROUP

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The mechanisms actuation is based on the motor groups. The kinematic analysis of the motor groups uses several methods such as: the closed-loops method, bars method, the matrix method, etc.

In this paper, the kinematic analysis of the RTaRT motor group by using the closed-loops method (Chr. Pelecudi) is presented. To ease the numerical calculations of the kinematic parameters of the RTaRT motors group, a procedure was established using the MATLAB®. This procedure was used in a program in order to determine the kinematic parameters of a mechanism for drawing a line mathematically precise.

Keywords: motor groups, kinematic analysis, kinematic parameters.

List of symbols

S_k: the variable parameter asociated to the prismatic joint
$\dot{S}_k$: the relative speed asociated to the prismatic joint
$\ddot{S}_k$: the relative acceleration asociated to the prismatic joint
φ_k : position angle related to revolute joint
$\dot{\varphi}_k$, ω_k: the angular velocity related to revolute joint
$\ddot{\varphi}_k$, ε_k: angular acceleration related to revolute joint
f_1, f_2: the functions of the nonlinear equations system
W: the functional matrix [Jacobian matrix] of the system
W^{-1}: the inverse matrix
θ: the angle between the unit vector's OX axis and the direction of translational motion of the link
$\dot{\theta}$: the first derivative of the angle θ
$\ddot{\theta}$: the second derivative of the angle θ

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1. Introduction

The kinematic analysis of the mechanism can be accomplished by considering modular groups separately and combine the results in order to study the entire mechanism [2, 5, 6, 7, 9, 10, 11, 12] or by taking into account the linkage as a whole [1, 3, 4, 14, 15, 16]. If the modular groups are utilized for the analysis of mechanisms, the number of equations is small, leading to a reduced calculation time. The kinematic analysis of mechanisms using the matrix method is successfully addressed in papers [12, 13].

In this paper, the kinematic analysis of the *RTaRT* motor group is fulfilled, a special procedure using the MATLAB® being achieved for this goal.

2. Kinematic analysis of the *RTaRT* motors group

To determine the kinematic parameters of the *RTaRT* motors group, the following data are known:

- $d = CD$ – distance from point *C* to the direction of the motion of the exterior prismatic joint;
- XA, YA, XP, YP - coordinates of points *A* and *P*;
- $\dot{XA}, \dot{YA}, \dot{XP}, \dot{YP}$ - projections of linear velocities of points *A* and *P*;
- $\ddot{XA}, \ddot{YA}, \ddot{XP}, \ddot{YP}$ - projections of linear accelerations of points *A* and *P*;
- S_1 - independent parameter of the *B* active joint;
- $\dot{S}_1$- linear relative velocity between the links **1** and **2** (the relative velocity of the active joint *B*);
- $\ddot{S}_1$- linear relative acceleration between the links **1** and **2** (the relative acceleration of active joint *B*);
- φ_1 - the approximate angle between the *OX* axis unit vector and the $\overline{AC}$ vector;
- S_3 - the approximate distance *PD*, measured in the positive sense of θ;

Figure 1 shows the kinematic schematics of the *RTaRT* motors group.

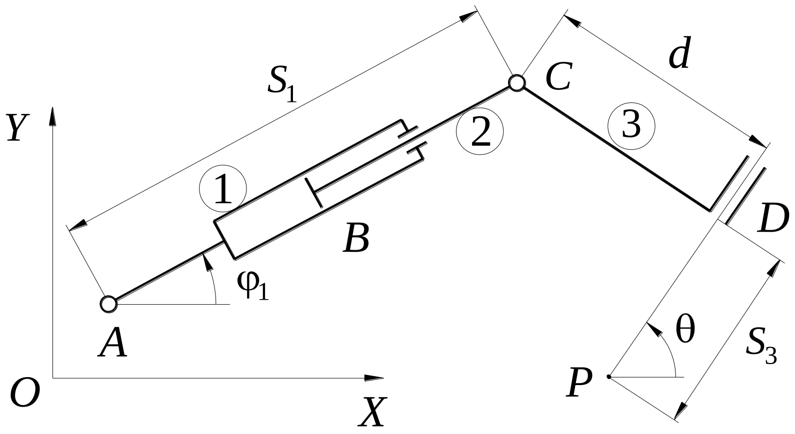


Fig. 1. The kinematic scheme of the *RTaRT* motors group

It is required to determine:

- φ_1 - angle related to revolute joint *A*, between the *OX* axis unit vector and the $\overline{AC}$ vector;
- S_3 - the variable parameter asociated to the prismatic joint *D*;
- $\dot{\varphi}_1$ - the angular velocity related to revolute joint *A* (of the links **1** and **2)**;
- $\dot{S}_3$ - the relative speed asociated to the prismatic joint *D* ;
- $\ddot{\varphi}_1$ - angular acceleration related to revolute joint *A* (of the links **1** and **2)**;
- $\ddot{S}_3$ - the relative acceleration asociated to the prismatic joint *D*;
- XC, YC - the coordinates of the center of the joint *C*;
- $\dot{XC}, \dot{YC}$ - projections of the linear velocity of the center of the joint *C*;
- $\ddot{XC}, \ddot{YC}$ - projections of the linear acceleration of the center of the joint C

2.1. Positions analysis of the *RTaRT* motors group

The positions system of equations is obtained by projecting the vectorial equation $\overline{OA} + \overline{AC} = \overline{OP} + \overline{PD} + \overline{DC}$ on the coordanate system axes, namely:

$$\begin{cases} S_1 \cdot \cos\varphi_1 - S_3 \cdot \cos\theta + d \cdot \sin\theta - k = 0; \\ S_1 \cdot \sin\varphi_1 - S_3 \cdot \sin\theta - d \cdot \cos\theta - h = 0, \end{cases} \tag{1}$$

where: *k* =*XP* - *XA*, *h* =*YP* - *YA*.

The nonlinear system of position equations with the unknowns φ_1 and S_3, is solved by the Newton-Raphson iterative method, starting from a given initial solution [8].

The system solution for iteration (i + 1) is:

$$\begin{Vmatrix} \varphi_1 \\ S_3 \end{Vmatrix}^{(i+1)} = \begin{Vmatrix} \varphi_1 \\ S_3 \end{Vmatrix}^{(i)} - W(\varphi_1^{(i)}, S_3^{(i)}) \begin{Vmatrix} f_1(\varphi_1^{(i)}, S_3^{(i)}) \\ f_2(\varphi_1^{(i)}, S_3^{(i)}) \end{Vmatrix}, \tag{2}$$

where:

$$f_1(\varphi_1, S_3) = S_1 \cos\varphi_1 - S_3 \cos\theta + d \cdot \sin\theta - k;$$
$$f_2(\varphi_1, S_3) = S_1 \sin\varphi_1 - S_3 \sin\theta - d \cdot \cos\theta - h;$$

are the functions of the nonlinear equations system, and

$$W = \begin{Vmatrix} -S_1 \sin\varphi_1 & -\cos\theta \\ AB\cos\varphi_1 & -\cos\theta \end{Vmatrix}$$

is the functional matrix [Jacobian matrix] of the system.

The iterative calculation process stops when the difference of two consecutive calculated solutions is less than an imposed ε,

$$\left|\varphi_1^{i+1} - \varphi_1^{i}\right| \le \varepsilon; \qquad \left|S_3^{i+1} - S_3^{i}\right| \le \varepsilon.$$

After determining the unknowns φ_1 and S_3, the exact coordinates of point *C* are calculated, using equations:

$$\begin{aligned} XC &= XA + S_1 \cos\varphi_1; \\ YC &= YA + S_1 \sin\varphi_1. \end{aligned} \tag{3}$$

2.2. Velocities analysis of the active group *RTaRT*

The position equations system (1) is derived with respect to time, to yield:

$$\begin{cases} -S_1 \sin\varphi_1 . \dot{\varphi}_1 - \dot{S}_3 \cos\theta . = A1; \\ S_1 \cos\varphi_1 . \dot{\varphi}_1 - \dot{S}_3 \cos\theta = A2, \end{cases} \tag{4}$$

where: $A1 = \dot{k} - \dot{S}_1 \cos\varphi_1 - (S_3 \sin\theta + d\cos\theta)\dot{\theta}$;

$A2 = \dot{h} - \dot{S}_1 \sin\varphi_1 + (S_3 \cos\theta - d\sin\theta)\dot{\theta}$;

$\dot{k} = \dot{X}D - \dot{X}A$; $\dot{h} = \dot{Y}D - \dot{Y}A$.

The system is linear with the unknowns $\dot{\varphi}_1$ and $\dot{S}_3$. Using the method of the inverse matrix, is obtained:

$$\begin{Vmatrix} \dot{\varphi}_1 \\ \dot{S}_3 \end{Vmatrix} = W^{-1} \begin{Vmatrix} A1 \\ A2 \end{Vmatrix}. \tag{5}$$

The projections of the linear velocity of point *C* are:

$\dot{X}C = \dot{X}A + \dot{S}_1 \cos\varphi_1 - S_1 \cdot \dot{\varphi}_1 \sin\varphi_1$;

$\dot{Y}C = \dot{Y}A + \dot{S}_1 \sin\varphi_1 + S_1 \cdot \dot{\varphi}_1 \cos\varphi_1$.

2.3. Acceleration analysis of the motodyad ***RTaRT***

By deriving the system of velocity equations (4) with respect to time, it results:

$$\begin{cases} -S_1 \sin\varphi_1 . \ddot{\varphi}_1 - \ddot{S}_3 \cos\theta . = B1; \\ S_1 \cos\varphi_1 . \ddot{\varphi}_1 - \ddot{S}_3 \sin\theta = B2, \end{cases}, \tag{6}$$

where:

$$B1 = \ddot{k} - \ddot{S}_1 \cos\varphi_1 + 2\dot{S}_1 \dot{\varphi}_1 \sin\varphi_1 - (S_3 \sin\theta + d\cos\theta)\ddot{\theta} -$$
$$- 2\dot{S}_3\dot{\theta}\sin\theta - (S_3\cos\theta - d\sin\theta)\dot{\theta}^2 + S_1\dot{\varphi}_1^2 \cos\varphi_1;$$

$$B2 = \ddot{h} - \ddot{S}_1 \sin\varphi_1 - 2\dot{S}_1 \dot{\varphi}_1 \cos\varphi_1 + (S_3\cos\theta - d\sin\theta)\ddot{\theta} +$$
$$+ 2\dot{S}_3\dot{\theta}\cos\theta - (S_3 \sin\theta + d\cos\theta)\dot{\theta}^2 + S_1\dot{\varphi}_1^2 \sin\varphi_1;$$

$\ddot{k} = \ddot{X}P - \ddot{X}A$; $\ddot{h} = \ddot{Y}P - \ddot{Y}A$.

The system is linear with the unknowns $\ddot{\varphi}_1$ and $\ddot{\varphi}_2$, so it results:

$$\begin{Vmatrix} \ddot{\varphi}_1 \\ \ddot{S}_3 \end{Vmatrix} = W^{-1} \begin{Vmatrix} B1 \\ B2 \end{Vmatrix} \tag{7}$$

The projections of the linear acceleration of the point C are:

$$\ddot{X}C = \ddot{X}A + \ddot{S}_1 \cos\varphi_1 - 2\dot{S}_1 \cdot \dot{\varphi}_1 \sin\varphi_1 - S_1 \cdot \dot{\varphi}_1^2 \cos\varphi_1 - S_1 \cdot \ddot{\varphi}_1 \sin\varphi_1;$$

$$\ddot{Y}C = \ddot{Y}A + \ddot{S}_1 \sin\varphi_1 + 2\dot{S}_1 \cdot \dot{\varphi}_1 \sin\varphi_1 - S_1 \cdot \dot{\varphi}_1^2 \sin\varphi_1 + S_1 \cdot \ddot{\varphi}_1 \cos\varphi_1;$$

Accordingly to the presented algorithm, a computational function was generated using MATLAB®, function that will be called and accesed by the main computational program, to determine the kinematic parameters of the motor group *RTaRT*.

The definition of this function is:

function [fi1,s3,C] = A2PVA_T(A,P,s1,th,fi1,s3,d).

The output parameters have the following meanings:

φ_1 – the vector that contains the angle between the vector attached to segment *AC* and the positive sense of *OX axis*, the angular velocity of the links 1 and 2, as well as the angular acceleration of the same links;

S_3 – the vector that contains the distance between the points *P* and *D* (in the pozitive sense of θ), the relative velocity of the prismatic joint *D*, as well as the relative acceleration of the same joint;

C – the vector that contains the components of the positions, the velocities and the accelerations of the point *C* in the coordinate system;

The input parameters have the following meanings:

A – the vector that contains the components of the positions, the velocities and the accelerations of the point *A* in the coordinate system;

P – the vector that contains the components of the positions, the velocities and the accelerations of the point *P* in the coordinate system;

S_1 – the vector that contains the independent parameter from the active joint *B*, the relative velocity between the links 1 and 2, as well as the angular acceleration of the same links;

θ – vector that contains the angle between the direction of linear motion of joint *B* and the pozitive sense of the *OX* axis, the angular velocity of S_3 (the direction of linear motion of joint *D*), as well as the angular acceleration of S_3;

φ_1 – the approximate angle between the vector attached to segment *AC* with the sense of the *OX* axis;

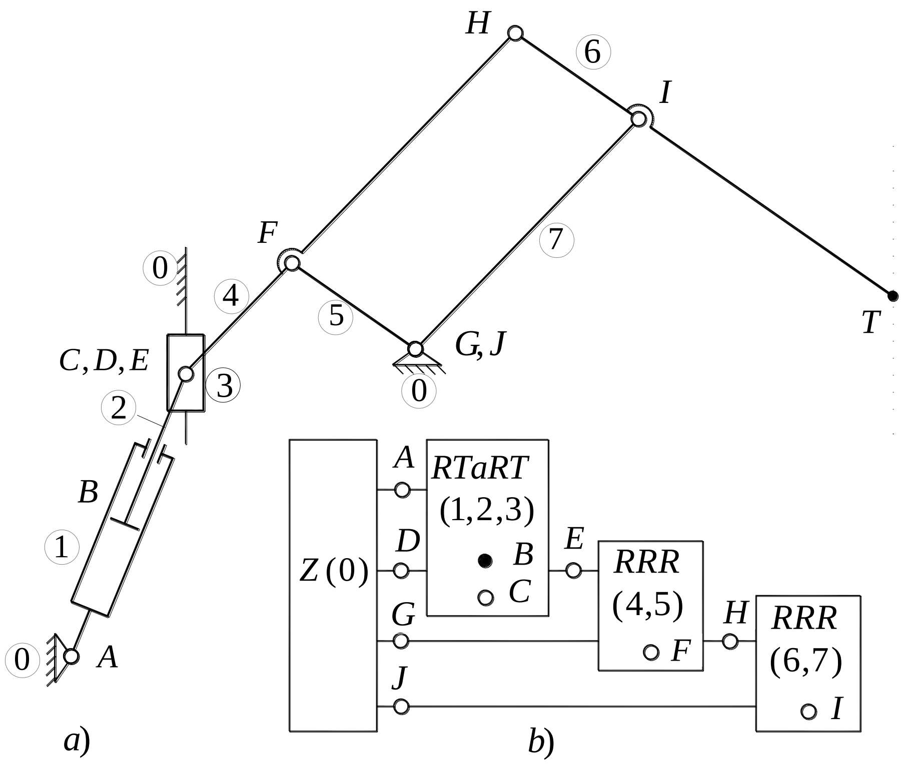
S_3 – the approximate distance between the points *P* and *D*;

d – the length of the perpendicular from point *C* on the *PD* line.

3. Computational example

A pantograph mechanism actuated by the motor group *RTaRT* is considered. The kinematic scheme of the mechanism is presented in Figure 2.a.

As it can be seen from the multipolar scheme (Figure 2.b) and from the structural relation (Figure 2.c), the mechanism is formed from the base $Z(0)$, the motor group $RTaRT(1,2,3)$, and the 1st aspect dyads: $RRR(4,5)$, $RRR(6,7)$.



$$Z(0) + RTaRT(1,2,3) + RRR(4,5) + RRR(6,7)$$

c)

Fig. 2. Kinematic scheme: *a*), multipolar scheme *b*), structural relation *c*)

The kinematic parameters are emphasized in the kinematic scheme from Figure 3.

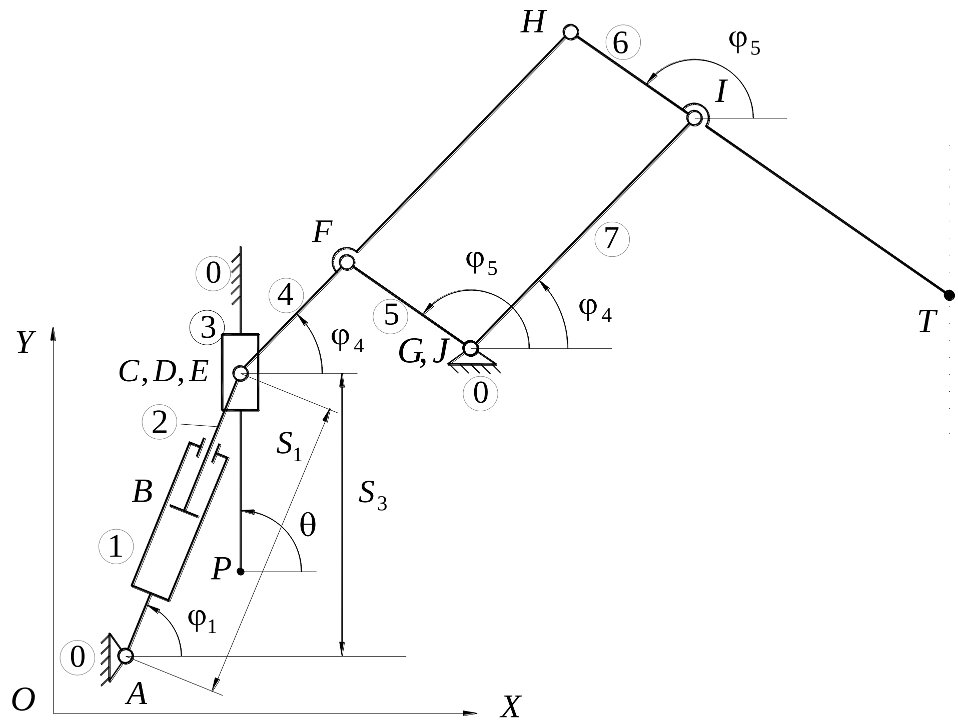


Fig. 3. The kinematic scheme of the mechanism, with the emphasis of the kinematic position parameters

For the kinematic analysis of the mechanism, are known:

- $XA = 0$; $YA = 0$; $XG = 0.7$ [m]; $YG = 0.75$ [m]; [m]; $YP = 0.2$ [m] - coordinates of the points *A*, *G*, *P*;
- $CP = 0.3$ [m] – the piston stroke;
- $S_{10} = 0.5$ [m]- the initial distance between points *A* and *C*;
- S_1 - the independent parameter in the prismatic joint *B*;
- $\dot{S}_1$ - the relative velocity between links 1 and 2;
- $\ddot{S}_1$ - the relative acceleration between links 1 and 2;
- θ - the angle between the OX axis unit vector and the direction of motion of link 3;
- $\dot{\theta}$ - the first derivative of angle θ;
- $\ddot{\theta}$ - the second derivative angle θ;
- $EF = 0.3$ [m]; $FG = 0.4$ [m]; $EH = 1.3$ [m]; $HI = 0.4$ [m]; $IJ = 1$ [m]; $HT = 1.7333$ [m] – the kinematic dimensions of the mechanisms' links.

The kinematic parameters of the mechanism are required.

Using the procedure proposed by the authors for the modular group *RTaRT*, as for the proper procedure of the dyad *RRR* [7, 12], a computational program has been accomplished in MATLAB®.

In table 1 are presented the kinematic parameters of the motor group *RTaRT*, which depends on the independent parameter s_1.

Table 1

poz	s1	fi1	om1	eps1	s3	vs3	as3
0	0.5000	1.1593	0.0873	-0.0382	0.4583	0.1091	-0.0042
1	0.5300	1.1838	0.0769	-0.0314	0.4908	0.1080	-0.0034
2	0.5600	1.2056	0.0683	-0.0262	0.5231	0.1071	-0.0028
3	0.5900	1.2250	0.0611	-0.0220	0.5551	0.1063	-0.0023
4	0.6200	1.2423	0.0550	-0.0188	0.5869	0.1056	-0.0020
5	0.6500	1.2580	0.0498	-0.0161	0.6185	0.1051	-0.0017
6	0.6800	1.2723	0.0453	-0.0139	0.6499	0.1046	-0.0015
7	0.7100	1.2852	0.0413	-0.0121	0.6812	0.1042	-0.0013
8	0.7400	1.2971	0.0379	-0.0107	0.7125	0.1039	-0.0011
9	0.7700	1.3080	0.0349	-0.0094	0.7436	0.1036	-0.0010
10	0.8000	1.3181	0.0323	-0.0083	0.7746	0.1033	-0.0009

In table 2 are presented the kinematic parameters of the pantograph mechanism.

Table 2

poz	s1	fi4	om4	eps4	fi5	om5	eps5
0	0.5000	1.3851	-0.1964	-0.0938	2.6090	-0.2851	0.0122
1	0.5300	1.3221	-0.2234	-0.0862	2.5241	-0.2805	0.0185
2	0.5600	1.2513	-0.2479	-0.0766	2.4409	-0.2738	0.0268
3	0.5900	1.1737	-0.2692	-0.0653	2.3602	-0.2643	0.0362
4	0.6200	1.0902	-0.2869	-0.0530	2.2826	-0.2520	0.0459
5	0.6500	1.0019	-0.3010	-0.0407	2.2092	-0.2368	0.0552
6	0.6800	0.9099	-0.3114	-0.0292	2.1408	-0.2190	0.0632
7	0.7100	0.8153	-0.3187	-0.0193	2.0780	-0.1990	0.0697
8	0.7400	0.7190	-0.3232	-0.0113	2.0215	-0.1774	0.0745
9	0.7700	0.6216	-0.3257	-0.0058	1.9717	-0.1545	0.0776
10	0.8000	0.5237	-0.3269	-0.0026	1.9289	-0.1309	0.0795

The variations of the angles φ_1, φ_4 and φ_5 are presented in Figure 4.

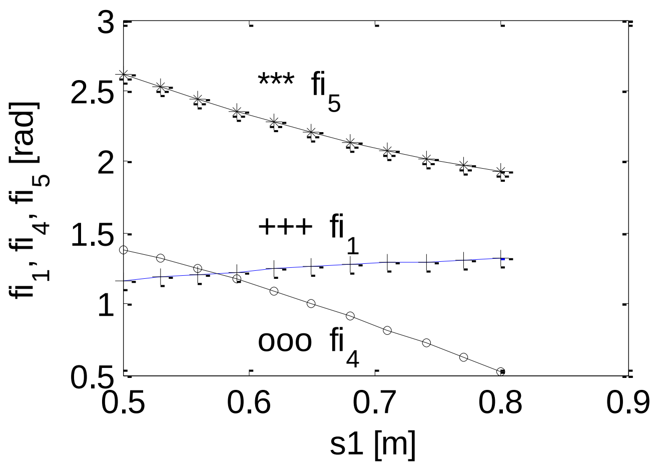


Fig. 4. The variations of the angles φ_1, φ_4 and φ_5

The variations of the angular velocities ω_1, ω_4 and ω_5 are shown in Figure 5

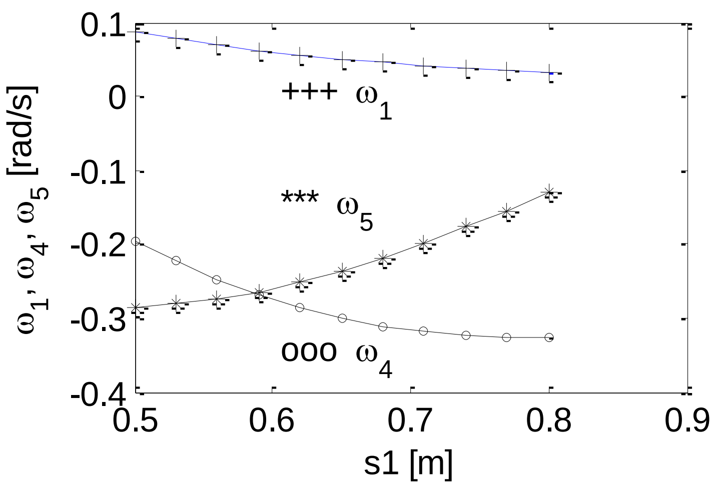


Fig. 5. The variations of the angular velocities ω_1, ω_4 and ω_5

In Figure 6, the variations of the angular accelerations ε_1, ε_4 and ε_5 are displayed.

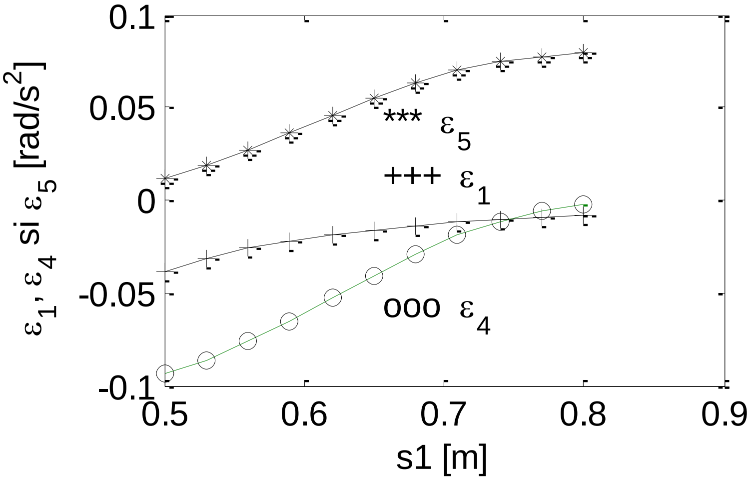


Fig. 6. The variations of the angular accelerations ε_1, ε_4 and ε_5.

4. Conclusions

Using the closed-loops method the kinematic analysis of the motor group *RTaRT* has been fulfilled. This modular group can be found in the composition of different mechanisms used in many areas of activity such as: the actuation of some agricultural machinery, the actuation of some folding tables, the actuation of hydraulic excavators etc.

The computational procedure, completed in MATLAB®, simplifies a lot the kinematic analysis of these mechanisms.

The results were validated in Turbo-Pascal, C++ and Matlab programs. In order to get a clearer picture on the analyzed mechanism, a simulation of its functionality was made using Matlab, and the results were the same as those obtained from numerical calculations.

R E F E R E N C E S

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