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SOFT M-SYSTEMS IN A CLASS OF SOFT NON-ASSOCIATIVE RINGS

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Molodtsov introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainty. In this paper, we study soft non-associative rings and explore some of its algebraic properties. The notions of soft M-systems, soft P-systems, soft I-systems, soft Quasi-prime ideals, soft Quasi-semiprime ideals, soft Irreducible and soft Strongly irreducible ideals are introduced and several related properties are investigated.

Keywords: Soft LA-ring, Soft M-system, Soft P-system, Soft Quasi prime, Soft strongly irreducible ideals.
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1. Introduction

If we look into past, we can see many theories which have been developed to deal with uncertainties. For instance, in [28], the theory of fuzzy sets has been introduced and later on, the theories like, intuitionistic fuzzy sets [4], vague sets and interval mathematics [5], [8] and rough sets [14] were explored by many researchers. Though many techniques have been developed as a result of these theories, yet difficulties are seem to be there.

In [16], D. Molodtsov introduced the concept of soft set theory and it has received much attention since its inception. Molodtsov presented the fundamental results of new theory and successfully applied it into several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability etc. A soft set is a collection of approximate description of an object. He also showed how soft set theory is free from parametrization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. Soft systems provide a very general framework with the involvement of parameters. Research works on soft set theory and its applications in various fields are progressing rapidly in these years.

An application of soft sets in decision making problems was presented by Maji [14], [15], and was based on the reduction of parameters to keep the optimal choice

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objects. Chen [6] presented a new definition of soft set parametrization reduction and a comparison of it with attributes reduction in rough set theory. Pei and Miao [17] showed that soft sets are a class of special information systems. Kong [13] introduced the notion of normal parameter reduction of soft sets and its use to investigate the problem of sub-optimal choice and added parameter set in soft sets. In [3], some new operations are defined on soft sets and some old operations are redefined. Application of soft set theory in algebraic structures was introduced by Aktaş and Çağman [2]. They discussed the notion of soft groups and derived some basic properties. They also showed that soft groups extends the concept of fuzzy groups. Jun [9], [10] investigated soft BCK/BCI-algebras and its application in ideal theory.

The concept of Left almost semigroups (LA-semigroups) has been introduced by Kazim and Naseeruddin [12]. A groupoid S is called an LA-semigroup if it satisfies the left invertive law: $(ab)c = (cb)a$ for all $a, b, c \in S$. This structure is also known as Abel-Grassmann's groupoid (abbreviated as AG-groupoid) [18, 19]. An AG-groupoid is the midway structure between a commutative semigroup and a groupoid. Later, in [11], Kamran extended the notion of LA-semigroup to left almost group (LA-group). A groupoid G is called a left almost group (LA-group), if there exists left identity $e \in G$ (that is $ea = a$ for all $a \in G$), for $a \in G$ there exists $b \in G$ such that $ba = e$ and left invertive law holds in G.

Left Almost Ring (LA-ring) is actually an off shoot of LA-semigroup and LA-group. It is a non-commutative and non-associative structure and gradually due to its peculiar characteristics it has been emerging as useful non-associative class which intuitively would have reasonable contribution to enhance non-associative ring theory. By an LA-ring, we mean a non-empty set R with at least two elements such that $(R, +)$ is an LA-group, $(R, \cdot)$ is an LA-semigroup, both left and right distributive laws hold. For example, from a commutative ring $(R, +, \cdot)$, we can always obtain an LA-ring $(R, \oplus, \cdot)$ by defining for all $a, b \in R$, $a \oplus b = b - a$ and $a \cdot b$ is same as in the ring.

In [25], Shah and Rehman have discussed left almost ring (LA-ring) of finitely nonzero functions which is in fact a generalization of a commutative semigroup ring. Recently Shah and Rehman [26], discussed some properties of non-associative rings through their ideals and intuitively ideal theory would be a gate way for investigating the application of fuzzy sets, intuitionistics fuzzy sets and soft sets in non-associative-rings. For example, Shah et al. [23], have applied the concept of intuitionistic fuzzy sets and established some useful results. In [21], some computational work through Mace4, has been done and some interesting characteristics of non-associative-rings have been explored. In [22], some concepts of soft sets are applied on non-associative rings and its ideals. For some more study of non-associative-rings, we refer the readers to see ([20], [27], [24]).

In this paper, by introducing soft M-system, soft P-system and related results in soft non-associative-rings, we make a new approach to apply the Molodstove's soft set theory to a class of non-associative rings and its ideals. We do provide number of examples to illustrate the concepts of soft M-system, soft P-system and soft I-system in soft non-associative-rings.

2. **Preliminaries**

In this section, we recall some basic notions relevant to soft sets.

Definition 2.1. [16] *Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.*

In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of $\varepsilon-$approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Definition 2.2. [15] *For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if*
(i) $A \subseteq B$ and
(ii) for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations.

We write $(F, A)\widetilde{\subset}(G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A)\widetilde{\supset}(G, B)$.

Definition 2.3. [15] *Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).*

Definition 2.4. [15] *A soft set (F, A) over U is said to be a $NULL$ soft set denoted by Φ if for all $\varepsilon \in A$, $F(\varepsilon) = \emptyset$ (null set).*

Definition 2.5. [1] *Let (μ, A) and (σ, B) be soft sets over a common universe U. The bi-intersection of (μ, A) and (σ, B) is defined as the soft set (ρ, C) satisfying the following conditions:*
(i) $C = A \cap B$.
(ii) For all $x \in C$, $\rho(x) = \mu(x) \cap \sigma(x)$.

In this case, we write $(\mu, A) \,\widetilde{\sqcap}\, (\sigma, B) = (\rho, C)$.

Definition 2.6. [7] *Let $(\mu_i, A_i)_{i \in I}$ be a nonempty family of soft sets over a common universe U. The bi-intersection of these soft sets is defined to be the soft set (ρ, C) such that $C = \cap_{i\in I} A_i$, and $\rho(x) = \cap_{i\in I}\mu_i$ for all $x \in C$. So here we write $\widetilde{\sqcap}_{i\in I}(\mu_i, A_i) = (\rho, C)$.*

Definition 2.7. [1] *Let (β, A) and (γ, B) be soft sets over a common universe U. The union of (β, A) and (γ, B) is defined as the soft set (ψ, C) satisfying the following conditions:*
(i) $C = A \cup B$.
(ii) For all $x \in C$,

$$\psi(x) = \begin{cases} \beta(x) & \text{if } x \in A - B, \\ \gamma(x) & \text{if } x \in B - A, \\ \beta(x) \cup \gamma(x) & \text{if } x \in A \cap B. \end{cases}$$

In this case, it is denoted by $(\beta, A) \widetilde{\cup} (\gamma, B) = (\psi, C)$.

Definition 2.8. [1] *Let $(\alpha_i, A_i)_{i\epsilon I}$ be a nonempty family of soft sets over a common universe U. The union of these soft sets is defined as the soft set (β, C) satisfying the followings conditions:*

(i) $C = \cup_{i_\epsilon I} A_i$.

(ii) For all $x \in B$, $\beta(x) = \cup_{i\epsilon I}\alpha_i(x)$, where $I(x) = \{i \in I \mid x \in A_i\}$.

It is denoted by $\widetilde{\cup}_{i\epsilon I}(\alpha_i, A_i) = (\beta, C)$.

Definition 2.9. [1] *Let (α, A) and (β, B) be soft sets over a common universe U, then "(α, A) AND (β, B)" denoted by $(\alpha, A) \widetilde{\wedge} (\beta, B)$ is defined as $(\alpha, A) \widetilde{\wedge} (\beta, B) = (\gamma, C)$, where $C = A \times B$ and $\gamma(x, y) = \alpha(x) \cap \beta(y)$ for all $(x, y) \in C$.*

Definition 2.10. [1] *Let (η, A) and (ρ, B) be soft sets over a common universe U, then "(η, A) OR (ρ, B)" denoted by $(\eta, A) \widetilde{\vee} (\beta, B)$ is defined as $(\eta, A) \widetilde{\vee} (\beta, B) = (\xi, C)$, where $C = A \times B$ and $\xi(x, y) = \eta(x) \cup \beta(y)$ for all $(x, y) \in C$.*

Definition 2.11. [7] *Let $(\alpha_i, A_i)_{i\in I}$ be a nonempty family of soft sets over a common universe U. The AND-soft set $\widetilde{\wedge}_{i\in I}(\alpha_i, A_i)$ of these soft sets is defined to be the soft set (β, B) such that $B = \Pi_{i\in I} A_i$ and $\beta(x) = \cap_{i\in I}\alpha_i(x_i)$ for all $x = (x_i)_{i\in I} \in B$.*

Similarly, the OR-soft set $\widetilde{\vee}_{i\in I}(\alpha_i, A_i)$ of these soft sets is defined to be the soft set (γ, B) such that $B = \Pi_{i\in I} A_i$ and $\gamma(x) = \cup_{i\in I}\alpha_i(x_i)$ for all $x = (x_i)_{i\in I} \in B$.

Definition 2.12. [7] *Let (α, A) be a soft set .The set supp $(\alpha, A) = \{x \in A \mid \alpha(x) \neq \phi\}$ is called the support of the soft set (α, A). A soft set is said to be non null if its support is not equal to the empty set.*

3. **Soft M-systems, Soft P-systems, and Soft I-systems in Soft LA-rings**

In this section, we discuss soft M-system, soft P-system, and soft I-system in soft LA-ring (F, A). We prove the equivalent conditions for soft left ideal to be soft M-system, soft P-system, soft I-system and establish that every soft M-system of elements of soft LA ring (F, A) is soft P-system.

Definition 3.1. *Let (F, A) be a soft LA-ring over R. A non null soft set (ξ, M) is said to be soft M-system if it satisfies the following conditions:*

(i) (ξ, M) is subset of soft LA-ring (F, A).

(ii) For $\xi(a), \xi(b) \in (\xi, M)$ then $\exists$ $F(x)$ in (F, A) such that $\xi(a)(F(x)\xi(b)) \in (\xi, M)$.

Throughout in this paper, R is considered to be an LA-ring.

Example 3.1. *Let $R = A = \{0,1,2,3,4,5,6,7\}$ is an LA-ring. Now consider the set-valued function $F : \{0,1,2,3,4,5,6,7\} \longrightarrow P(R)$ given by $F(x) = \{y \in R \mid x \cdot y \in \{0,4\}\}$*

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	2	0	3	1	6	4	7	5
2	1	3	0	2	5	7	4	6
3	3	2	1	0	7	6	5	4
4	4	5	6	7	0	1	2	3
5	6	4	7	5	2	0	3	1
6	5	7	4	6	1	3	0	2
7	7	6	5	4	3	2	1	0

·	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	4	4	0	0	4	4	0
2	0	4	4	0	0	4	4	0
3	0	0	0	0	0	0	0	0
4	0	3	3	0	0	3	3	0
5	0	7	7	0	0	7	7	0
6	0	7	7	0	0	7	7	0
7	0	3	3	0	0	3	3	0

Then $F(0) = F(1) = F(2) = F(3) = \{0,1,2,3,4,5,6,7\}$ and $F(4) = F(5) = F(6) = F(7) = \{0,3,4,7\}$, which are sub LA-rings of R and hence (F,A) is soft LA-ring over R. Let $M = \{0,2,3,4,7\}$ and consider the set valued function $\xi : M \longrightarrow P(R)$ given by $\xi : \{y \in R \mid x \cdot y \in \{0,2,4\}\}$. Then $\xi(0) = \xi(2) = \xi(3) = \{0,1,2,3,4,5,6,7\}$ and $\xi(4) = \xi(7) = \{0,3,4,7\}$. Here it can be seen that if $\xi(a), \xi(b) \in (\xi, M)$ then $\exists$ $F(x)$ in (F,A) such that $\xi(a)(F(x)\xi(b)) \in (\xi, M)$. Hence (ξ, M) is a soft M-system over a soft LA-ring (F,A).

Definition 3.2. *Let (ϕ, I) be a soft left ideal of soft LA-ring (F,A). Then (ϕ, I) is said to be soft quasi-prime ideal if $(\phi_1, H)(\phi_2, K) \subseteq (\phi, I)$ implies that either $(\phi_1, H) \subseteq (\phi, I)$ or $(\phi_2, K) \subseteq (\phi, I)$, where (ϕ_1, H) and (ϕ_2, K) are any soft left ideals of (F,A). If for any soft left ideal (ϕ_1, H) of (F,A) such that $(\phi_1, H)^2 \subseteq (\phi, I)$, we have $(\phi_1, H) \subseteq (\phi, I)$, then (ϕ, I) is said to be soft quasi-semiprime ideal of soft LA-ring (F,A).*

Example 3.2. *Let $R = A = \{0,1,2,3,4,5,6,7\}$ is an LA-ring. Now consider the set-valued function $F : \{0,1,2,3,4,5,6,7\} \longrightarrow P(R)$ defined by $F(x) = \{y \in R \mid x \cdot y \in \{0,4\}\}$*

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	0	3	2	6	7	4	5
2	2	3	0	1	5	4	7	6
3	3	2	1	0	7	6	5	4
4	4	6	5	7	0	2	1	3
5	5	7	4	6	2	0	3	1
6	6	4	7	5	1	3	0	2
7	7	5	6	4	3	1	2	0

·	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	2	0	2	0	0	2	2
2	0	4	0	4	0	0	4	4
3	0	5	0	5	0	0	5	5
4	0	0	0	0	0	0	0	0
5	0	4	0	4	0	0	4	4
6	0	2	0	2	0	0	2	2
7	0	5	0	5	0	0	5	5

Then $F(0) = F(2) = F(4) = F(5) = \{0,1,2,3,4,5,6,7\}$, $F(1) = F(3) = F(6) = F(7) = \{0,2,4,5\}$, which are sub LA-rings of R and hence (F,A) is soft LA-ring over R. Let $I = \{0,2,4\}$. Now consider the set valued function $\phi : I \longrightarrow P(R)$

given by $\phi : \{y \in R \mid 3x + y \in \{0, 2\}\}$. Then $\phi(0) = \phi(2) = \phi(4) = \{0, 2\}$. Here it is observed that (ϕ, I) is a soft left ideal of soft LA-ring (F, A). Now consider $H = \{0, 2\}$ and $K = \{0, 5\}$ defined by the set valued functions $\phi_1 : H \longrightarrow P(R)$ and $\phi_2 : K \longrightarrow P(R)$ respectively. Given by $\phi_1 : \{y \in R \mid 3x + y \in \{0, 2\}\}$ and $\phi_2 : \{y \in R \mid 3x+y \in \{0, 5\}\}$ respectively. Then $\phi_1(0) = \phi_1(2) = \{0, 2\}$ and $\phi_2(0) = \phi_2(2) = \{0, 5\}$. Here it can be easily observed that (ϕ_1, H) and (ϕ_2, K) are any soft left ideals of a soft LA-ring (F, A). Also it can be seen that if $(\phi_1, H)(\phi_2, K) \subseteq (\phi, I)$ implies that either $(\phi_1, H) \subseteq (\phi, I)$ or $(\phi_2, K) \subseteq (\phi, I)$, where (ϕ_1, H) and (ϕ_2, K) are any soft left ideals of (F, A). Hence (ϕ, I) is a soft quasi-prime ideal over a soft LA-ring (F, A).

Proposition 3.1. *Let (ϕ, I) be a soft left ideal of (F, A), then the following statements are equivalent:*

(1) (ϕ, I) *is soft quasi-prime ideal.*

(2) $(\phi_1, H)(\phi_2, K) = \langle(\phi_1, H)(\phi_2, K)\rangle \subseteq (\phi, I)$ *which implies that either* $(\phi_1, H) \subseteq (\phi, I)$ *or* $(\phi_2, K) \subseteq (\phi, I)$, *where* (ϕ_1, H) *and* (ϕ_2, K) *are any soft left ideals of* (F, A).

(3) *If* $(\phi_1, H) \nsubseteq (\phi, I)$ *and* $(\phi_2, K) \nsubseteq (\phi, I)$, *then* $(\phi_1, H)(\phi_2, K) \nsubseteq (\phi, I)$, *where* (ϕ_1, H) *and* (ϕ_2, K) *are any soft left ideals of* (F, A).

(4) *If* $F(a), F(b)$ *are elements of* (F, A) *such that* $F(a) \notin I$ *and* $F(b) \notin I$, *then* $\langle F(a)\rangle\langle F(b)\rangle \nsubseteq (\phi, I)$.

(5) *If* $F(a), F(b)$ *are elements of* (F, A) *satisfying* $F(a)((F, A)F(b)) \subseteq (\phi, I)$, *implies that either* $F(a) \in (\phi, I)$ *or* $F(b) \in (\phi, I)$.

Proof. (1) $\Leftrightarrow$ (2) Let (ϕ, I) is soft quasi-prime ideal. Now by Definition 3.2, if $(\phi_1, H)(\phi_2, K) = \langle(\phi_1, H)(\phi_2, K)\rangle \subseteq (\phi, I)$, then it implies that either $(\phi_1, H) \subseteq (\phi, I)$ or $(\phi_2, K) \subseteq (\phi, I)$ for all soft left ideals (ϕ_1, H) and (ϕ_2, K) of a soft LA-ring (F, A).. Converse is trivial by Definition 3.2. (2) $\Leftrightarrow$ (3) is obvious. (1) $\Rightarrow$ (4) Let (ϕ, I) is soft quasi-prime ideal, if $\langle F(a)\rangle\langle F(b)\rangle \subseteq (\phi, I)$, then by hypothesis $\langle F(a)\rangle \subseteq (\phi, I)$ or $\langle F(b)\rangle \subseteq (\phi, I)$, which further implies that either $F(a) \in (\phi, I)$ or $F(b) \in (\phi, I)$, then clearly we can say that if $F(a), F(b)$ are elements of (F, A) such that $F(a) \notin I$ and $F(b) \notin I$ then $\langle F(a)\rangle\langle F(b)\rangle \nsubseteq (\phi, I)$. (4) $\Rightarrow$ (2) Let $(\phi_1, H)(\phi_2, K) \subseteq (\phi, I)$. If $F(a) \in (\phi_1, H)$ and $F(b) \in (\phi_2, K)$, then $\langle F(a)\rangle\langle F(b)\rangle \subseteq (\phi, I)$ and hence it implies that either $F(a) \in (\phi, I)$ or $F(b) \in (\phi, I)$. This implies either $(\phi_1, H) \subseteq (\phi, I)$ or $(\phi_2, K) \subseteq (\phi, I)$. (1) $\Leftrightarrow$ (5) Let $F(a)((F, A)F(b)) \subseteq (\phi, I)$, then $(F, A)(F(a)((F, A)F(b))) \subseteq (F, A)(\phi, I) \subseteq (\phi, I)$. Now consider

$$\begin{aligned}
(F, A)[F(a)\{(F, A)F(b)\}] &= [(F, A)(F, A)][F(a)\{(F, A)F(b)\}] \\
&= [(F, A)F(a)][(F, A)\{(F, A)F(b)\}], \text{ by medial law} \\
&= [(F, A)F(a)][\{(F, A)(F, A)\}\{(F, A)F(b)\}] \\
&= [(F, A)F(a)][F(b)(F, A)][(F, A)(F, A)], \text{ by paramedial law} \\
&= [(F, A)F(a)][\{(F, A)(F, A)\}F(b)], \text{ by left invertive law} \\
&= [(F, A)F(a)][(F, A)F(b)] \subseteq (\phi, I).
\end{aligned}$$

Since $(F,A)F(a)$ and $(F,A)F(b)$ are soft left ideals for all $F(a)\in(\phi_1,H)$ and $F(b)\in(\phi_2,K)$, hence either $F(a)\in(\phi,I)$ or $F(b)\in(\phi,I)$. Conversely, let $(\phi_1,H)(\phi_2,K)\subseteq(\phi,I)$ where (ϕ_1,H) and (ϕ_2,K) are any soft left ideals of soft LA-ring (F,A). Let $(\phi_1,H)\nsubseteq(\phi,I)$ then there exists $F(c)\in(\phi_1,H)$ such that $F(c)\notin(\phi,I)$. For all $F(d)\in(\phi_2,K)$, we have $F(c)((F,A)F(d))\subseteq(\phi_1,H)((F,A)(\phi_2,K))\subseteq(\phi_1,H)(\phi_2,K)\subseteq(\phi,I)$. This implies that $(\phi_2,K)\subseteq(\phi,I)$ and hence (ϕ,I) is a soft quasi-prime ideal of (F,A). □

Proposition 3.2. *A soft left ideal (ϕ,I) of soft LA-ring (F,A) is soft quasi-prime if and only if $(F,A)\backslash(\phi,I)$ is soft M-system.*

Proof. Suppose (ϕ,I) is a soft quasi-prime ideal. Let $\alpha,\beta\in(F,A)\backslash(\phi,I)$ which implies that $\alpha\notin(\phi,I)$ and $\beta\notin(\phi,I)$. So by Proposition 3.1, $\alpha((F,A)\beta)\nsubseteq(\phi,I)$. This implies that there exists some $F(r)\in(F,A)$ such that $\alpha(F(r)\beta)\notin(\phi,I)$ which further implies that $\alpha(F(r)\beta)\in(F,A)\backslash(\phi,I)$. Hence $(F,A)\backslash(\phi,I)$ is a soft M-system. Conversely, let $(F,A)\backslash(\phi,I)$ is a soft M-system. Suppose that $\alpha(F(r)\beta)\subseteq(\phi,I)$ and let $\alpha\notin(\phi,I)$ and $\beta\notin(\phi,I)$. This implies that $\alpha,\beta\in(F,A)\backslash(\phi,I)$. Since $(F,A)\backslash(\phi,I)$ is a soft M-system so there exists $F(r)\in(F,A)$ such that $\alpha(F(r)\beta)\in(F,A)\backslash(\phi,I)$ which implies that $\alpha((F,A)\beta)\nsubseteq(\phi,I)$, which is a contradiction. Hence either $\alpha\in(\phi,I)$ or $\beta\in(\phi,I)$. This shows that (ϕ,I) is a soft quasi-prime ideal. □

Definition 3.3. *A nonempty subset (ρ,I) of a soft LA-ring (F,A) is called soft P-system if for all $\rho(a)\in(\rho,I)$, there exists $F(r)\in(F,A)$ such that $\rho(a)(F(r)\rho(a))\in(\rho,I)$.*

Example 3.3. *Let $R=\{0,1,2,3,4,5,6,7,8\}$ be an LA-ring of order 9, as defined by the cayley table given below. Let we take $A=R=\{0,1,2,3,4,5,6,7,8\}$ and $I=\{0,2,4\}\subset A$. Define a set-valued function $F:A\longrightarrow P(R)$ by $F(x)=\{y\in R\mid x\cdot y\in\{0,3,8\}\}$.*

+	0	1	2	3	4	5	6	7	8
0	3	4	6	8	7	2	5	1	0
1	2	3	7	6	8	4	1	0	5
2	1	5	3	4	2	0	8	6	7
3	0	1	2	3	4	5	6	7	8
4	5	0	4	2	3	1	7	8	6
5	4	2	8	7	6	3	0	5	1
6	7	6	0	1	5	8	3	2	4
7	6	8	1	5	0	7	4	3	2
8	8	7	5	0	1	6	2	4	3

·	0	1	2	3	4	5	6	7	8
0	3	1	6	3	1	6	6	1	3
1	0	3	0	3	8	8	3	0	8
2	8	1	5	3	7	2	6	4	0
3	3	3	3	3	3	3	3	3	3
4	0	6	7	3	5	4	1	2	8
5	8	6	4	3	2	7	1	5	0
6	8	3	8	3	0	0	3	8	0
7	0	1	2	3	4	5	6	7	8
8	3	6	1	3	6	1	1	6	3

From table, $F(0)=F(1)=F(2)=F(4)=F(5)=F(7)=F(8)=\{0,3,8\}$ and $F(3)=F(6)=\{0,1,2,3,4,5,6,7,8\}$. It can be easily seen that all these sets are subLA-rings over R. Hence (F,A) is soft LA-ring over R. On the other hand consider the mapping $\varphi:I\longrightarrow P(R)$ defined by $\varphi(x)=\{y\in R\mid xy+y=3\}\}$. Then $\varphi(0)=\varphi(2)=\{1,3,6\}$ and $\varphi(4)=\{0,3,8\}$. Here it can be seen that if

$\varphi(a) \in (\varphi, I)$, then $\exists\ F(r)$ in (F, A) such that $\varphi(a)(F(r)\varphi(a)) \in (\varphi, I)$. Hence (φ, I) is a soft P-system over a soft LA-ring (F, A).

Proposition 3.3. *Let (ϕ, I) be a soft left ideal of soft LA-ring (F, A), then the following assertions are equivalent:*

(1) *(ϕ, I) is a soft quasi-semiprime.*

(2) *$(\phi_1, H)^2 = \langle(\phi_1, H)^2\rangle \subseteq (\phi, I) \Rightarrow (\phi_1, H) \subseteq (\phi, I)$, where (ϕ_1, H) is any soft left ideal of (F, A).*

(3) *For any soft left ideal (ϕ_1, H) of (F, A) such that $(\phi_1, H) \nsubseteq (\phi, I) \Rightarrow (\phi_1, H)^2 \nsubseteq (\phi, I)$.*

(4) *If $F(a)$ is any element of (F, A) such that $\langle F(a)\rangle^2 \subseteq (\phi, I)$, then it implies that $F(a) \in (\phi, I)$.*

(5) *For all $F(a) \in (F, A)$ such that $F(a)((F, A)F(a)) \subseteq (\phi, I) \Rightarrow F(a) \in (\phi, I)$.*

Proof. (1) $\Leftrightarrow$ (2) $\Leftrightarrow$ (3) is trivial. (1) $\Rightarrow$ (4) Let $\langle F(a)\rangle^2 \subseteq (\phi, I)$. But by hypothesis (ϕ, I) is soft quasi-semiprime, so it implies that $\langle F(a)\rangle \subseteq (\phi, I)$ which further implies that $F(a) \in (\phi, I)$. (4) $\Rightarrow$ (2) For all soft left ideals (ϕ_1, H) of (F, A) let $(\phi_1, H)^2 = \langle(\phi_1, H)^2\rangle \subseteq (\phi, I)$. If $F(a) \in (\phi_1, I)$, then by (4) $\langle F(a)\rangle^2 \subseteq (\phi, I)$ implies that $F(a) \in (\phi, I)$. Hence it shows that $(\phi_1, H) \subseteq (\phi, I)$. (1) $\Leftrightarrow$ (5) is obvious. □

Proposition 3.4. *A soft left ideal (ϕ, I) of soft LA-ring (F, A) is a soft quasi-semiprime if and only if $(F, A)\backslash(\phi, I)$ is a soft P-system.*

Proof. Let (ϕ, I) is a soft quasi-semiprime ideal of (F, A) and let $\alpha \in (F, A)\backslash(\phi, I)$. On contrary suppose that there does not exist an element $F(r) \in (F, A)$ such that $\alpha(F(r)\alpha) \in (F, A)\backslash(\phi, I)$. This implies that $\alpha(F(r)\alpha) \in (\phi, I)$. Since (ϕ, I) is a soft quasi-semiprime, so by Proposition 3.3, $\alpha \in (\phi, I)$ which is a contradiction. Thus there exists $F(r) \in (F, A)$ such that $\alpha(F(r)\alpha) \in (F, A)\backslash(\phi, I)$. Hence $(F, A)\backslash(\phi, I)$ is a soft P-system. Conversely, suppose for all $\alpha \in (F, A)\backslash(\phi, I)$ there exists $F(r) \in (F, A)$ such that $\alpha(F(r)\alpha) \in (F, A)\backslash(\phi, I)$. Let $\alpha((F, A)\alpha) \subseteq (\phi, I)$. This implies that there does not exist $F(r) \in (F, A)$ such that $\alpha(F(r)\alpha) \in (F, A)\backslash(\phi, I)$ which implies that $\alpha \in I$. Hence by Proposition 3.3, (ϕ, I) is a soft quasi-semiprime ideal. □

Lemma 3.1. *A soft M-system of elements of soft LA-ring (F, A) is a soft P-system.*

Proof. Let (ψ, B) be a nonempty subset of (F, A) such that (ψ, B) is a soft M-system. Then for all $\psi(a), \psi(b) \in (\psi, B)$, there exists an element $F(r) \in (F, A)$ such that $\psi(a)(F(r)\psi(b)) \in (\psi, B)$. If we take $\psi(b) = \psi(a)$, then $\psi(a)(F(r)\psi(a)) \in (\psi, B)$ which implies that (ψ, B) is a soft P-system. □

Remark 3.1. *Converse of Lemma 3.1 need not to be true always. We illustrate this fact in the following example.*

Example 3.4. *3.3 Let $A = R = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and $I = \{0, 2, 4\} \subset A$. Define a set-valued function $F : A \longrightarrow P(R)$ by $F(x) = \{y \in R \mid x \cdot y \in \{0, 3, 8\}\}$. Using Example 3.3, it is can be shown that (F, A) is a soft LA-ring. Define a mapping*

$\varphi : I \longrightarrow P(R)$ *defined by* $\varphi(x) = \{y \in R \mid xy{+}y = 3\}$. *Then* $\varphi(0) = \varphi(2) = \{1, 3, 6\}$ *and* $\varphi(4) = \{0, 3, 8\}$. *By Example 3.3 it is soft P-system. It is not hard to see that* $\varphi(4)(F(1)\varphi(0)) \notin \varphi(4)$, *so the condition of soft M-system is not true in this case. Hence every soft P-system need not to be soft M-system.*

Definition 3.4. *A soft ideal* (ϕ, I) *of a soft LA-ring* (F, A) *is soft strongly irreducible if and only if for soft ideals* (ϕ_1, H) *and* (ϕ_2, K) *of* (F, A), $(\phi_1, H) \tilde{\cap} (\phi_2, K) \subseteq (\phi, I)$ *implies that* $(\phi_1, H) \subseteq (\phi, I)$ *or* $(\phi_2, K) \subseteq (\phi, I)$ *and* (ϕ, I) *is said to be soft irreducible if for soft ideals* (ϕ_1, H) *and* (ϕ_2, K), $(\phi, I) = (\phi_1, H) \tilde{\cap} (\phi_2, K)$ *implies that* $(\phi, I) = (\phi_1, H)$ *or* $(\phi, I) = (\phi_2, K)$.

Example 3.5. *Let* $R = \{0, 1, 2, 3, 4, 5, 6, 7\}$ *be an LA-ring of order 8 defined as follows:*

+	0	1	2	3	4	5	6	7
0	2	3	4	5	6	7	0	1
1	3	2	5	4	7	6	1	0
2	0	1	2	3	4	5	6	7
3	1	0	3	2	5	4	7	6
4	6	7	0	1	2	3	4	5
5	7	6	1	0	3	2	5	4
6	4	5	6	7	0	1	2	3
7	5	4	7	6	1	0	3	2

·	0	1	2	3	4	5	6	7
0	2	7	2	7	2	7	2	7
1	2	7	2	7	2	7	2	7
2	2	2	2	2	2	2	2	2
3	2	2	2	2	2	2	2	2
4	2	7	2	7	2	7	2	7
5	2	7	2	7	2	7	2	7
6	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2

Let $R = A = \{0, 1, 2, 3, 4, 5, 6, 7\}$ *and* $I = \{2, 3, 6, 7\} \subset A$. *Now consider set valued function* $F : A \longrightarrow P(R)$ *defined by* $F(x) = \{y \in R \mid x \cdot y \in \{1, 2, 7\}\}$. *Then* $F(0) = F(1) = F(2) = F(3) = F(4) = F(5) = F(6) = F(7) = \{0, 1, 2, 3, 4, 5, 6, 7\}$, *it can be seen that all these sets are sub LA-rings over R. Hence* (F, A) *is soft LA-Ring over R. On the other hand consider the function* $\varphi : I \longrightarrow P(R)$ *given by* $\varphi(x) = \{y \in R \mid x{+}y \in \{2, 3, 6, 7\}\}$. *Since* $\varphi(2) = \varphi(3) = \varphi(6) = \varphi(7) = \{2, 3, 6, 7\}$. *Here it is observed that* (ϕ, I) *is soft ideal of soft LA-ring* (F, A). *Now consider* $H = \{2, 7\}$ *and* $K = \{2, 6\}$ *defined by the set valued functions* $\varphi_1 : H \longrightarrow P(R)$ *and* $\varphi_2 : K \longrightarrow P(R)$ *respectively. Given by* $\varphi_1 : \{y \in R \mid x + y \in \{2, 7\}\}$ *and* $\varphi_2 : \{y \in R \mid x + y \in \{2, 6\}\}$ *respectively. Then* $\varphi_1(2) = \varphi_1(7) = \{2, 7\}$ *and* $\varphi_2(2) = \varphi_2(6) = \{2, 6\}$. *Here it can be easily observed that* (φ_1, H) *and* (φ_2, K) *are any soft ideals of a soft LA-ring* (F, A). *Also it can be seen that if* $(\varphi_1, H) \tilde{\cap} (\varphi_2, K) \subseteq (\varphi, I)$ *implies that either* $(\varphi_1, H) \subseteq (\varphi, I)$ *or* $(\varphi_2, K) \subseteq (\varphi, I)$, *where* (φ_1, H) *and* (φ_2, K) *are any soft left ideals of* (F, A). *Hence* (φ, I) *is a soft strongly irreducible ideal over a soft LA-ring* (F, A).

Lemma 3.2. *Every soft strongly irreducible ideal of soft LA-ring* (F, A) *is soft irreducible.*

Proof. Proof is straight forward. □

Definition 3.5. *An ideal* (φ, I) *of soft LA-ring* (F, A) *is said to be soft prime ideal if and only if* $(\varphi_1, H)(\varphi_2, K) \subseteq (\varphi, I)$ *implies either* $(\varphi_1, H) \subseteq (\varphi, I)$ *or* $(\varphi_2, K) \subseteq$

(φ, I), where (φ_1, H) and (φ_2, K) are ideals in (F, A) and it is called soft semi-prime if for any ideal (ψ, G) of (F, A), $(\psi, G)^2 \subseteq (\varphi, I)$ implies that $(\psi, G) \subseteq (\varphi, I)$.

Proposition 3.5. *A soft ideal (φ, I) of a soft LA-ring (F, A) is soft prime if and only if it is soft semiprime and soft strongly irreducible.*

Proof. Let (φ, I) is a soft prime ideal of soft LA-ring (F, A), then (φ, I) is tivially soft semi-prime ideal of soft LA-ring (F, A). Now to prove irreducible condition, let (φ_1, H) and (φ_2, K) be ideals of soft LA-ring (F, A) such that $(\varphi_1, H) \,\widetilde{\cap}\, (\varphi_2, K) \subseteq (\varphi, I)$, as $(\varphi_1, H)(\varphi_2, K) \subseteq (\varphi_1, H)(F, A) \subseteq (\varphi_1, H)$ and $(\varphi_1, H)(\varphi_2, K) \subseteq (F, A)(\varphi_2, K) \subseteq (\varphi_2, K) \Rightarrow (\varphi_1, H)(\varphi_2, K) \subseteq (\varphi_1, H) \,\widetilde{\cap}\, (\varphi_2, K) \subseteq (\varphi, I) \Rightarrow (\varphi_1, H)(\varphi_2, K) \subseteq$

$(\varphi, I) \Rightarrow (\varphi_1, H) \subseteq (\varphi, I)$ or $(\varphi_2, K) \subseteq (\varphi, I)$. Hence (φ, I) is soft strongly irreducible ideal. Conversely, suppose that (φ, I) is a soft semi-prime and soft strongly irreducible ideal. Now let $(\varphi_1, H)(\varphi_2, K) \subseteq (\varphi, I)$, where (φ_1, H) and (φ_2, K) are ideals of soft LA-ring (F, A). Consider $(\varphi_1, H) \,\widetilde{\cap}\, (\varphi_2, K) \subseteq (\varphi_1, H)$ and $(\varphi_1, H) \,\widetilde{\cap}\, (\varphi_2, K) \subseteq (\varphi_2, K) \Rightarrow ((\varphi_1, H) \,\widetilde{\cap}\, (\varphi_2, K))^2 \subseteq (\varphi_1, H)(\varphi_2, K) \subseteq (\varphi, I) \Rightarrow ((\varphi_1, H)\widetilde{\cap}(\varphi_2, K))^2 \subseteq (\varphi, I)$. Since (φ, I) is a soft semi-prime, so $(\varphi_1, H)\widetilde{\cap}(\varphi_2, K) \subseteq (\varphi, I) \Rightarrow (\varphi_1, H) \subseteq (\varphi, I)$ or $(\varphi_2, K) \subseteq (\varphi, I) \Rightarrow (\varphi, I)$ is a soft prime ideal of soft LA-ring (F, A). □

Definition 3.6. *A nonempty soft subset (ξ, I) of a soft LA-ring (F, A) is called a soft I-system if for all $\xi(a), \xi(b) \in (\xi, I)$, $(\langle \xi(a)\rangle \,\widetilde{\cap}\, \langle \xi(b)\rangle) \,\widetilde{\cap}\, (\xi, I) \neq \phi$.*

Proposition 3.6. *The following conditions on a soft ideal (ξ, I) of a soft LA-ring (F, A) are equivalent:*

(1) *(ξ, I) is a soft strongly irreducible ideal.*

(2) *For all $F(a), F(b) \in (F, A) : \langle F(a)\rangle \,\widetilde{\cap}\, \langle F(b)\rangle \subseteq (\xi, I)$ implies that either $F(a) \in (\xi, I)$ or $F(b) \in (\xi, I)$.*

(3) *$(F, A)\backslash(\xi, I)$ is a soft I-system.*

Proof. (1) $\Rightarrow$ (2) is obvious. (2) $\Rightarrow$ (3) Let $\alpha, \beta \in (F, A)\backslash(\xi, I)$. Let $(\langle\alpha\rangle \,\widetilde{\cap}\, \langle\beta\rangle) \,\widetilde{\cap}\, (F, A)\backslash(\xi, I) = \phi$. This implies that $\langle\alpha\rangle \,\widetilde{\cap}\, \langle\beta\rangle \subseteq (\xi, I)$ and so by hypothesis either $\alpha \in (\xi, I)$ or $\beta \in (\xi, I)$ which is a contradiction. Hence $(\langle\alpha\rangle \,\widetilde{\cap}\, \langle\beta\rangle) \,\widetilde{\cap}\, (F, A)\backslash(\xi, I) \neq \phi$. (3) $\Rightarrow$ (1) Let (ξ_1, I_1) and (ξ_2, I_2) be soft ideal of (F, A) such that $(\xi_1, I_1) \,\widetilde{\cap}\, (\xi_2, I_2) \subseteq (\xi, I)$. Suppose (ξ_1, I_1) and (ξ_2, I_2) are not contained in (ξ, I), then there exist elements α, β such that $\alpha \in (\xi_1, I_1)\backslash(\xi, I)$ and $\beta \in (\xi_2, I_2)\backslash(\xi, I)$. This implies that $\alpha, \beta \in (F, A)\backslash(\xi, I)$. So by hypothesis $(\langle\alpha\rangle \,\widetilde{\cap}\, \langle\beta\rangle) \,\widetilde{\cap}\, (F, A)\backslash(\xi, I) \neq \phi$ which implies that there exists an element $\gamma \in \langle\alpha\rangle \,\widetilde{\cap}\, \langle\beta\rangle$ such that $\gamma \in (F, A)\backslash(\xi, I)$. It shows that $\gamma \in \langle\alpha\rangle \,\widetilde{\cap}\, \langle\beta\rangle \subseteq (\xi_1, I_1) \,\widetilde{\cap}\, (\xi_2, I_2) \subseteq (\xi, I)$ which further implies that $(\xi_1, I_1) \,\widetilde{\cap}\, (\xi_2, I_2) \nsubseteq (\xi, I)$. A contradiction. Hence either $(\xi_1, I_1) \subseteq (\xi, I)$ or $(\xi_2, I_2) \subseteq (\xi, I)$ and so (ξ, I) is soft strongly irreducible ideal. □

4. **Conclusions**

In this paper, we have initiated a step to apply soft sets (in Molodtsove's sense) by considering a set of parameters as a non-associative structure and have studied several related properties. We have introduced the notions of soft M-system, soft P-system, and soft I-system in soft LA-ring. By defining soft quasi-prime ideals, soft quasi-semiprime ideals and soft strongly irreducible ideals, we have investigated various related properties and illustrated these notions by number of corresponding examples. To extend this study one can investigate soft direct sums, soft subtractive and soft LA modules.

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