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# PERIPHERAL SURFACE PROFILING OF THE HOB CUTTER FOR SCREW COMPRESSOR ROTORS COMPONENTS II. MALE ROTOR

Camelia POPA[1], Nicolae OANCEA[2]

*Profilarea suprafeţei periferice a sculei melc reciproc înfăşurătoare cu suprafeţele elicoidale ale flancurilor rotoarelor compresorului elicoidal, constituie o problemă de generare de speţa a II-a (referitoare la suprafeţele reciproc înfăşurătoare cu contact punctiform). În lucrare, pe baza teoremelor referitoare la descompunerea unei mişcări elicoidale, se propune o metodă de profilare a suprafeţei periferice primare a sculei melc generatoare a rotorului conducător din componenţa compresorului elicoidal. Sunt prezentate formele constructive ale lobilor rotorului conducător, forma cremalierei generatoare şi forma profilului axial al sculei melc, reciproc înfăşurătoare acesteia. De asemenea, pe baza unui produs soft dedicat, se prezintă exemple numerice ale formelor axiale ale sculelor melc generatoare pentru rotorul conducător pentru un compresor cu raport de transmitere 4/6, respectiv 3/5.*

*Peripheral surface profiling of the hob cutter represents a second degree generation problem referring to point contact surfaces. In this paper, according to theorems referring to the decomposition of a helical motion, we propose a surface profiling method of the peripheral surface of the hob cutter for the male rotor, screw compressor component. The constructive form of the male rotor lobes, the shape of the generating rack and the shape of the axial profile of the hob cutter are presented. Also, we present frontal applications of the axial shapes of the hob cutter generating for the male rotor, screw compressor component, gear ratio 4/6, and 3/5 respectively.*

**Keywords***:* helical surfaces, hob cutter, meshing surfaces

## 1. Introduction

Methods of profiling tool drives, envelope a vortex of surfaces associated with a couple in rolling axodes, are: Oliver II theorem, for enveloping surfaces with contact point, and Gohman theorem, through the intermediate surface; helical movement decomposition method, Nicolaev [1],[3],[4],[6],[8]. We also proposed 2D and 3D graphic solutions in order to solve these problems [7], [9].

[1] Assist , Department of Machine Elements and Graphics, "Dunărea de Jos" University of Galati, Galati, Romania, e-mail: lpopa@ugal.ro

[2] Prof., Department of Manufacturing, Robotics and Welding Engineering, , "Dunărea de Jos" University of Galati, Galati, Romania, e-mail: noancea@ugal.ro

Using Bézier polynomials, surface representation is possible to be done, precisely enough. This is the solution for the helical surfaces of screw compressor rotors, male and female.

## 2. Coordinates systems; generation motion

In Fig. 1, it is represented the axes system, and the absolute movements of the reference systems associated to these axes.

The reference systems are the following:

- $x_0\ y_0\ z_0$ is the global system with axis $z_0$ , axis of rotation of the axis associated to the generating surfaces roller;

- $x_{01}y_{01}z_{01}$ – the global system, with axis $y_{01}$ superposed to the primary peripheral surface axis of the hob cutter;

- $X_1Y_1Z_1$ – the relative system attached to the axis of the axode, $A_1$;

- $\xi\eta\zeta$ – the relative system attached to the axis of the rack generating (flat surface superposed to the plane $\eta\zeta$), $A_2$;

- $X_{1S}Y_{1S}Z_{1S}$ – the relative system associated to the peripheral surface of the hob cutter.

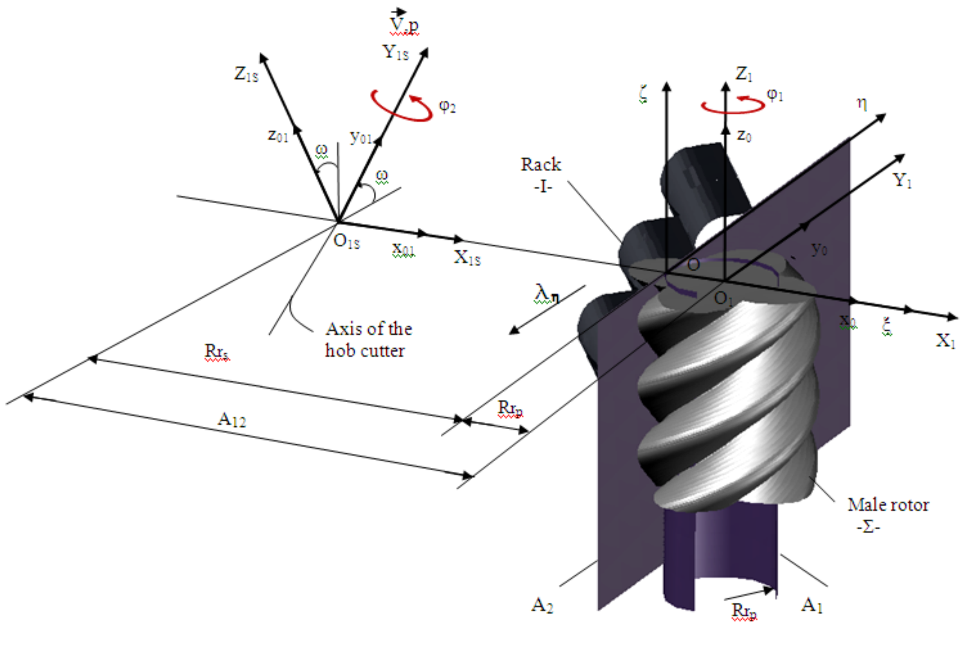


Fig. 1. Coordinates systems; generation motions

The kinematics of the generation process is known as follow:

• rotation of the axode $A_1$ (radius $R_{rp}$), $\varphi_1$– angular motion parameter,

$$x_0 = \omega_3^T(\varphi_1)X_1. \tag{1}$$

•displacement of the axode $A_2$ - $l_\eta$ motion parameter,

$$x_0 = \xi - a \quad , \quad a = \begin{Vmatrix} R_{r_p} \\ \lambda_\eta \\ 0 \end{Vmatrix} \quad ; \tag{2}$$

• rotation of the system $X_{1S}Y_{1S}Z_{1S}$ around the axis $Y_{01}$, with $\varphi_2$ angular motion parameter,

$$x_{01} = \omega_3^T(\varphi_2)X_{1S}. \tag{3}$$

The following conditions are known:

• condition of rolling of the axodes $A_1$ and $A_2$,

$$\lambda_\eta = R_{r_p}\varphi_2 \ ; \tag{4}$$

• dependence generated by the shape of the peripheral surface of the hob cutter (constant steep parallel-type helix, with p – helical parameter),

$$\lambda_\eta = p\varphi_2 cos\omega \, ; \tag{5}$$

• transformation between fixed coordinates systems,

$$\begin{Vmatrix} x_{01} \\ y_{01} \\ z_{01} \end{Vmatrix} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & cos\omega & -sin\omega \\ 0 & sin\omega & cos\omega \end{Vmatrix} \cdot \left( \begin{Vmatrix} x_0 \\ y_0 \\ z_0 \end{Vmatrix} - \begin{Vmatrix} A_{12} \\ 0 \\ 0 \end{Vmatrix} \right), \tag{6}$$

where $A_{12}$ is the distance between the axes, respectively, of axode $A_1$ and of the helical surface $\vec{V}$.

The relative motion of the system attached to the axode $A_1$, of the generating surface XYZ, compared with the reference system associated to the rack generating space $\xi\eta\zeta$, is given by the transformation:

$$\xi = \omega_3^T(\varphi_1)X - a \, . \tag{7}$$

The current point on the generating surface, as a cylindrical helical surface is:

$$\Sigma \quad \begin{aligned} X_1 &= X_1(u); \\ Y_1 &= Y_1(u); \\ Z_1 &= p_1 \upsilon \, , \end{aligned} \tag{8}$$

for u discrete variable known through a reduced number of values (3 or 4 points); $\upsilon$ angular rotation parameter around the axis $Z_1$; $p_1$ – helical parameter of the conductive rotor.

### 3. Method of definition for the shape of the generating rack surface

According to (7) and (8) we define:

$$\begin{Vmatrix}\xi\\ \eta\\ \zeta\end{Vmatrix}=\begin{Vmatrix}cos\varphi_1 & sin\varphi_1 & 0\\ sin\varphi_1 & cos\varphi_1 & 0\\ 0 & 0 & 1\end{Vmatrix}\begin{Vmatrix}X_1(u)\\ Y_1(u)\\ p_1 u\end{Vmatrix}\quad\begin{Vmatrix}R_{r_p}\\ R_{r_p}\varphi_1\\ 0\end{Vmatrix}. \tag{9}$$

The enveloping condition is

$$\left[X_1 - X_1(u)\right]X'_{1u}+\left[Y_1 - Y_1(u)\right]Y'_{1u}=0, \tag{10}$$

where:

$$\begin{aligned}X_1 &= R_{r_p}\cos\varphi_1;\\ Y_1 &= R_{r_p}\sin\varphi_1.\end{aligned} \tag{11}$$

Equation (10) represents “condition of normal” [1] ($R_{rp}$ is the radius of the cylindrical axode $A_1$, (Fig. 2)).

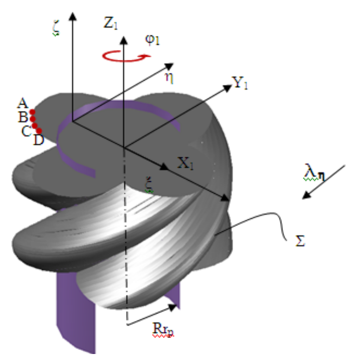


Fig.2. Surface $\Sigma$, generating surface known through 4 points : $A[X_A,X_A]$, $B[X_B,X_B]$,$C[X_C,X_C]$, $D[X_D,X_D]$

We can accept that, the directrix of the surface representing the cylindrical flank of the generating rack, (equations (1)…(8)), can be substituted by a Bézier polynomial, given by the following equations :

$$I:\begin{cases}\xi=\lambda^3\cdot A_\xi+3\lambda^2(1-\lambda)\cdot B_\xi+3\lambda(1-\lambda)^2\cdot C_\xi+(1-\lambda)^3\cdot D_\xi;\\ \eta=\lambda^3\cdot A_\eta+3\lambda^2(1-\lambda)\cdot B_\eta+3\lambda(1-\lambda)^2\cdot C_\eta+(1-\lambda)^3\cdot D_\eta.\end{cases} \tag{12}$$

All the points which determine the directrix of the cylindrical surface (the flank of the blank) is given by: $A[x_A, y_A]$, $B[x_B, y_B]$, $C[x_C, y_C]$, $D[x_D,y_D]$, ( Fig. 2).

## 4. Peripheral surface profiling method for the hob cutter

Knowing the surface of the generating rack flank (in approximate shape, as Bézier polynomial), we propose to determine the characteristic (contact curve) using the method of decomposition of the helical motion [5], (Fig. 3).

The helical movement of peripheral surface of the hob cutter, ($\vec{V}$, p) decomposes into a sum of equivalent movements: a translation, upon the unit vector $\vec{t}$ of the generatrix of cylindrical surface of the flank rack and the rotation of the axis $\vec{A}$.
The distance:

$$a = p \cdot tan\left(\frac{\pi}{2} - \omega\right), \tag{13}$$

represents the distance to the helical surface axis, $\vec{V}$, ( Fig. 3).

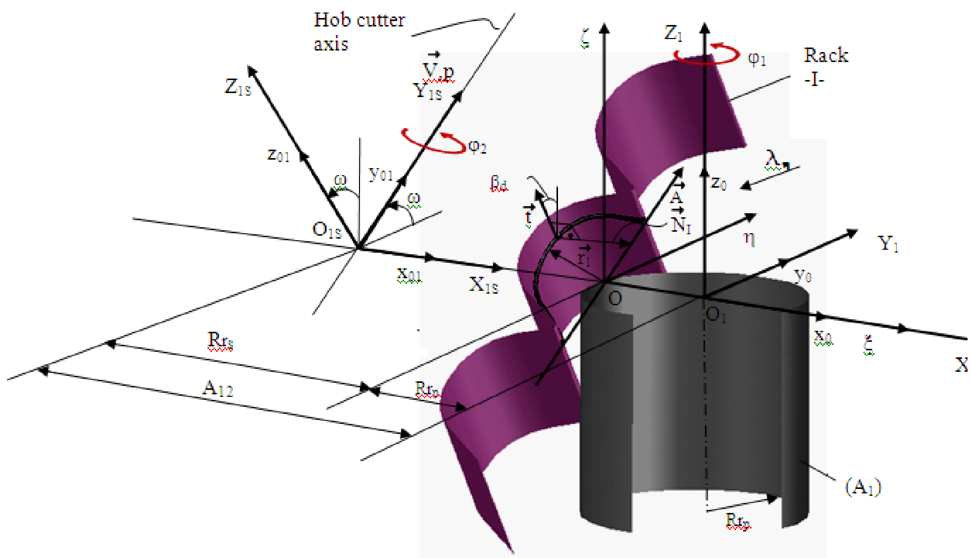


Fig. 3. Helical movement decomposition method; coordinates systems

The condition to determine the characteristic curve is provided as follows:

$$\vec{N} \cdot \vec{t} \equiv 0\,. \tag{14}$$

The normal vector to the cylindrical surface *I*, is always normal to its own generatrix; characteristic curve depends only on the rotation around the axis $\vec{A}$. Therefore, the characteristic curve, during the helical movement of axis $\vec{V}$ and the parameter p, is defined as the projection axis $\vec{A}$ on the surface *I*.

We define, see Fig. 3:

- the axis $\vec{A}$, in the $x_0y_0z_0$ system,

$$\vec{A} = cos\omega j + sin\omega k\ , \tag{15}$$

- the normal on the surface I,

$$\vec{N}_I = N_{x_0} \cdot \bar{i} + N_{j_0} \cdot \bar{j} + N_{z_0} \cdot \bar{k}\ , \tag{16}$$

or :

$$\vec{N}_I = \left( \frac{\partial x_0}{\partial \lambda} \bar{i} + \frac{\partial y_0}{\partial \lambda} \bar{j} + \frac{\partial z_0}{\partial \lambda} \bar{k} \right) \times \left( \frac{\partial x_0}{\partial t} \bar{i} + \frac{\partial y_0}{\partial t} \bar{j} + \frac{\partial z_0}{\partial t} \bar{k} \right), \tag{17}$$

(see formula (20) flank surface of the tool-rack);

- the vector

$$\bar{r}_i = O_1O \cdot \bar{i} + \bar{r}\ , \tag{18}$$

where, $\vec{r}_I$ is the vector of the cylindrical surface I.

According (2), the coordinate transformation $x_0y_0z_0$,

$$\begin{Vmatrix} x_0 \\ y_0 \\ z_0 \end{Vmatrix} = \begin{Vmatrix} \xi \\ \eta \\ \zeta \end{Vmatrix} - \begin{Vmatrix} -R_{r_p} \\ -t \cdot \sin\beta_d \\ t\cos\beta_d \end{Vmatrix}, \tag{19}$$

leads to the expression of surface shape of the rack – I in the system $x_0y_0z_0$:

$$\begin{cases} x_0 = \lambda^3 \cdot A_\xi + 3\lambda^2(1-\lambda) \cdot B_\xi + 3\lambda(1-\lambda)^2 \cdot C_\xi + (1-\lambda)^3 \cdot D_\xi + R_{r_p}; \\ y_0 = \lambda^3 \cdot A_\eta + 3\lambda^2(1-\lambda) \cdot B_\eta + 3\lambda(1-\lambda)^2 \cdot C_\eta + (1-\lambda)^3 \cdot D_\eta + t \cdot \sin\beta_d; \\ z_0 = -t \cdot \cos\beta_d. \end{cases} \tag{20}$$

The size of the parameter ω is, see Fig. 4:

$$\omega = \beta_d - \omega_s, \tag{21}$$

where:

-$\omega_s$ is the angle of hob cutter axis, in relation to the front plane of the male rotor (right screw);

-$\beta_d$ - angle of the male rotor helix, cylinder radius $R_{rp}$ ;

-ω - tilt angle of the helix of hob cutter, with the radius $R_{rs}$,

$$\omega_s = arcsin \left[ \frac{R_{r_p}}{R_{r_s}} \cdot \frac{cos\ \beta}{z_{lobs}} \right]. \tag{22}$$

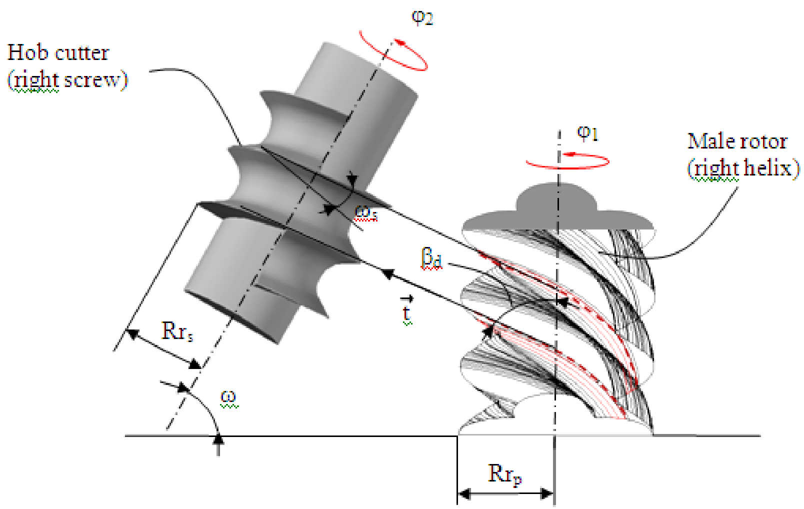


Fig.4. Tilt axis of hob cutter

The enveloping condition is:

$$\left(\vec{A},\vec{N}_I,\vec{r}_i\right)=0\,. \tag{23}$$

or,

$$\begin{vmatrix} N_{x_0} & N_{y_0} & N_{z_0} \\ x_0(\lambda,t) \;\; a & y_0(\lambda,t) & t\cos\beta_d \\ 0 & \cos\omega & \sin\omega \end{vmatrix}=0\,. \tag{24}$$

The condition (24) is a function of parameters $\lambda$ and t:

$$q\left(\lambda,t\right)=0\,, \tag{25}$$

where, $0\leq\lambda\leq 1$ .

The equations (21) and (25) are a locus surface I. The values of the parameters λ and t fulfilling (25), replaced in Bézier form for rack flanks, determine the matrix:

$$C_I=\left\|x_{0_i}\quad y_{0_i}\quad z_{0_i}\right\|^T, \tag{26}$$

representing the coordinates of the characteristic curve $C_I$.

Helical movement ($\vec{V}$,p) of the curve $C_I$, generates peripheral surface of the hob cutter.

The coordinate transformation, see Fig. 4,

$$\begin{Vmatrix} X_{1S} \\ Y_{1S} \\ Z_{1S} \end{Vmatrix} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & cos\omega & sin\omega \\ 0 & sin\omega & cos\omega \end{Vmatrix} \begin{Vmatrix} x_{0_i} + \left[R_{r_p} + R_{r_s}\right] \\ y_{0_i} \\ z_{0_i} \end{Vmatrix}_{i=1,\dots,n}, \tag{27}$$

so that, the helical movement of ($\vec{V}$,p) of the characteristic $C_I$, becomes:

$$\Pi \begin{vmatrix} X_{1S} = X_{1S}\left[x_{0_i}, y_{0_i}, z_{0_i}\ \varphi_2\right]; \\ Y_{1S} = Y_{1S}\left[x_{0_i}, y_{0_i}, z_{0_i}\ \varphi_2\right]; \\ Z_{1S} = Z_{1S}\left[x_{0_i}, y_{0_i}, z_{0_i}\ \varphi_2\right] \end{vmatrix} \quad i=1\dots n. \tag{28}$$

and represents the primary peripheral surface of the hob cutter - surface Π.

Giving the surface Π the condition

$$Z_{1S} = 0, \tag{29}$$

the axial section $\Pi_A$ is obtained in the form:

$$\Pi \begin{vmatrix} X_{1S} = X_{1S}\left[x_{0_i}, y_{0_i}, z_{0_i}, \varphi_2\right]; \\ Y_{1S} = Y_{1S}\left[x_{0_i}, y_{0_i}, z_{0_i}, \varphi_2\right] \end{vmatrix} \quad i=1\dots n, \tag{30}$$

the variable $[\phi_2]$ representing the size of the parameter corresponding to the axial section, see (29).

## 5. Numerical applications

The profile of generating rack of male rotor (see also Fig. 5) has the following profile portions.

- First application (screw compressor, ratio 4/6)

*Table 1*

**The constructive dates of the reference rack, version I ( Fig. 5)**

| $R_0$ [mm] | $r_0$ [mm] | $u_{max}$ [mm] | $\psi_{max}$ [$^0$] | $\upsilon_{max}$ [$^0$] | $\upsilon_{1max}$ [$^0$] | $u_{1max}$ [mm] | $L_p$ [mm] | $Rr_2$ [mm] | $c_0$ [mm] |
|---|---|---|---|---|---|---|---|---|---|
| 22.000 | 1.100 | 10.30 | 63.40 | 63.40 | 58.285 | 6.451 | 50.265 | 32.00 | 4.000 |

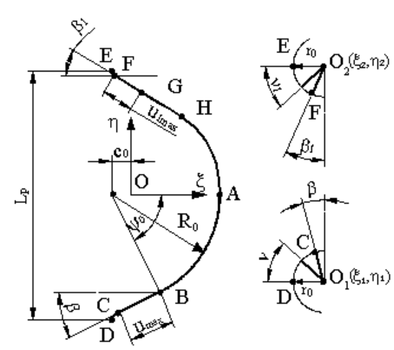


Fig. 5. Rack generating profile

Table 2 and Fig. 6 describe numerically the axial section coordinates of the primary peripheral surface of the hob cutter.

*Table 2*

**Axial section of the primary peripheral surface of the hob cutter**

| | $X_{1S}$ [mm] | $Y_{1S}$ [mm] | | | $X_{1S}$ [mm] | $Y_{1S}$ [mm] |
|---|---|---|---|---|---|---|
| 1 | 76.00005 | -24.84933 | …. | 691 | 76.0076 | 24.46793 |
| 2 | 76.00012 | -24.84312 | …. | 692 | 76.0063 | 24.48611 |
| 3 | 76.00042 | -24.82196 | …. | 693 | 76.00487 | 24.49183 |
| 4 | 76.00084 | -24.81578 | …. | 694 | 76.00383 | 24.51004 |
| 5 | 76.00148 | -24.79446 | …. | 695 | 76.00275 | 24.51563 |
| 6 | 76.00225 | -24.78835 | …. | 696 | 76.00206 | 24.53233 |
| 7 | 76.00323 | -24.76715 | …. | 697 | 76.00131 | 24.53815 |

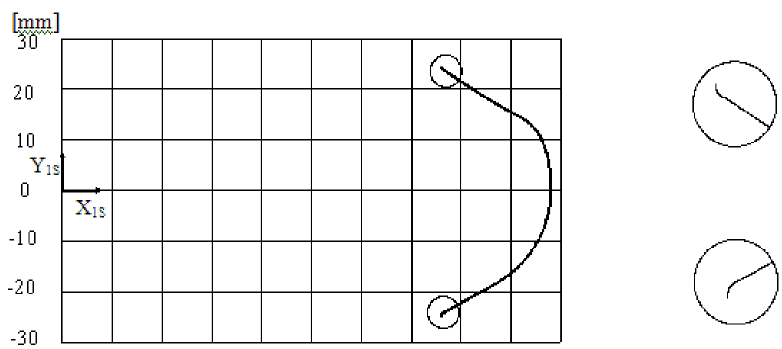


Fig. 6. The axial section profile of the hob cutter

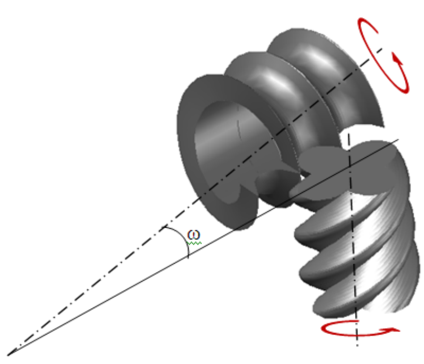


Fig. 7. The relative position between the hob cutter and the male rotor

The Fig.7, represents the relative position between the hob cutter and the male rotor.

- Second application (screw compressor, ratio 3/5)

*Table 3*

**The constructive dates of the generating rack ( Fig. 1)**

| $R_0$ [mm] | $r_0$ [mm] | $u_{max}$ [mm] | $\psi_{max}$ [$^0$] | $\upsilon_{max}$ [$^0$] | $\upsilon_{1max}$ [$^0$] | $u_{1max}$ [mm] | $L_p$ [mm] | $c_0$ [mm] | $Rr_2$ [mm] |
|---|---|---|---|---|---|---|---|---|---|
| 22.00 | 2.00 | 7.045 | 70.300 | 70.30 | 35.053 | 7.774 | 62.832 | 4.00 | 30.00 |

In Table 4, the axial section coordinates of the hob cutter peripheral surface are described.

*Table 4*

**The axial section of the hob cutter's primary peripheral surface**

| | $X_1$[mm] | $Y_1$[mm] | | | $X_1$[mm] | $Y_1$[mm] |
|---|---|---|---|---|---|---|
| 1 | 49.54200 | -23.70950 | … | 691 | 49.54596 | 35.82811 |
| 2 | 49.54217 | -23.67561 | … | 692 | 49.54517 | 35.83319 |
| 3 | 49.54266 | -23.65962 | … | 693 | 49.54455 | 35.85216 |
| 4 | 49.54348 | -23.62574 | … | 694 | 49.54391 | 35.85655 |
| 5 | 49.54462 | -23.60976 | … | 695 | 49.54345 | 35.87594 |
| 6 | 49.54609 | -23.57584 | … | 696 | 49.54297 | 35.87987 |

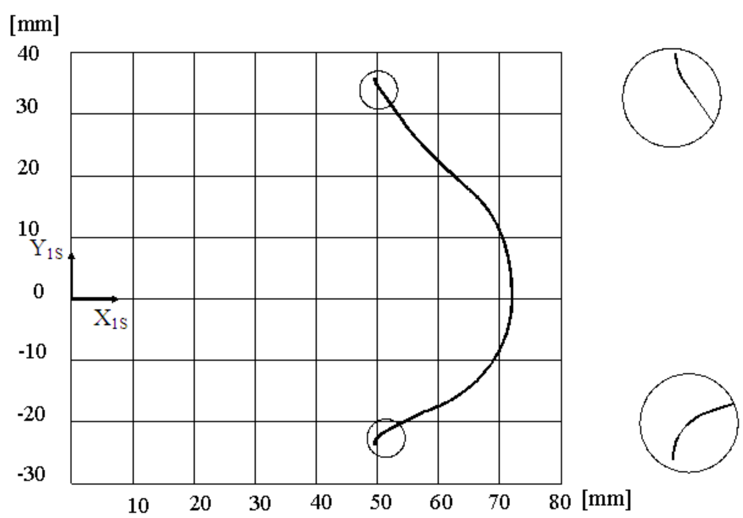


Fig. 9. The shape of the axial section for the male hob cutter

In Fig. 9 is represented the axial section shape of the hob cutter for the main rotor. In Fig. 10 is represented, in 3D modeling, the relative position between the hob cutter and the male rotor.

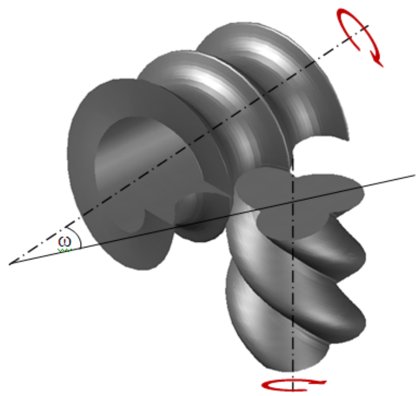

Fig. 10 . The relative position between the hob cutter and the male rotor

## 6. Conclusions

1. The helical surfaces that edges the screw compressors component rotors, are complex surfaces (cylindrical helical surfaces and constant step).
Generating this type of surfaces is done with a specific rack, whose transverse profile is determined from technological and functional considerations in order to obtain an optimum yield.

2. The male rotor generators can be expressed by simple analytical forms, or approximation of Bézier polynomials.

3. The hob cutter profiles are generated according to the composition and decomposition of the helical motion, using the intermediate surfaces method. The methods 3D of the tools' primary peripheral surfaces were also represented.

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