CENTRALIZED AND DISTRIBUTED H_∞ STATE FEEDBACK CONTROL LAWS FOR MULTI-AGENT SYSTEMS WITH TIME-DELAY COMMUNICATION NETWORKS

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The main objective of this paper is to design a distributed controller for multi-agent networks. The design of this controller involves solving two specific Riccati equations and its structure depends on the connection between agents. Using the H_∞ control method, the influence of communication time delays on system stability is analyzed. The characteristics of the designed controller are highlighted through two configurations with a variable number of identical agents and with distinct possibilities of their interconnection. Through the presented case study, it is shown that the structure of the obtained distributed controller complies with the interconnection mode of the agents.

Keywords: multi-agent systems, H_∞ type control, distributed controller, time delays.

1. Introduction

Over the last decades, the interest in multi-agent systems (MAS) development has increased significantly. Due to their capabilities to solve complex problems, MAS are found in multiple applications of engineering control. Therefore, these systems are widely used in various aerial and space missions like search and rescue, surveillance and monitoring. Recent progress of this topic is treated in surveys, such as [1], [2], [3], [4], where many aspects of these systems are described. A few challenges faced by the control theory of multi-agent systems are presented in [5].

In recent literature, the solutions proposed for networked control systems refer to centralized and distributed controllers. A comparative study of these methods is presented in [6]. The centralized controller design involves interconnection of all agents. This fact implies process data difficulties as a control decision-maker has to access information from all networked agents. This type of control requires high performance of the central controller, becoming ineffective for a large number of agents. Therefore, a single error of the central controller influences the behavior of the entire network.

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Compared to the centralized case, distributed control requires a particular structure, namely, the information transmission is achieved between certain agents. The communication manner is described by specific matrix forms of graphs. Data transmission is provided by communication channels. Many significant theoretical results in distributed control of multi-agent systems are reviewed in [7].

Although the last period has contributed to the appearance of a considerable amount of works regarding the distributed control, only a few of them are surveyed in this paper. [8] uses the robustness properties of Linear Quadratic Regulator (LQR) to guarantee the robust stability of multi-agent systems. This approach is based on the distributed LQR results given in [9], where it is proved that the optimal solution depends on the stabilizing solutions of two Riccati equations. It provides a numerical example for a network of a large number of identical and dynamically decoupled agents. The determination of the solution for the LQR problem for both centralized and distributed control is treated in [10]. In order to emphasize the differences between these approaches, two different configurations are considered as a case study.

Distributed control has been treated in various works such as [11], [12], [13], [14], [15], using different approaches and assumptions. For instance, the results presented in [14] focuses on coupled Linear Matrix Inequalities to design a distributed feedback controller to achieve H_∞ performances. The aim of [15] is to determine a distributed control law for a formation of autonomous vehicles with double-integrator dynamics.

Due to recent developments in communication theory, the applications field of distributed systems has been considerably extended. The present paper focuses on distributed control features for multi-agent systems. To design the control algorithms for networked control systems, the H_∞ design method is used, taking into consideration potential time delays in the communication channels.

In this paper, the distributed control characteristics are analyzed for two different types of flight formation configurations, with variable number of agents and distinct possibilities of interconnection. To reveal the capabilities of this design approach, the presented numerical simulations use the decoupled dynamics of an agent. Therefore, the performances of the networked system members for the longitudinal motion are analyzed.

This paper is divided in several sections as follows. A few relevant notions regarding H_∞ standard problem that are used throughout this work are briefly reminded in the second part. The following section concerns on the problem formulation, specifying the required steps in centralized and distributed controller design. The next part presents the proposed approach illustrated by a comparative analysis of two different network configurations. The numerical simulations reveal the time evolutions for each agent taking into consideration the time delays in communication channels. The concluding remarks are stated in the last section of the paper.

2. Preliminaries

Consider a network of identical agents for which the dynamics of each one is written as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1u_1(t) + B_2u_2(t) \\ y_1(t) &= Cx(t) + Du_2(t) \\ y_2(t) &= x(t), t \geq 0 \end{aligned} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^{\boldsymbol{n}}$ is the state vector, $\boldsymbol{u_1} \in \mathbb{R}^{\boldsymbol{m_1}}$ denotes the exogenous input vector, $\boldsymbol{u_2} \in \mathbb{R}^{\boldsymbol{m_2}}$ represents the control variable, $\boldsymbol{y_1} \in \mathbb{R}^{\boldsymbol{p_1}}$ is the quality output and $\boldsymbol{y_2}$ stands for the measured output. Furthermore, two conditions are assumed to be true: $\boldsymbol{C^TD = 0}$ and $\boldsymbol{D^TD = I}$. For $\boldsymbol{D^TD}$ invertible, if the previous assumptions are not satisfied, the control variable $\boldsymbol{u}$ can be changed and written as follows:

$$u = -(D^TD)^{-1}D^TCx + (D^TD)^{-\frac{1}{2}}\tilde{u} \tag{2}$$

Thus, the above conditions are fulfilled for the new control variable $\boldsymbol{\tilde{u}}$. In order to determine the solution of the H_∞ problem for the system (1), the following theorem is proved in [16] for the more general case when the system (1) is corrupted with state dependent noise.

Theorem 1. There exists a state-feedback gain $\boldsymbol{F} \in \mathbb{R}^{\boldsymbol{m_2 \times n}}$ so that the closed-loop system obtained from (1) with $\boldsymbol{u(t) = Fy_2(t)}$, namely

$$\begin{aligned} \dot{x}(t) &= (A + B_2F)x(t) + B_1u_1(t) \\ y_1(t) &= (C + DF)x(t) \end{aligned} \tag{3}$$

is stable and it has the property that for $\boldsymbol{x(0) = 0}$ and for a given value $\boldsymbol{\gamma > 0}$,

$$\int_0^\infty (|y_1(t)|^2 - \gamma^2|u_1(t)|^2)\, dt < 0 \tag{4}$$

for all $\forall \boldsymbol{u_1} \in \boldsymbol{\mathcal{L}^2}([\boldsymbol{0}, \infty), \mathbb{R}^{\boldsymbol{m_1}})$ where $\boldsymbol{\mathcal{L}^2}([\boldsymbol{0}, \infty), \mathbb{R}^{\boldsymbol{m_1}})$ denotes the space of all $\boldsymbol{m_1}$-dimensional square-integrable functions $\boldsymbol{f(t)}$ if the Riccati equation

$$A^TX + XA + \gamma^{-2}XB_1B_1^TX - XB_2B_2^TX + C^TC = 0 \tag{5}$$

has a stabilizing solution $\boldsymbol{X \geq 0}$ and

$$F = -B_2^TX. \tag{6}$$

3. Multi agent systems state feedback H∞ Control; Centralized and Distributed Structures

Consider a network consisting of N identical agents with dynamics of form (1), namely:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + B_1u_{1i}(t) + B_2u_{2i}(t) \\ y_{1i}(t) &= Cx_i(t) + Du_{2i}(t) \\ y_{2i}(t) &= x_i(t), t \geq 0, i = 1, \dots, N \end{aligned} \tag{7}$$

holding the two conditions $C^T D = 0$ and $D^T D = I$. The above dynamic system can be written in a compact form as:

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{A}\tilde{x}(t) + \tilde{B}_1\tilde{u}_1(t) + \tilde{B}_2\tilde{u}_2(t) \\ \tilde{y}_1(t) &= \tilde{C}\tilde{x}(t) + \tilde{D}\tilde{u}_2(t) \\ \tilde{y}_2(t) &= \tilde{x}(t), t \geq 0 \end{aligned} \tag{8}$$

where $\tilde{A} = I_N \otimes A$, $\tilde{B}_1 = I_N \otimes B_1, \tilde{B}_2 = I_N \otimes B_2, \tilde{C} = I_N \otimes C$, $\tilde{D} = I_N \otimes D, \tilde{x} = [x_1^T \dots x_N^T]^T$, $\tilde{u}_1 = [u_{1_1}^T \dots u_{1_N}^T]^T$, $\tilde{u}_2 = [u_{2_1}^T \dots u_{2_N}^T]^T$, $\tilde{y}_1 = [y_{1_1}^T \dots y_{1_N}^T]^T$, $\tilde{y}_2 = [y_{2_1}^T \dots y_{2_N}^T]^T$, where $\otimes$ denotes the Kronecker product.

For $\gamma > 0$, the following cost function is defined:

$$\begin{aligned} J(u_{1_1}, \dots, u_{1_N}, u_{2_1}, \dots, u_{2_N}) = \int_0^\infty [\textstyle\sum_{i=1}^N (|y_{1i}(t)|^2 - \gamma^2 |u_{1i}(t)|^2) \; + \\ + \tfrac{1}{2} \textstyle\sum_{i=1}^N \sum_{j=1, j\neq i}^N (x_i(t) - x_j(t))^T Q_{ij} (x_i(t) - x_j(t))] dt \end{aligned} \tag{9}$$

where Q_{ij}, $i, j = 1, \dots, N$ are positive semidefinite weighting matrices. Using the previous notations, one may check that (9) can be rewritten as follows:

$$\begin{aligned} J(\tilde{u}_1, \tilde{u}_2) &= \int_0^\infty \left(|\tilde{y}_1(t)|^2 - \gamma^2 |\tilde{u}_1(t)|^2 + \tilde{x}^T(t)\tilde{Q}\tilde{x}(t) \right) dt \\ &= \int_0^\infty \left(\tilde{x}^T(t)\tilde{Q}\tilde{x}(t) - \gamma^2 \tilde{u}_1^T(t)\tilde{u}_1(t) + \tilde{u}_2^T(t)\tilde{u}_2(t) \right) dt \end{aligned} \tag{10}$$

with

$$\begin{aligned} \tilde{Q}_{ii} &= C^T C + \sum_{j=1, j\neq i}^N Q_{ij} \\ \tilde{Q}_{ij} &= -Q_{ij}, i \neq j, i, j = 1, \dots, N. \end{aligned} \tag{11}$$

Choosing $\boldsymbol{Q_{ij}}$ of form $\boldsymbol{Q_{ij} = P^T P, i, j = 1, \dots, N, i \neq j}$ with $\boldsymbol{P \geq 0}$, the matrix $\widetilde{\boldsymbol{Q}}$ becomes:

$$\tilde{Q} = \begin{bmatrix} C^T C + (N-1)P^T P & -P^T P & \cdots & -P^T P \\ -P^T P & C^T C + (N-1)P^T P & \cdots & -P^T P \\ \vdots & \vdots & \ddots & \vdots \\ -P^T P & -P^T P & \cdots & C^T C + (N-1)P^T P \end{bmatrix} \tag{12}$$

Following [16], we can define $\widetilde{\boldsymbol{\mathcal{P}}} \in \mathbb{R}^{\boldsymbol{n \cdot N \times n \cdot N}}$ satisfying the equality

$$\tilde{\mathcal{P}}^T \tilde{\mathcal{P}} = \tilde{Q} - I_N \otimes C^T C. \tag{13}$$

Then direct algebraic computations show that the cost function (10) may be rewritten as

$$J(\tilde{u}_1, \tilde{u}_2) = \int_0^\infty (|\tilde{z}(t)|^2 - \gamma^2 |\tilde{u}_1(t)|^2) dt \tag{14}$$

where $\tilde{\boldsymbol{z}}(\boldsymbol{t}) = \widetilde{\boldsymbol{\mathcal{C}}}\tilde{\boldsymbol{x}}(\boldsymbol{t}) + \widetilde{\boldsymbol{\mathcal{D}}}\tilde{\boldsymbol{u}}_{\boldsymbol{2}}(\boldsymbol{t})$ and where

$$\tilde{\mathcal{C}} := \left[\frac{\tilde{\mathcal{P}}}{I_N \otimes C}\right] \text{ and } \tilde{\mathcal{D}} := \left[\frac{O_{n \cdot N \times m_2 \cdot N}}{I_N \otimes D}\right] \tag{15}$$

The matrices $\widetilde{\boldsymbol{\mathcal{C}}}$ and $\widetilde{\boldsymbol{\mathcal{D}}}$ defined above satisfy the conditions $\widetilde{\boldsymbol{\mathcal{C}}}^{\boldsymbol{T}}\widetilde{\boldsymbol{\mathcal{D}}} = \boldsymbol{0}$ and $\widetilde{\boldsymbol{\mathcal{D}}}^{\boldsymbol{T}}\widetilde{\boldsymbol{\mathcal{D}}} = \boldsymbol{I}$ and thus one may use Theorem 1 for the multi-agent system

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{A}\tilde{x}(t) + \tilde{B}_1 \tilde{u}_1(t) + \tilde{B}_2 \tilde{u}_2(t) \\ \tilde{z}(t) &= \tilde{\mathcal{C}}\tilde{x}(t) + \tilde{\mathcal{D}}\tilde{u}_2(t) \\ \tilde{y}_2(t) &= \tilde{x}(t), t \geq 0 \end{aligned} \tag{16}$$

Using a similar reasoning as in [16] where it was assumed that the agents dynamics include state-dependent noises, one obtains the following result concerning the structure of the centralized H∞ controller.

Theorem 2. There exists $\widetilde{\boldsymbol{F}} \in \mathbb{R}^{\boldsymbol{m_2 N \times nN}}$ *so that the closed loop system obtained from* (*16*) *with* $\tilde{\boldsymbol{u}}(\boldsymbol{t}) = \widetilde{\boldsymbol{F}}\tilde{\boldsymbol{x}}(\boldsymbol{t})$, *namely*

$$\begin{aligned} \dot{\tilde{x}}(t) &= (\tilde{A} + \tilde{B}_2 \tilde{F})\tilde{x}(t) + \tilde{B}_1 \tilde{u}_1(t) \\ \tilde{z}(t) &= (\tilde{C} + \tilde{\mathcal{D}}\tilde{F})\tilde{x}(t) \end{aligned} \tag{17}$$

has the property that for $\tilde{\boldsymbol{x}}(\boldsymbol{0}) = \boldsymbol{0}$ *and for a given value* $\boldsymbol{\gamma > 0}$,

$$\int_0^\infty (|\tilde{z}(t)|^2 - \gamma^2 |\tilde{u}_1(t)|^2)\, dt < 0 \tag{18}$$

for all $\tilde{\mathbf{u}}_1 \in \mathcal{L}^2_{\mathbf{u}_1}([\mathbf{0},\infty), \mathbb{R}^{m_1 N})$ *if the Riccati type equation*

$$\tilde{A}^T\tilde{X} + \tilde{X}\tilde{A} + \gamma^{-2}\tilde{X}\tilde{B}_1\tilde{B}_1^T\tilde{X} - \tilde{X}\tilde{B}_2\tilde{B}_2^T\tilde{X} + \tilde{Q}^T\tilde{Q} = 0 \tag{19}$$

has a stabilizing solution $\tilde{\mathbf{X}} \geq \mathbf{0}$*, and in this case,*

$$\tilde{F} = -\tilde{B}_2^T\tilde{X}. \tag{20}$$

Furthermore, the stabilizing solution of the Riccati type equation (19) has the following structure:

$$\tilde{X} = \begin{bmatrix} \tilde{X}_1 & \tilde{X}_2 & \cdots & \tilde{X}_2 \\ \tilde{X}_2 & \tilde{X}_1 & \cdots & \tilde{X}_2 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{X}_2 & \tilde{X}_2 & \cdots & \tilde{X}_1 \end{bmatrix} \tag{21}$$

where $\tilde{\mathbf{X}}_1 = \mathbf{X}_1 + (\mathbf{N}-\mathbf{1})\mathbf{X}_2$ *and* $\mathbf{X}_1$ *is the stabilizing positive semidefinite solution of the Riccati equation*

$$A^TX_1 + X_1A + X_1(\gamma^{-2}B_1B_1^T - B_2B_2^T)X_1 + C^TC = 0 \tag{22}$$

and $\tilde{\mathbf{X}}_2 = \mathbf{X}_2$*, where* $\mathbf{X}_2$ *is the stabilizing solution of the following Riccati equation:*

$$\begin{aligned}(A + (\gamma^{-2}B_1B_1^T - B_2B_2^T)X_1)^TX_2 + X_2(A + (\gamma^{-2}B_1B_1^T - B_2B_2^T)X_1) \\ + NX_2(\gamma^{-2}B_1B_1^T - B_2B_2^T)X_2 + P^TP = 0.\end{aligned} \tag{23}$$

Then the centralized H∞ state-feedback gain of the multi-agent system has the expression

$$\tilde{F} = \begin{bmatrix} F_1 & F_2 & \cdots & F_2 \\ F_2 & F_1 & \cdots & F_2 \\ \vdots & \vdots & \ddots & \vdots \\ F_2 & F_2 & \cdots & F_1 \end{bmatrix}, \tag{24}$$

where

$$F_1 = -B_2^T(X_1 + (N-1)X_2) \tag{25}$$

$$F_2 = B_2^T X_2.$$

In the following, one will focus the attention on the distributed controller of the multi-agent system. The systems interconnection is defined by graph theory, the communication way between agents being described by a matrix form. Therefore, a data communication network is established between its agents, defined as a graph described by the pair $\boldsymbol{\mathcal{G}} = (\boldsymbol{\mathcal{V}}, \boldsymbol{E})$ where $\boldsymbol{\mathcal{V}}$ represents the set of nodes that define the network agents, $\boldsymbol{\mathcal{V}} = \{\mathbf{1}, \mathbf{2}, \dots, \boldsymbol{N}\}$, and $\boldsymbol{E} \subseteq \boldsymbol{\mathcal{V}} \times \boldsymbol{\mathcal{V}}$ is the set of edges that represent the interconnection between a pair of members, $\boldsymbol{E} \subseteq \{(\boldsymbol{i}, \boldsymbol{j}): \boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{\mathcal{V}}, \boldsymbol{j} \neq \boldsymbol{i}\}$. Each edge is marked by a pair of different nodes $(\boldsymbol{\mathcal{V}_i}, \boldsymbol{\mathcal{V}_j})$. If $(\boldsymbol{\mathcal{V}_i}, \boldsymbol{\mathcal{V}_j}) \in \boldsymbol{E} \Leftrightarrow (\boldsymbol{\mathcal{V}_j}, \boldsymbol{\mathcal{V}_i}) \in \boldsymbol{E}$, the graph is called symmetric (undirected) [18].

Several notions regarding graph theory and matrix properties are treated in [18], [19], [20]. Some necessary specific matrix forms are briefly mentioned in this section. According to [8], if $\boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{\mathcal{V}}$ and $\boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{E}$, then the agent $\boldsymbol{i}$ and the agent $\boldsymbol{j}$ indicate two adjacent nodes.

The degree matrix, denoted $\boldsymbol{\mathcal{D}}(\boldsymbol{\mathcal{G}}) \in \mathbb{R}^{\boldsymbol{N} \times \boldsymbol{N}}$, is a diagonal matrix consisting of the number of connections for each agent. The adjacency matrix, $\boldsymbol{\mathcal{A}}(\boldsymbol{\mathcal{G}}) \in \mathbb{R}^{\boldsymbol{N} \times \boldsymbol{N}}$, indicates the mode of connection between the nodes, namely if the pair of agents is interconnected. The Laplacian matrix, $\boldsymbol{L}(\boldsymbol{\mathcal{G}}) \in \mathbb{R}^{\boldsymbol{N} \times \boldsymbol{N}}$, defines the connection way of the graph, given by $\boldsymbol{L}(\boldsymbol{\mathcal{G}}) = \boldsymbol{\mathcal{D}}(\boldsymbol{\mathcal{G}}) - \boldsymbol{\mathcal{A}}(\boldsymbol{\mathcal{G}})$. These matrix forms are explained as follows:

$$\mathcal{D}(\mathcal{G}) = \begin{cases} deg(\mathcal{V}_i), i = j \\ 0, i \neq j \end{cases}; \quad \mathcal{A}(\mathcal{G}) = \begin{cases} a_{ii} = 0, \quad \forall i \in \mathcal{V} \\ a_{ij} = 0, (i,j) \notin E, \forall i, j \in \mathcal{V}, i \neq j; \\ a_{ij} = 1, (i,j) \in E, \forall i, j \in \mathcal{V}, i \neq j \end{cases}$$
$$L(\mathcal{G}) = \begin{cases} deg\,(\mathcal{V}_i), i = j \\ -1, i \neq j, (\mathcal{V}_i, \mathcal{V}_j)\; adjacent \\ 0, otherwise \end{cases} \tag{26}$$

The distributed control requires a certain structure, rather, the information transmission is possible only between certain agents. Due to the limited interconnection between agents, the feedback gain expression may be written using the adjacency matrix defined above, as follows:

$$\tilde{F}_D = I_N \otimes F_1 + \mathcal{A}(\mathcal{G}) \otimes F_2 \tag{27}$$

The presence of null terms in the adjacency matrix introduces a new feature, namely, if the obtained distributed controller guarantees the system stability and the required H$_\infty$ performances. Adopting as in [9] the parameterization

$$\tilde{F}_D = I_N \otimes F_1 + aI_N \otimes F_2 + b\mathcal{A}(\mathcal{G}) \otimes F_2 \tag{28}$$

one will determine the domain of the parameters $\boldsymbol{a}$ and $\boldsymbol{b}$ for which the multi-agent closed loop system is stable. One can notice that for $\boldsymbol{a}=\mathbf{0}$ and $\boldsymbol{b}=\mathbf{1}$, the gain expressed in (28) coincides with (24). Using the expressions (25), it follows that:

$$\tilde{F}_D=-I_N\otimes(B_2^TX_1)-((N_L-1-a)I_N-b\mathcal{A}(\mathcal{G}))\otimes(B_2^TX_2) \quad (29)$$

where $\boldsymbol{N_L}=\mathbf{1}+\boldsymbol{d_{max}}$ and $\boldsymbol{d_{max}}$ is the maximum number of connections for an agent.
Denoting $\boldsymbol{\mathcal{N}_{a,b}}=(\boldsymbol{N_L}-\mathbf{1}-\boldsymbol{a})\boldsymbol{I_N}-\boldsymbol{b}\boldsymbol{\mathcal{A}}(\boldsymbol{\mathcal{G}})$, the above expression becomes:

$$\tilde{F}_D=-I_N\otimes(B_2^TX_1)-\mathcal{N}_{a,b}\otimes(B_2^TX_2). \quad (30)$$

Taking into account that $\widetilde{\boldsymbol{B}}_2(\boldsymbol{\mathcal{N}_{a,b}}\otimes(\boldsymbol{B}_2^T\boldsymbol{X}_2))=\boldsymbol{\mathcal{N}_{a,b}}\otimes(\boldsymbol{B}_2\boldsymbol{B}_2^T\boldsymbol{X}_2)$, it follows that:

$$\tilde{A}_D=I_N\otimes(A-B_2B_2^TX_1)-\mathcal{N}_{a,b}\otimes(B_2B_2^TX_2). \quad (31)$$

In order to determine the domain $(\boldsymbol{a},\boldsymbol{b})$ for which the matrix $\widetilde{\boldsymbol{A}}_{\boldsymbol{D}}$ is Hurwitz one may use Proposition 2 of [9] which states that if $\overline{\boldsymbol{A}}=\boldsymbol{I_n}\otimes\boldsymbol{A}$ and $\overline{\boldsymbol{C}}=\boldsymbol{B}\otimes\boldsymbol{C}$ where $\boldsymbol{A},\boldsymbol{C}\in\mathbb{R}^{m\times m}$ and $\boldsymbol{B}\in\mathbb{R}^{n\times n}$, then $\boldsymbol{\Lambda}(\overline{\boldsymbol{A}}+\overline{\boldsymbol{C}})=\cup_{i=1}^{n}\boldsymbol{\Lambda}(\boldsymbol{A}+\boldsymbol{\lambda_i}(\boldsymbol{B})\boldsymbol{C})$ in which $\boldsymbol{\Lambda}(\cdot)$ denotes the spectrum of $(\cdot)$ and $\boldsymbol{\lambda_i}(\boldsymbol{B})$ stands for the $\boldsymbol{i-th}$ eigenvalue of $\boldsymbol{B}$. Denoting by $\boldsymbol{\lambda_i},\boldsymbol{i}=\mathbf{1},\dots,\boldsymbol{N}$ the eigenvalues of $\boldsymbol{\mathcal{N}_{a,b}}$, from the above-mentioned result it follows that

$$\boldsymbol{\Lambda}(\widetilde{\boldsymbol{A}}_{\boldsymbol{D}})=\cup_{i=1}^{N}\boldsymbol{\Lambda}\left(\boldsymbol{A}-\boldsymbol{B_2}\boldsymbol{B}_2^T\boldsymbol{X_1}-\boldsymbol{\lambda_i}\boldsymbol{B_2}\boldsymbol{B}_2^T\boldsymbol{X_2}\right). \quad (32)$$

On the other hand, using the definition of $\boldsymbol{\mathcal{N}_{a,b}}$ it results that its eigenvalues have the expressions:

$$\lambda_i=N_L-1-a-b\mu_i, i=1,\dots,N \quad (33)$$

where $\boldsymbol{\mu_i},\boldsymbol{i}=\mathbf{1},\dots,\boldsymbol{N}$ denote the eigenvalues of the adjacency matrix $\boldsymbol{\mathcal{A}}(\boldsymbol{\mathcal{G}})$. Then the following algorithm proposed in [16] is used to determine the two parameters.
Step 1. Determine $\boldsymbol{\delta_1}<\mathbf{0}$ and $\boldsymbol{\delta_2}>\mathbf{0}$ such that $\boldsymbol{\Lambda}\left(\widetilde{\boldsymbol{A}}_{\boldsymbol{D}}\right)\in\mathbb{C}^-,\forall\boldsymbol{\delta}\in[\boldsymbol{\delta_1},\boldsymbol{\delta_2}]$;
Step 2. Solve the systems of inequalities:

$$\begin{array}{l}\delta_1+1-N_L+a+b\mu_2<0\\ \delta_2+1-N_L+a+b\mu_1>0\\ b>0\end{array} \quad (34)$$

and

$$\delta_1+1-N_L+a+b\mu_1<0$$

$$\delta_2 + 1 - N_L + a + b\mu_2 > 0 \quad (35)$$
$$b < 0$$

where $\boldsymbol{\mu_1} = \boldsymbol{min_i \mu_i}$ and $\boldsymbol{\mu_2} = \boldsymbol{max_i \mu_i}$.

Although time delays represent a recently treated subject, the challenge consists in developing command algorithms for multi-agent systems taking into consideration their influences on the behavior of its members. According to [17], the delays in the communication channels are defined as the time difference between the moment when the information is transmitted and the one when it is correctly received. Hence, in order to analyze their influence, one considers the first-order delay modeled using the Padé approximation, whose transfer function is defined as follows:

$$e^{-\tau s} \simeq \frac{2 - \tau s}{2 + \tau s} \quad (36)$$

where $\boldsymbol{\tau}$ is the time delay. Since the term $\boldsymbol{x_i(t-\tau)}$ is needed, it is obtained as:

$$x_i(t-\tau) = \frac{4}{\tau} x_{p_i}(t) - x_i(t) \quad (37)$$

with $\boldsymbol{x_{p_i}}$ – the state vector of the Padé approximation for agent $\boldsymbol{i}$ and $\boldsymbol{\dot{x}_{p_i}(t)} = -\frac{\boldsymbol{2}}{\boldsymbol{\tau}}\boldsymbol{x_{p_i}(t)} + \boldsymbol{x_i(t)}$.

4. Case studies

This part includes a comparative analysis of the time evolutions for each agent. The influence of time delays in case of increasing the number of members and modifying their interconnection is studied. Therefore, the two different network configurations presented in Fig. 1 are considered.

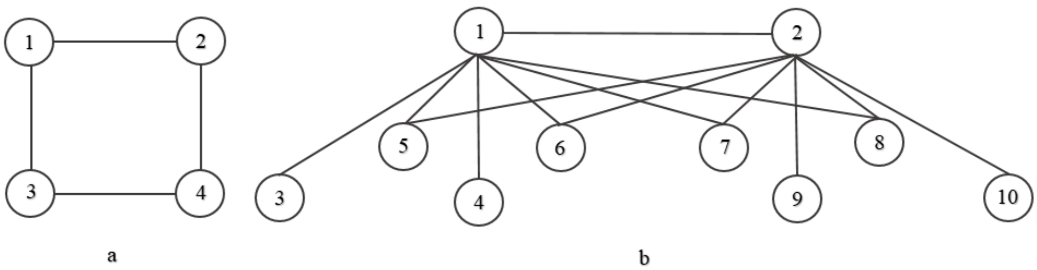

Fig. 1 Network configurations

In order to emphasize the characteristics of this approach, the numerical simulations use the decoupled dynamics of the system. In references as [21], [22], the complete dynamics of an agent is elaborated. Furthermore, the linearized model of an air vehicle given in [23] is considered. Thus, the performances of the agents are analyzed for the longitudinal dynamics characterized by the state vector $\boldsymbol{x} = [\boldsymbol{u} \quad \boldsymbol{w} \quad \boldsymbol{q} \quad \boldsymbol{\theta} \quad \boldsymbol{h}]^{\boldsymbol{T}}$ approximated by the linear system:

$$[\dot{u} \quad \dot{w} \quad \dot{q} \quad \dot{\theta} \quad \dot{h}]^T = A_{long}[u \quad w \quad q \quad \theta \quad h]^T + B_{long}[\delta_E \quad \delta_T]^T \tag{38}$$

The capacity of the agents to maintain imposed values for certain states ($\boldsymbol{u}$ and $\boldsymbol{h}$) is guaranteed by the introduction of the integrators whose states are denoted by $\boldsymbol{\eta_1}$ and $\boldsymbol{\eta_2}$. The resulting system of form (1) is written as:

$$\begin{aligned}
\begin{bmatrix} \dot{x} \\ \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} &= \underbrace{\begin{bmatrix} A_{long} & 0 & 0 \\ -C_u & 0 & 0 \\ -C_h & 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x \\ \eta_1 \\ \eta_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_{B_1} \begin{bmatrix} u_{com} \\ h_{com} \end{bmatrix} + \underbrace{\begin{bmatrix} B_{long} \\ 0 \\ 0 \end{bmatrix}}_{B_2} \begin{bmatrix} \delta_E \\ \delta_T \end{bmatrix} \\
\begin{bmatrix} \eta_1 \\ \eta_2 \\ \delta_E \\ \delta_T \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_E \\ \delta_T \end{bmatrix} \\
y_2 &= [x^T \quad \eta_1 \quad \eta_2]^T
\end{aligned} \tag{39}$$

Defining the system in this way, one can check that the two conditions $\boldsymbol{C^T D = 0}$ and $\boldsymbol{D^T D = I}$ are fulfilled. The adjacency matrix corresponding to the configuration in Fig. 1a is given below.

$$\mathcal{A}(\mathcal{G}) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \tag{40}$$

In order to obtain the distributed controller, the Riccati equations (22) and (23) are solved for $\boldsymbol{\gamma = 100}$. The set of parameters $(\boldsymbol{a}, \boldsymbol{b})$ satisfying the inequalities (34) are $\boldsymbol{a = 2}$ and $\boldsymbol{b = 0.1}$, with $\boldsymbol{\delta_1 = -0.3}$, $\boldsymbol{\delta_2 = 0.3}$. The stability of the network is proved by $\boldsymbol{Re(\lambda) < 0}$ for the eigenvalues of the matrix (31) corresponding to the closed loop system. The distributed controller obtained for this configuration has the following structure:

$$\tilde{F} = \begin{bmatrix} F_1 & F_2 & F_2 & 0 \\ F_2 & F_1 & 0 & F_2 \\ F_2 & 0 & F_1 & F_2 \\ 0 & F_2 & F_2 & F_1 \end{bmatrix} \tag{41}$$

For the obtained simulations, null initial conditions (altitudes and velocities) of the agents are considered. The performances of the controller are to achieve and maintain certain imposed constrains ($\boldsymbol{h = 10\ m}$ and $\boldsymbol{u = 3\ m/s}$) during flight simulation. The time evolutions are identical for all members due to the equal number of connections for each agent and the same initial conditions. The offset described by the red line (denoted by the index $\boldsymbol{D}$) is caused by the introduction of time delay in the communication channels. The delay has been introduced using the Padé approximation (36) with $\boldsymbol{\tau = 0.15\ sec}$. These features are illustrated in the comparative representations of velocity (Fig. 2) and altitude (Fig. 3).

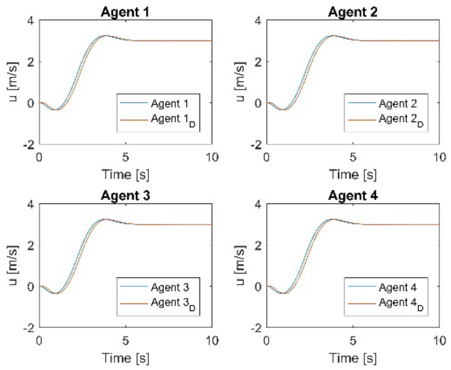

Fig. 2 Time response of velocity

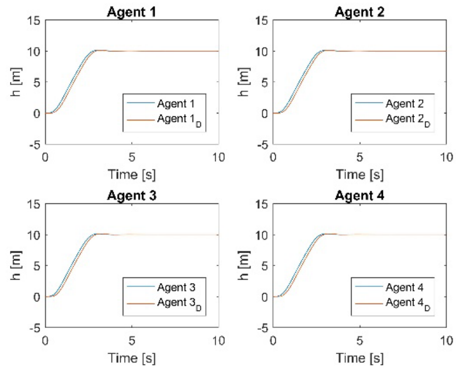

Fig. 3 Time response of altitude

For the configuration in Fig. 1b where $d_{max} = 7$, the way of agents' interconnection is defined by the following adjacency matrix.

$$\mathcal{A}(\mathcal{G}) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{42}$$

The values of the parameters for which inequalities (34) hold are: $a = 6.92, b = 0.15, \delta_1 = -0.5, \delta_2 = 0.5$. The structure of the distributed controller is obtained of form (43).

Analyzing the configuration in Fig. 1b, one can see that agent 1, with the maximum number of connections, is not connected with agents 9 and 10. This fact is denoted by the terms $\tilde{F}(1,9) = 0$ and $\tilde{F}(1,10) = 0$ in the controller structure. For all positions corresponding to failure communication, the related terms in form (43) are null.

$$\tilde{F}=\begin{bmatrix} F_1 & F_2 & F_2 & F_2 & F_2 & F_2 & F_2 & F_2 & 0 & 0 \\ F_2 & F_1 & 0 & 0 & F_2 & F_2 & F_2 & F_2 & F_2 & F_2 \\ F_2 & 0 & F_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ F_2 & 0 & 0 & F_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ F_2 & F_2 & 0 & 0 & F_1 & 0 & 0 & 0 & 0 & 0 \\ F_2 & F_2 & 0 & 0 & 0 & F_1 & 0 & 0 & 0 & 0 \\ F_2 & F_2 & 0 & 0 & 0 & 0 & F_1 & 0 & 0 & 0 \\ F_2 & F_2 & 0 & 0 & 0 & 0 & 0 & F_1 & 0 & 0 \\ 0 & F_2 & 0 & 0 & 0 & 0 & 0 & 0 & F_1 & 0 \\ 0 & F_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & F_1 \end{bmatrix} \quad (43)$$

Maintaining the same imposed conditions, the comparative time representations of the two states (altitude and velocity) illustrated in Fig. 4 and Fig. 5 are obtained. One can note the offset caused by the time delay, without affecting the network stability and the achievement of imposed objectives.

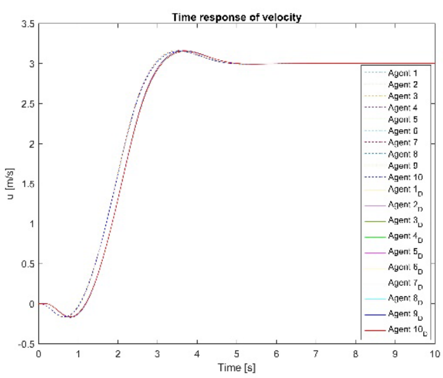

Fig. 4 Comparative time response of velocity

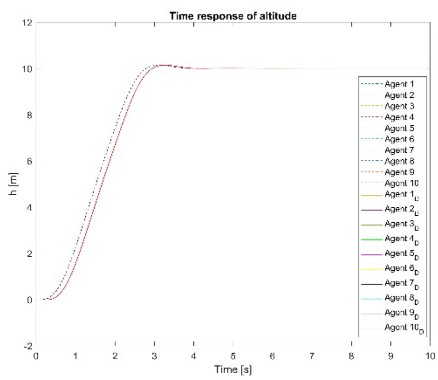

Fig. 5 Comparative time response of altitude

5. Conclusions

The present paper focuses on the solution of H_∞ problem for multi-agent systems. The distributed controller design takes into consideration the presence of time delays in communication channels. The numerical results presented in this work assume the comparative analysis of the agents' behavior. In order to highlight the characteristics of the obtained controller, two different configurations with variable numbers of agents and with distinct interconnection possibilities are considered.

The new issue introduced by the failure communication refers to the capability of the designed controller to guarantee the system stability. Regarding the structure of distributed controller for both analyzed configurations, it can be observed that the absence of connections between agents according to the corresponding adjacency matrices is denoted by the null terms in the gain matrices.

The results show the influence of time delays on the two states stabilization, their introduction being described by the offset between the case with communication delays and the ideal one. Regarding the required time to stabilize at certain imposed values, the sensitivity of the latter configuration to time delays is observed, for which slower evolutions are identified. Furthermore, it is proved that regardless the presence or absence of time delays, the multi-agent system stability is obtained.

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