INTERACTION BETWEEN A MOVING OSCILLATOR WITH DRY FRICTION AND AN INFINITE EULER-BERNOULLI BEAM ON VISCOELASTIC FOUNDATION – GREEN'S FUNCTION METHOD

Traian MAZILU[1], Vlăduț-Marian DINU[2]

The paper presents a new application of Green's function method in the moving load problem aiming to point out the basic features of the interaction between a moving two-degree oscillator with dry friction and an infinite Euler-Bernoulli beam on viscoelastic foundation in the presence of a random irregularity in contact point. Using an explicit numerical method to solve the equations of motion of the oscillator and Green's function of the beam, the contact equation is solved, and the contact force is calculated via the convolution integral. Stick-slip vibration in oscillator exhibits components of higher frequency than those induced by the irregularity. The static friction force in the dry friction element of the oscillator influences the acceleration of the upper body of the oscillator which can reach minimum value in particular conditions. This aspect could be interesting from viewpoint of the ride quality of a wagon.

Keywords: oscillator, dry friction, stick-slip, beam, viscoelastic foundation, Green's function method

1. Introduction

In this paper, the interaction between a moving two-degree of freedom oscillator with dry friction and an infinite uniform Euler-Bernoulli beam resting on viscoelastic foundation is studied in order to point out the basic features of the dynamic behaviour of a wagon running along a straight track in the presence of the track irregularity. This topic is interesting in railway field from many practical viewpoints: ride quality, rolling noise, rail corrugation and wheel out-of-roundness, ballast settlement etc.

To this end, Green's function method is applied following a new approach to include the nonlinear effect of the dry friction. Green's function method is usually applied to solve many applications of the moving load problem in railway: dynamic wheel/rail interaction [1-5] and simulation of vertical dynamic vehicle-track interaction in a railway crossing [6-7].

[1] Prof., Department of Railway Vehicles, University POLITEHNICA of Bucharest, Romania, e-mail: traian.mazilu@upb.ro

[2] PhD student, Doctoral School of Transports, University POLITEHNICA of Bucharest, Romania, e-mail: marius.gheti@upb.ro

Applying Green's function method, both track and vehicle models are linear, and the time-domain Green's functions of rail (track) and wheel (vehicle) meet in the nonlinear contact equation via the convolution integral. Solving numerically this equation, the wheel/rail contact force results and then the wheel and rail displacement are obtained using once again the convolution integral.

This time, the vehicle (oscillator) model is nonlinear due to the damping element with dry friction and a different procedure is applied: (a) the displacements of the oscillator bodies are calculated following an explicit numerical method; (b) the beam displacement is analytically calculated using Green's function of the beam in the contact point and the convolution integral and inserted in the contact equation; (c) the contact force results from contact equation; (d) beam displacement is obtained using the convolution integral.

Using the above approach, the oscillator response to a random irregularity in the contact point is calculated and analysed.

2. Mechanical model

Fig. 1 presents the interaction model between a moving oscillator with dry friction and an infinite Euler-Bernoulli beam resting on viscoelastic foundation.

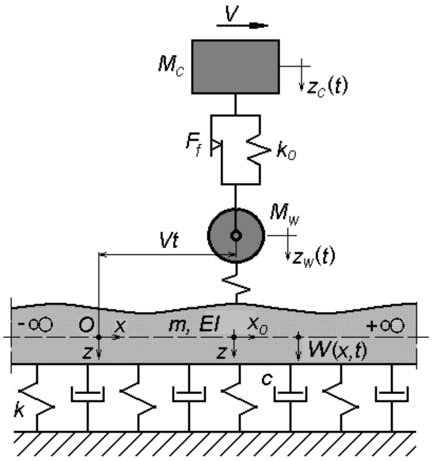

Fig. 1. Mechanical model of the moving oscillator along a beam on viscoelastic foundation.

The oscillator has two degrees of freedom and consists of two rigid bodies of M_c and M_w mass, respectively, and two elastic elements of k_o and k_H stiffness. Dry friction element of F_f friction force works between the two bodies, in parallel with the k_o elastic element.

The oscillator represents one of the simplest models of a wagon in which the upper body is for the suspended mass of the wagon that rests on one wheel (1/2 wheelset) via suspension, and the lower body is for the wheel.

The oscillator is moving with the constant velocity V along an infinite uniform Euler-Bernoulli beam of EI bending stiffness (E – Young's modulus and I – area moment of inertia) and of m mass per length unit, resting on a viscoelastic foundation of k stiffness and c damping constant per length unit.

Displacements of the oscillator bodies are $z_c(t)$ and $z_w(t)$, and the beam displacement is $w(x,t)$ in respect to the reference Oxz; t denotes the time moment.

Applying Newton's second law, the equations of motion for the oscillator can be written as follows:

- the equation of motion of the upper body

$$M_c \ddot{z}_c = F_e + F_f \tag{1}$$

- the equation of motion of the lower body

$$M_w \ddot{z}_w = Q_o - Q - F_e - F_f \,, \tag{2}$$

where F_e is the elastic force in suspension

$$F_e = -k(z_c - z_w) \,, \tag{3}$$

F_f is the dry friction force, Q_o is the static load

$$Q_o = (M_c + M_w) g \,, \tag{4}$$

where g is the gravitational acceleration and Q is the wheel-rail contact force.

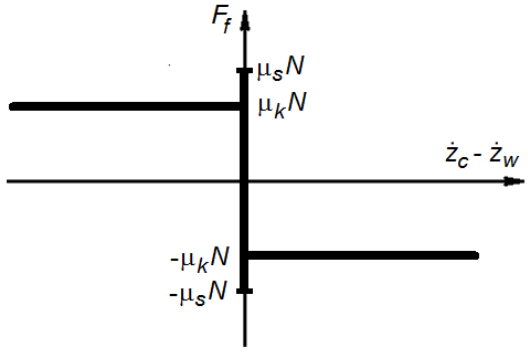

Fig. 2. Friction force.

Considering Coulomb friction law (fig. 2), the friction force can be calculated using the following equations

$$\begin{aligned} F_f &= -\mu_k N \operatorname{sgn}(\dot{z}_c - \dot{z}_w) \quad \text{for} \quad \dot{z}_c - \dot{z}_w \neq 0 \\ \left| F_f \right| &= -\mu_s N \quad \text{for} \quad \dot{z}_c - \dot{z}_w = 0 \end{aligned} \tag{5}$$

where N is the normal pressing force, μ_s - static friction coefficient and μ_k - kinematic friction coefficient.

Equation of motion for the Euler-Bernoulli beam is

$$EI \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + kw = Q \delta(x - Vt) \tag{6}$$

where $\delta(.)$ is Dirac's delta function.

The boundary conditions associated to Eq. (6) are

$$\lim_{|x-Vt|\to\infty} w(x,t)=0\,. \tag{7}$$

If the dynamic component of the contact force $\Delta Q = Q - Q_o$ is low comparing to the static load, the contact equation is

$$\Delta Q(t)=k_H\left[z_w(t)-w(Vt,t)-r(Vt)\right], \tag{8}$$

where k_H is the contact stiffness and $r(Vt)$ is the irregularity between the lower body and the beam against the contact point.

Introducing the moving reference frame $O_o x_o z$, Eqs. (6-8) become

$$EI\frac{\partial^4 w}{\partial x_o^4}+mV^2\frac{\partial^2 w}{\partial x_o^2}-cV\frac{\partial w}{\partial x_o}-2mV\frac{\partial^2 w}{\partial x_o\partial t}+m\frac{\partial^2 w}{\partial t^2}+c\frac{\partial w}{\partial t}+kw=Q\delta(x_o), \tag{9}$$

$$\lim_{|x_o-Vt|\to\infty} w(x_o,t)=0\,, \tag{10}$$

$$\Delta Q(t)=k_H\left[z_w(t)-w(0,t)-r(0)\right], \tag{11}$$

where $w(0, t)$ and $r(0)$ are the beam displacement and the irregularity between the lower body and beam at the contact point.

Next, the equations of motion are solved in terms of the displacement in respect to the steady-state position; these quantities are: $\Delta z_w(t)$, $\Delta z_c(t)$ and $\Delta w(x_o,t)$.

Beam displacement at the contact point can be calculated using the convolution integral

$$\Delta w(0,t)=\int_0^t\int_{-\infty}^{\infty} g(0,\xi,t-\tau)\Delta Q(\tau)\delta(\xi)\mathrm{d}\xi\mathrm{d}\tau=\int_0^t g(0,0,t-\tau)\Delta Q(\tau)\mathrm{d}\tau\,, \tag{12}$$

where $g(x_o, \xi, t-\tau)$ is the time-domain Green's function of the beam in the moving reference frame. This function describes the beam response in the section x_o at the $t-\tau$ time moment due to a unit impulse force applied at the section ξ at the τ time moment.

Considering a time partition – $t_0, t_1, \ldots, t_n$ (with $t_0 = 0$, $t_n = t$ and $\Delta t = t_i - t_{i-1}$ where $i = 1 \div n$), the beam displacement at the contact point can be calculated applying the following equation

$$\Delta w_n=\Delta t\sum_{j=1}^{n}\left\{\frac{g_{j-1}\Delta Q_j+g_j\Delta Q_{j-1}}{2}+\frac{(g_j-g_{j-1})(\Delta Q_j-\Delta Q_{j-1})}{3}\right\} \tag{13}$$

where $\Delta w_n = \Delta w(0, t_n)$, $g_j = g(0, 0, t_n - t_j)$, $\Delta Q_j = \Delta Q(t_j)$, and both the Green's function of the beam and dynamic component of the contact force are linear time functions.

Knowing the displacement of the lower body from an explicit integration method, the contact equation can be rewritten

$$\Delta Q_n=k_H(\Delta z_{wn}-\Delta w_n-r_n)\,, \tag{14}$$

where $\Delta z_{wn} = \Delta z_w(t_n)$ and $r_n = r(Vt_n) = r(x_o = 0)$.

Inserting Eq. (13) in (14), it results

$$\Delta Q_n = k_H \frac{z_{wn} - \frac{\Delta t}{6}\left[\sum_{j=1}^{n-1}(g_{j-1}+2g_j)\Delta Q_j + \sum_{j=1}^{n}(g_j+2g_{j-1})\Delta Q_{j-1}\right] - r_n}{1+\frac{\Delta t}{6}k_H(g_{n-1}+2g_n)}. \quad (15)$$

Finally, the beam displacement at the contact point results inserting ΔQ_n in Eq. (13).

Irregularity between the lower body and the beam can be synthesised starting from the power spectral density of the longitudinal level, recommended by the ORE report B176 [8] as representative for the European rail networks,

$$S(\Omega) = \frac{A\Omega_c^2}{(\Omega^2+\Omega_r^2)(\Omega^2+\Omega_c^2)}, \quad (16)$$

where Ω is the wave number, Ω_c = 0.8246 rad/m, Ω_r = 0.0206 rad/m, and A is a coefficient depending on the quality of the track, and applying the method recommended in ref. [9].

3. Numerical application

In this section, numerical results obtained with the model and method above are shown considering the following parameters: for oscillator - M_c = 9350 kg, M_w = 650kg, k_o = 2.5 MN/m, μ_k = 0.15, μ_s = 0.18, N = 36.699 kN, k_H= 1.68 GN/m, Q_o = 100 kN; for beam and viscoelastic foundation - EI = 6.4 MNm^2, m = 268 kg/m, k = 70 MN/m^2, c = 54.821 kNs/m^2 and Δt = 1/20000 s. Oscillator parameters correspond to 1/8 loaded wagon with Y25 bogie, and beam parameters are similar to 1/2 track with UIC 60 rail and concrete sleepers.

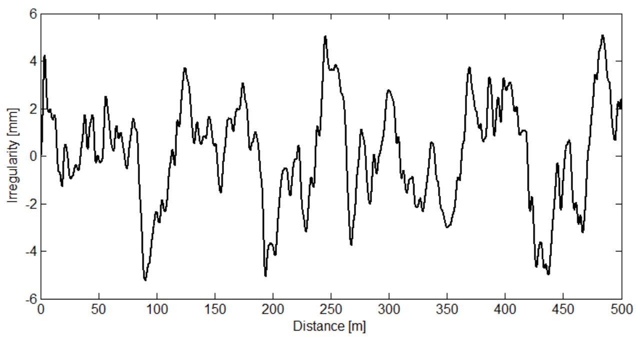

Fig. 3. Irregularity.

Fig 3 shows the irregularity between the lower body and beam for the 500 m section. Irregularity wavelength ranges from 3 to 120 m, with peak value of 5.235 mm and RMS value of 2.19 mm.

Figs. 4 and 5 present the time series and the spectrum of the upper body acceleration when the oscillator without dry friction is running with 60 km/h along the beam as reference variant. Time series of the acceleration exhibits continuous increasing due to the lack of damping.

Acceleration spectrum ranges from 0.14 to 5.55 Hz according to the frequencies induced by the irregularity at that velocity. Peak in acceleration spectrum appears at 2.56 Hz, the resonance frequency of the upper body on elastic element of the oscillator.

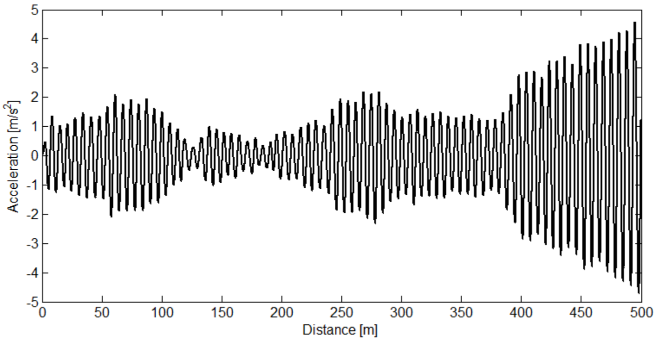

Fig. 4. Acceleration of the upper body at 60 km/h – no damping case.

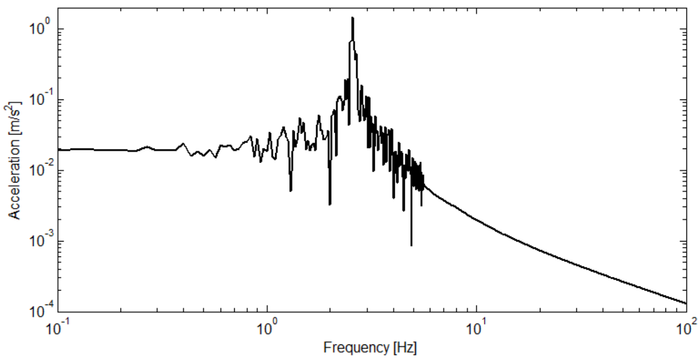

Fig. 5. Acceleration spectrum of the upper body at 60 km/h – no damping case.

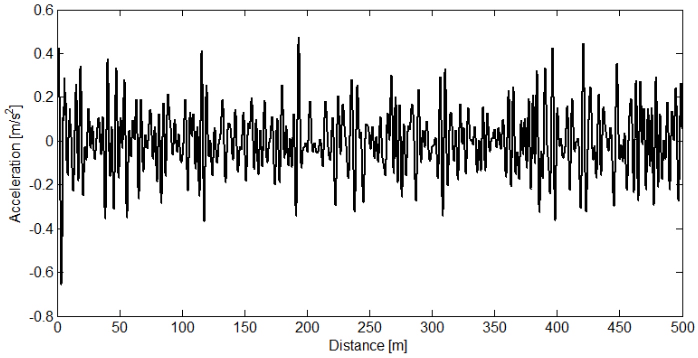

Fig. 6. Acceleration of the upper body at 60 km/h – with dry friction.

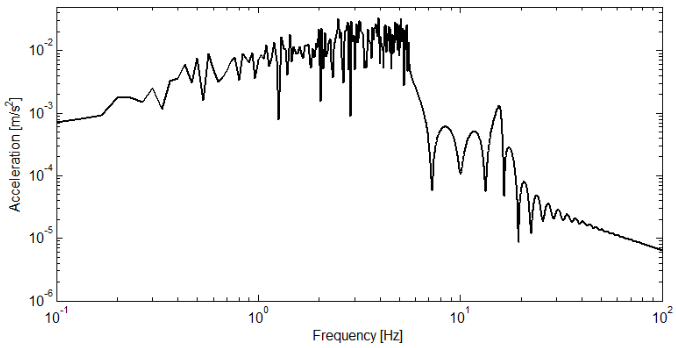

Fig. 7. Acceleration spectrum of the upper body at 60 km/h – with dry friction.

Figs. 6 and 7 present the same quantities calculated following the same conditions when the oscillator is damped. This time, the acceleration takes a stationary shape and its values are much lower. Acceleration spectrum has no peak. In fact, the suspension is locked because the friction force is lower than the static friction force in the dry friction element. This aspect can be observed in fig. 8 which displays the friction force and the elastic force in the k_o element; the elastic force is zero.

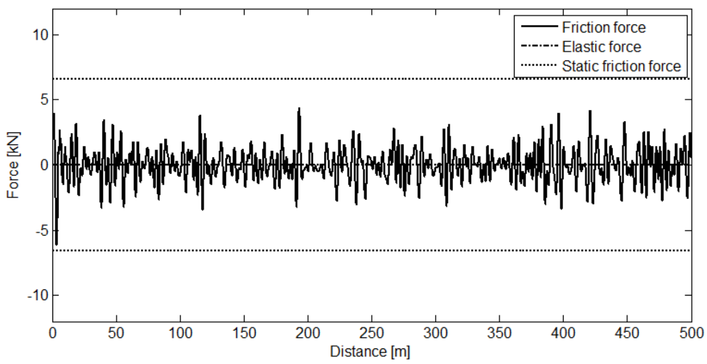

Fig. 8. Friction and elastic forces in suspension at 60 km/h – with dry friction.

As the oscillator velocity increases, the suspension begins to work, but only partially as can be seen in figs. 9 - 11 which shows the friction and elastic forces and the relative velocity between the upper and lower bodies of the oscillator at the velocity of 120 km/h. This time, the friction force reaches the value of the static friction force, meaning that the dynamic behaviour is characterised by alternating stick and slip phases. When the stick phase occurs, the elastic force does not change. This aspect is more visible in the diagram of the relative velocity between the upper and lower bodies when the relative velocity is zero. This fact happens along 248.357 m, which is about 50 % of the running distance.

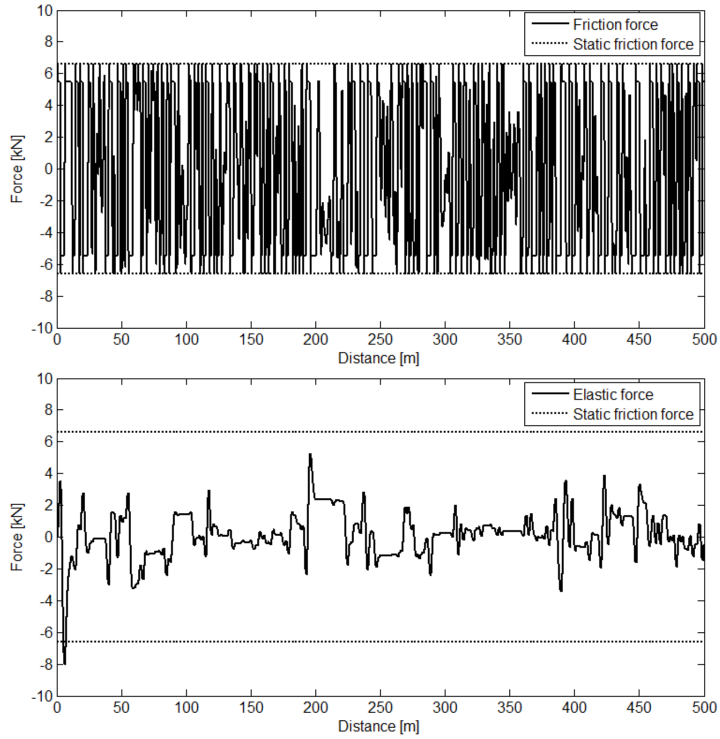

Fig. 9. Friction and elastic forces in suspension at 120 km/h – with dry friction.

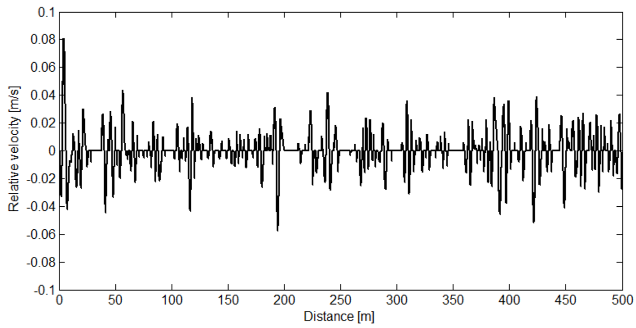

Fig. 10. Relative velocity between upper and lower bodies at 120 km/h – with dry friction.

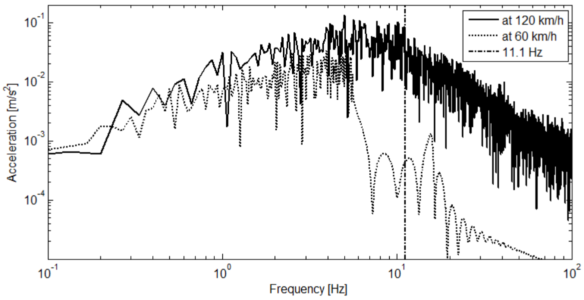

Fig. 11. Acceleration spectrum of the upper body.

Fig. 11 presents the acceleration spectrum of the upper body when the oscillator is running at 120 km/h. Acceleration spectrum of the upper body at 60 km/h is also displayed for comparison. Irregularity induces vibration with frequency between 0.28 and 11.1 Hz at 120 km/h. As shown, there is no stick-slip vibration at 60 km/h, and the acceleration spectrum has no component beyond the highest component induced by irregularity (5.56 Hz). It can be observed that the stick-slip vibration which emerges at 120 km/h has high and strong components.

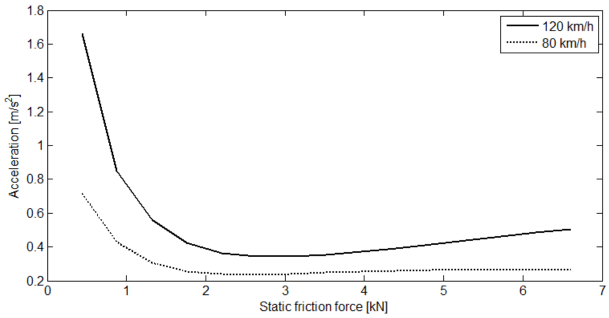

Fig. 12. Effective acceleration versus static friction force.

Fig. 12 show the upper body effective acceleration versus the static friction force when the oscillator is moving at 80 and 120 km/h, respectively. It should be mentioned that the effective acceleration of the upper body (carbody) reflects the ride quality of a railway vehicle and due to that, it is interesting to show how this parameter depends on the static friction force. Effective acceleration exhibits a pronounced decrease reaching a minimum point after which it begins to increase smoothly depending on the static friction force. Minimum point corresponds to the static friction force of 2.6 kN for both velocities, but the effective acceleration is higher at 120 km/h than at 80 km/h.

4. Conclusions

In this paper, the issue of the dynamic interaction of a two-degree oscillator with dry friction moving along an infinite uniform Euler-Bernoulli beam on viscoelastic foundation in the presence of the irregularity in the contact point is solved using Green's function method. This issue is interesting in the railway field, considering that the system consisting of the two-degree oscillator and the beam on viscoelastic foundation could be regarded as the simplified model of a wagon moving along the track.

A mixed method involving the numerical solution given by an explicit method of the nonlinear equations of the dry friction oscillator and the use of the Green function of the beam and of the convolution integral to solve the contact equation between the lower body and the beam is presented.

Numerical simulations have highlighted the appearance of the stick-slip vibrations in oscillator with higher frequency components than the frequency due to irregularities in the point of contact.

In terms of railway technique, the ride quality of the wagon depends on the static friction force in the dry friction element. A threshold of static friction force that minimizes the upper body acceleration has been identified. It is recommended that the static friction force in the dry friction element to be greater than the threshold value (and not smaller) because this does not significantly affect the ride quality.

Acknowledgments

The activity of PhD student Vlăduț-Marin Dinu in this work has been funded by the European Social Fund from the Sectorial Operational Programme Human Capital 2014-2020, through the Financial Agreement with the title "Scholarships for entrepreneurial education among doctoral students and postdoctoral researchers (Be Antreprenor!)", Contract no. 51680/09.07.2019 - SMIS code: 124539.

R E F E R E N C E S

[1]. *S. Zhang, G. Cheng, X. Sheng, D. J. Thompson*, Dynamic wheel-rail interaction at high speed based on time-domain moving Green's functions in Journal of Sound and Vibration, **vol. 488**, 8 December 2020, 115632.

[2]. X. Sheng, X. Xiao, S. Zhang, The time domain moving Green function of a railway track and its application to wheel–rail interactions in Journal of Sound and Vibration, **vol. 377**, 2016, pp 133-154.

[3]. *T, Mazilu*, Interaction between moving tandem wheels and an infinite rail with periodic supports – Green's matrices of the track method in stationary reference in Journal of Sound and vibration, **vol. 401**, 2017, pp 233-254.

[4]. *T. Mazilu, M. Dumitriu*, *C. Tudorache*, Instability of an oscillator moving along a Timoshenko beam on viscoelastic foundation in Nonlinear Dynamics, **vol. 67**, issue 2, January 2012, pp 1273-1293.

[5]. *T. Mazilu*, A dynamic model for the impact between the wheel flat and rail in UPB Scientific Bulletin, Series D: Mechanical Engineering **vol. 69**, issue 2, 2007, pp 45-58.

[6]. *X. Li, J.C.O. Nielsen, P.T. Torstensson,* Simulation of wheel–rail impact load and sleeper–ballast contact pressure in railway crossings using a Green’s function approach in Journal of Sound and Vibration, **vol. 463**, 2019, 114949.

[7]. *P.T. Torstensson, G. Squicciarini, M. Krüger, B.A. Pålsson, J.C.O. Nielsen, D.J. Thompson*, Wheel–rail impact loads and noise generated at railway crossings – Influence of vehicle speed and crossing dip angle in Journal of Sound and Vibration, **vol. 456**, 2019, pp 119-136.

[8] ORE B 176:1989 Bogies with steered or steering wheelsets Report No. 1: Specifications and preliminary studies, vol. 2. Specification for a bogie with improved curving characteristics.

[9] *M. Dumitriu*, Numerical synthesis of the track alignment and applications. Part I: the synthesis method in Transport Problems **vol. 11** (1) 2016, 19-28.