NUMERICAL MODELING THE DIMINUTION OF TRANSVERSE SECTION OF A SEWER PIPE FOR SEDIMENTS TRANSPORTED AS BED LOAD

Elena IATAN[1], Radu Mircea DAMIAN[2]

Sewer pipes designed to transport stormwater and domestic wastewater are suspect to form deposits at the pipe invert. The main problems caused by deposits are: diminution of flow capacity, augmentation of roughness and surcharging during storm events. In order to manage the effects of deposits an important parameter is the percentage of the cross section of the sewer which is reduced by the presence of the solids. A mathematical model based on the continuity equations, expressions for particle fall velocity, particle velocity and transport capacity of the sewer was numerically solved using Matlab and indicate the variation of sediment layer thickness on the pipe bed.

Keywords: sewer pipe, roughness coefficient, finite differences, Matlab.

1. Introduction

Because of its simplicity the Manning formula is very often used like design criteria for partially full sewer pipes. Usually the value of the roughness coefficient is estimated and not calculated, see [1] for values of the roughness coefficient for different pipe materials. Some design prescriptions, see [2], takes into account the differences between the laboratory conditions and installation conditions, resulting in a 30% augmentation of the roughness coefficient vis a vis the commercial presented values. The Manning roughness coefficient is expected to be affected also by the size and shape of the pipe, imperfect pipe alignment, Reynolds and Froude numbers, deposits on the pipe invert and biofilm formation, see [3].

There are also equations used to calculate the roughness coefficient, relations that can be divided in two categories: those that introduce the depth of water [3] and relations that do not introduce it (Strikler, 1923). In table 1 we may see typical values of the roughness coefficient obtained in different installation

[1] PhD, Hydraulics and Environmental Protection Department, Technical University of Civil of Bucharest, Romania, e-mail: iatan.elena@gmail.com
[2] Prof., Hydraulics and Environmental Protection Department, Technical University of Civil of Bucharest, Romania, e-mail: damian@astralnet.ro

conditions. All the experiments presented like biography of this study determine the roughness coefficient estimating the variation of water surface elevation.

We will numerically solve, using Matlab, the one dimensional mathematical model that contains conservation equations (for water and sediments) with three supplementary closure equations. The unknown bed elevation of sand deposits is calculated first for a Manning roughness coefficient of 0,01 (chosen like characteristic value for clean pipes that transport clear water) and secondly for a Manning roughness coefficient of 0,04 (value observed in laboratory with artificially created biofilm, in small diameter pipes).

Table 1

Typical values of the Manning roughness coefficient

Author	**Material, diameter (mm)**	**Pipe characteristics**	**n**	**water depth dependence**
K. Guzman & others (2007)	Pvc, 150, 200	Clean pipes, without and with sediment transport Biofilm, without and with sediment transport	0,011 0,014…0,043	Yes
Bureau of reclamation, hydraulic laboratory (1964)	Concrete, 914 106,7 121,9	Existing pipes in a functional sewer system	0,0122...0,0107 0,0193...0,0132 0,0235...0,0114 (bigger values for smaller flow rates)	Not studied directly but dependence of flow rate is observed
D. Bloodgood, J. Bell (1961)	Steel, Concrete, Asbestos. 100 – 200	Experimental clean pipes	0,004...0,013 0,0028...0,0185 0,0056...0,0199	Not studied directly but dependence of flow rate is observed. Influence of diameter not observed

2. One dimensional mathematical model and numerical solution

We consider a concrete pipe section with a transversal section of trapezoidal form, see figure 1 for dimensions. The length of the section is 500 m with a slope of 6‰. The boundary conditions, water level variation and bed elevation, are calculated knowing the flowrate respectively the concentration of sediments transported as bedload at the inlet of the pipe section. The water level and the bed elevation of sand deposits are simultaneously calculated at every time step over the entire pipe length. To this end additional information about sediment particle velocity, particle fall velocity and transport capacity of the sewer are needed.

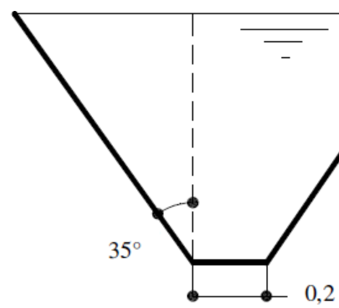

Fig. 1. Geometrical features of the cross section of the sewer pipe.

The system of equations is the one proposed by Taifur & Singh [4] in their kinematic wave theory.

Water continuity equation:

$$\frac{\partial h(1-c)}{\partial t} + \frac{\partial hu(1-c)}{\partial x} + p\frac{\partial z}{\partial t} = 0 \tag{1}$$

Sediment continuity equation:

$$\frac{\partial hc}{\partial t} + \frac{\partial huc}{\partial x} + (1-p)\frac{\partial z}{\partial t} + \frac{\partial q_{bs}}{\partial x} = 0 \tag{2}$$

Three supplementary closure equations are needed. The first one is derived from the momentum equation neglecting the local and convective acceleration therms :

$$S = S_i \tag{3}$$

The second supplementary equation is called Velikanov equation and relates flow variables and the concentration of sediments transported in suspension.

$$c = \frac{Ku^3}{gv_f h} \tag{4}$$

Velikanov observed that for given flow characteristics the transport capacity is not a fixed value but an interval delimited by a minimum and a maximum value. For a sewer system Combes (1982) suggest a minimum value for the coefficient K between 0,0005...0,002 and a maximum value for K between 0,002...0,007. In this study a value of 0,0005 is considered for the coefficient of sediment transport capacity.

Tayfur and Singh (2006) derived a sediment flux equation in terms of bed elevation starting from the equation proposed by Langbein & Leopold [5]. For a detailed demonstration of this relation see [4].

$$\frac{\partial q_{bs}}{\partial x} = (1-p)v_s\left(1-\frac{2z}{z_{\max}}\right)\frac{\partial z}{\partial x} \tag{5}$$

The system formed by equations (1) to (5) contain five relations and five unknowns h, u, c, z and q_{bs}.

To estimate the fall velocity we used in this study the relation proposed by Zhang which is said to be valid for laminar, transitory and turbulent flow regime. Other relations obtained by empirical and theoretical considerations can be found in the literature with their validity intervals.

$$v_f = \sqrt{\left(13.95\frac{\upsilon}{d_s}\right)^2 + 1.09\frac{\rho_s-\rho_f}{\rho_f}gd_s} - 13.95\frac{\upsilon}{d_s} \tag{6}$$

The water density and kinematic viscosity, sand grain diameter and density are: $\rho_s = 2650$ kg/m^3, $\rho_f = 1000$ kg/m^3, $d_s = 0{,}5$ mm and υ = 0.000001139 m^2/s. Particle diameter was considered uniform and the shape factor was considered equal to 1 (the sand grain is a sphere). The sediments are considered non cohesives.

To calculate the particle velocity we used the relation proposed by Chien and Wan (1999), relation 9.

$$v_s = u - \frac{(0{,}714u_c)^3}{u^2} \tag{7}$$

u_c is the critical flow velocity at incipient sediment motion and it is calculated from particle Reynolds number.

$$u_c = \begin{cases} \dfrac{2.5 v_f}{\log(\mathrm{Re}^*) - 0.06} + 0.66 v_f, & 1.2 < \mathrm{Re}^* < 70 \\ 2.05 v_f, & \mathrm{Re}^* > 70 \end{cases} \tag{8}$$

where: Re^* is the particle Reynolds number,

$$\mathrm{Re}^* = \frac{u_* d_s}{\upsilon} \tag{9}$$

and u_* is the shear velocity, $u_* = \sqrt{g \cdot h \cdot S}$ (10)

The correlation between the values obtained for shear velocity and fall velocity correspond to a transport of sediments as bedload.

The equations (1) to (5) forme a closed system of equations. Boundary conditions will be specified to solve the system. Initial conditions (at the moment $t = 0$) of the problem assume known values for water depth and for bed elevation.

$$h(t = 0, x) = h_0 \text{ and } z(t = 0, x) = z_0 \tag{11}$$

The upstream boundary conditions in therms of flowrate and inflow of sediments can be visualized in Fig. 2 and the values may correspond to a simplified rain event. The total time of 30 minutes for simulation is diveded in 3 equal intervals of 10 minutes each. The results obtained at the end of every interval represent initial conditions for the following one. Sediment inflow is considered only in the upstream section.

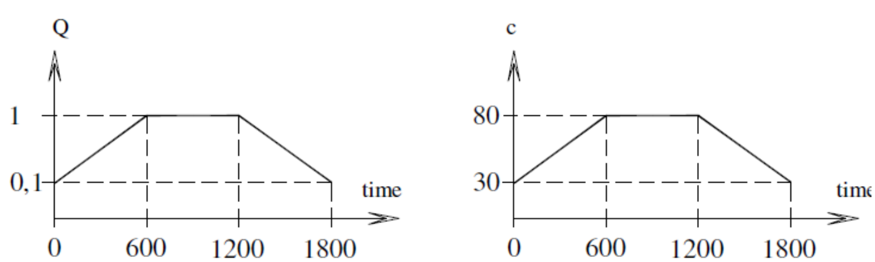

Fig. 2. Upstream boundary conditions for flowrate and inflow concentration of sediments. Time is in seconds, flowrate in m^3/s and the concentration of sediments kg/m^3.

The inflow boundary condition is written in terms of water depth using the Manning relation, (12). The areal concentration is written in terms of bed elevation using relation (13).

$$u = n^{-1} R_h^{2/3} S^{1/2} \tag{12}$$

$$c = (1 - p) z \rho_s \tag{13}$$

All the calculations where made for two values of the roughness coefficient n : 0,01 and 0,04, values that corresponds to a possible minimum and a possible maximum if we consider the cases presented in table 1.

For uniform flow the downstream boundary conditions are:

$$h(i+1, j+1) = h(i, j+1), \quad z(i+1, j+1) = z(i, j+1) \tag{14}$$

where: $i+1$ is the last point in the space domain and $j+1$ is the last time step for calculation.

The calculations where made using Matlab. The system of equations is discretized with finite differences and an explicit formulation is proposed for the unknowns $h(i, j+1)$ and $z(i, j+1)$. For the stability of the explicite form the Courant number criteria should be accomplished.

$$Cr = \frac{\left(u + \sqrt{gh}\right)\Delta t}{\Delta x} < 1 \tag{15}$$

Courant numbers between 0,3 and 0,7 has been imposed. This stability criterium ensure us that the dependence domain of the discretised equation is contained in the dependence domain of the partial differential equation but this is not a guarantee for stability of the solution. Missing experimental measurements for a sewer the results of the proposed numerical code has been tested for the geometry and boundary conditions of a river presented in [4] and the results have been considered satisfactory.

3. Bed elevation evolution

There is a need to predict the rate and magnitude by which sediments are deposited downstream of a sewer pipe and how it influences sewer bed morphology. The present paper reports the results of a small numerical investigation on the bed load sediment transport.

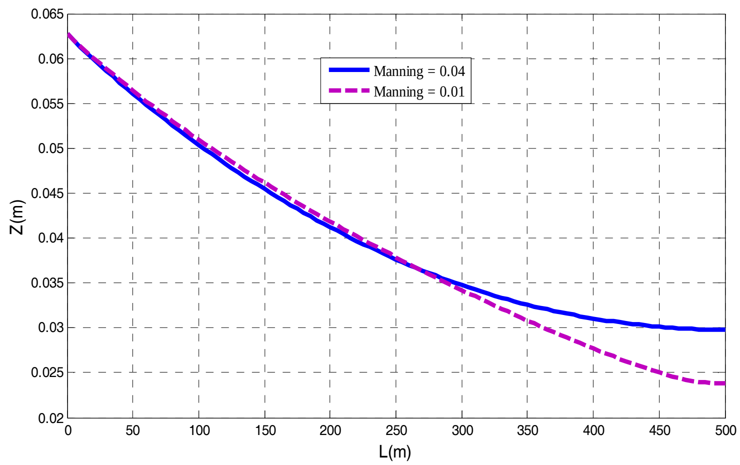

Fig. 3. Bed level evolution for the ascending interval of the boundary conditions for two different values of the Manning roughness coefficient.

The value of the roughness coefficient is an important parameter to consider because directly influences the flow velocity and the particle velocity, see relations (11) and (12). For the first 600 seconds of the boundary conditions presented in figure 2 we observe that the bed level is increasing all over the length of the sewer. The thickness of deposits is more important at the beginning of the sewer pipe. In the first 50% of the sewer pipe a slightly bigger bed elevation is observed in the case of the smaller roughness coefficient. In the last 50% of the sewer length the deposits are clearly more important for the bigger roughness coefficient.

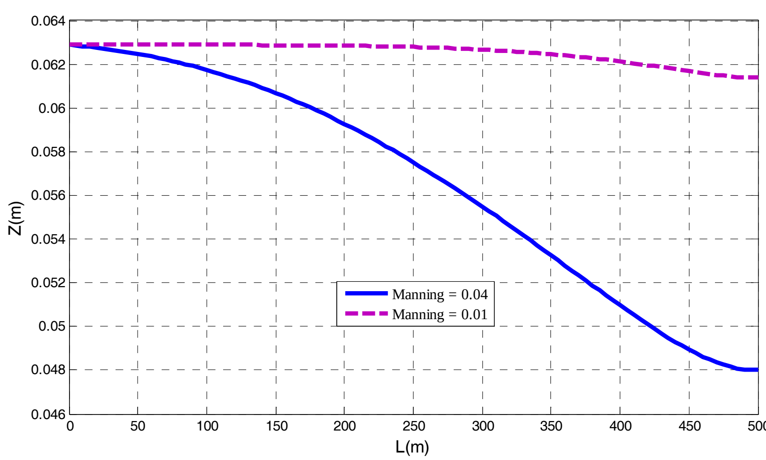

Fig. 4. Bed level evolution for the constant interval of the boundary conditions for two different values of the Manning roughness coefficient.

From figure 4 it can be seen more pronunced that for the lower values of the roughness coefficient (which correspond to a higher mean velocity) the sediments move faster in the downstream direction and the bed level is increasing in the entierly length of the pipe. The space step was chosen of 5m and the time step was chosen to obtain the Courant number smaller than 1. A similarly bed evolution is found in Tayfur and Singh (2006) in their experimental verified study concerning the movement of bed profiles in alluvial channels.

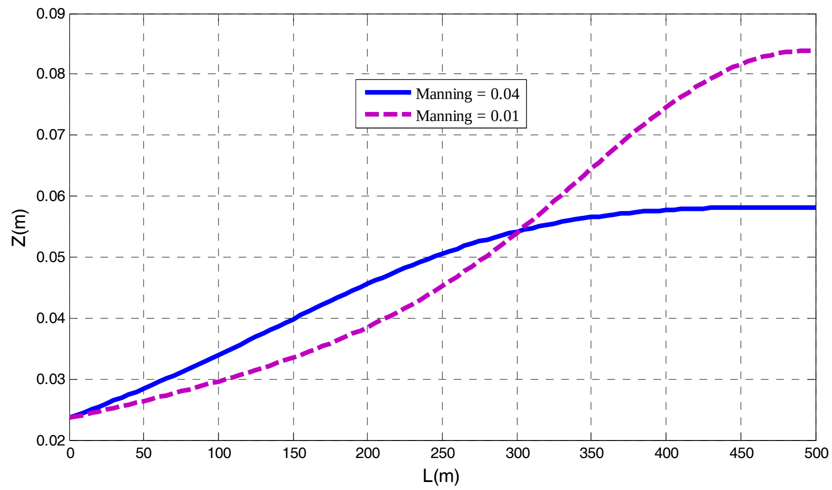

Fig. 5. Bed level evolution for the descending interval of the boundary conditions for two different values of the Manning roughness coefficient.

From figure 5 it can be seen that during the recession interval of the boundary conditions (the end of the storm event) in the case of a higher velocity (lower roughness coefficient) the sewer pipe is cleaner in the first 60% of its length. A higher velocity will transport the sediments faster in the downstream and the bed elevation is more important, in that case, at the end of the pipe.

4. Conclusions

In this model we proposed a uniform and unsteady waterflow without important suspended sediments concentration. The sediments are transported as bedload and there is no exchange of sediments between water phase and the bed load transport. The mathematical model is numerically solved using finite differences.

The results reveal that the roughness coefficient, which is usually not calculated but simply a proposed value, strongly affects the thickness of deposits at the pipe invert and the position of the pipe section where the risk of clog is more important.

Only the effect of the roughness on the mean velocity is considered. It should be beneficial to study the effects of the roughness on the Reynolds stress and turbulence intensity. Also, the studies that offer a relation to calculate the roughness coefficient in the case of bed load transport of sediments are not many and the results obtained at the laboratory model scale are difficult to extrapolate to the prototype scale.

Notations

n Manning roughness coefficient
h water deph (m)
R_h hydraulic radius (m)
z bed elevation of sediments transported as bed load (m)
$z_{\max}$ maximum bed elevation of sediments transported as bed load (m)
L length of sewer pipe (m)
p fixed deposited sediments porosity (m^3/m^3)
v_f fall velocity of sand grain (m/s)
v_s particle velocity when concentration of sediments tends to zero (m/s)
u water mean velocity (m/s)
u_c critical water velocity at incipient sediments motion (m/s)
u_* shear velocity (m/s)
g gravitational acceleration (m/s^2)
c concentration of sediments in water phase (m^3/m^3)
q_{bs} flux of sediments in bedload transport (m^2/s)
Q water flowrate (m^3/s)
S sewer pipe slope (%)
S_i friction slope (%)
K coefficient of sediment transport capacity
υ kinematic viscosity of water (m^2/s)
d_s diameter of transported grain (m)
ρ_f water density (m)
ρ_s sand grain density (kg/ m^3)
Re_* particle Reynolds number (-)
$t = 0$ initial time
Δt discretisation time step
Δx discretisation space step

j time increment
i space increment
Cr Courant number

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