U.P.B. Sci. Bull., Series C, Vol. 81, Iss. 3, 2019 ISSN 2286-3540

A NEW APPROACH FOR NONLINEAR ANALOG CIRCUITS ANALYSIS

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This paper presents a new version of the hybrid method for nonlinear resistive circuit analysis. The approach for the generation of the hybrid equation is the separation of linear and nonlinear part of the circuit.

The efficiency comes from reducing computing time and memory. The main advantage is to evaluate, only once, at the beginning of the iteration process those parameters in the circuit equations that depend on the linear element terms.

Keywords: hybrid equations, Maple, analog circuit, resistive circuit

1. Introduction

There are several types of equations that describe nonlinear resistive circuits. The nonlinear circuits are described by a several types of equations. Among them, hybrid equations [1-3] are frequent used in the theoretical study of nonlinear circuits because they have an easily analyzed structure. Hybrid equations present also the advantage of numerical analysis of circuits because they consist of a small number of variables and are separable. However, the hybrid equations are rarely used in practical applications because their formulation is quite difficult [3]. This paper considers new approaches for analysis a non-linear circuits: one using hybrid equations, Maplesoft and second one using Matlab for Chua's circuit analysis.

2. Description of algorithm for formulation of hybrid equations

Hybrid equations have an easy-to-analyze structure and have the advantage that they are separable and consist of a relatively small number of variables. Because of these points hybrid equations are applied in the theoretical study of nonlinear resistive circuits.

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Analog circuits must meet certain conditions in order to formulate hybrid equations, the main one is that the circuit should allow the existence of a normal tree. Normal tree is a tree with restrictions that satisfy the following conditions in this order of priority,[8] :

a) Contains all independent ideal sources and voltage controlled

b) all nonlinear items controlled in voltage

c) contains a maximum possible number of linear resistor

d) contains no ideal independent source or controlled by current and any nonlinear element that is current controlled

e) controlled dimensions of the controlled sources are associated with linear resistors or independent variables.

The hybrid method allows in the circuit a mixture of voltage-controlled (v.c.) nonlinear capacitors, current and voltage-controlled resistors, current-controlled (c.c.) nonlinear inductor, all four types of linear controlled sources and linear inductors, capacitors, resistors, independent sources.

The algorithm is to substitute all nonlinear elements in voltage controlled (c.u) or current controlled (c.i.) by ideal independent voltage sources respective ideal independent current sources and to replace the linear capacitors and inductors by resistive discrete circuit models associated with a numerical integration algorithm (for example, the backward Euler algorithm), in this way we obtain a linear and time-invariant circuit.

The method is more efficient if the analyzed circuit includes more linear circuit elements and controlled sources.

The hybrid equation notation, [8]:

$$X_{j+1}^{(m+1)} = \begin{bmatrix} i_{u,j+1}^{(m+1)} \\ u_{i,j+1}^{(m+1)} \end{bmatrix}; \qquad H = \begin{bmatrix} G_{u,u} & B_{u,i} \\ C_{i,u} & R_{i,i} \end{bmatrix};$$

$$x_{j+1}^{(m+1)} = \begin{bmatrix} u_{u,j+1}^{(m+1)} \\ i_{i,j+1}^{(m+1)} \end{bmatrix}; \; V_{j+1} = \begin{bmatrix} G_{u,e} & B_{u,j} \\ C_{i,e} & R_{i,j} \end{bmatrix} \begin{bmatrix} e_{j+1} \\ j_{j+1} \end{bmatrix}$$

$$V_{LC,j} = \begin{bmatrix} G_{u,L} & B_{u,C} \\ C_{i,L} & R_{i,C} \end{bmatrix} \begin{bmatrix} e_{L,j} \\ j_{C,j} \end{bmatrix} \; X_{j+1}^{(m+1)} = Hx_{j+1}^{(m+1)} + V_{j+1} + V_{LC,j}, \tag{1}$$

H represents the hybrid matrix of the circuit; *Vj+1* is the source vector corresponding to the independent voltage and current sources at the moment *tj+1*, and $V_{LC,j}$ represents the source vector corresponding to the companion models of the linear inductors and capacitors, at the previous time moment

$B_{u,i}$ $C_{i,u}$ are the current (voltage) gain matrix of the tree-branch (link) v.c. (c.c.) nonlinear elements in respect of the link (tree-branch) c.c. (v.c.) nonlinear elements;, $u_{u,(j+1)}^{(m+1)}$ $\left(i_{i,(j+1)}^{(m+1)}\right)$ the voltage (current) vector of the v.c. (c.c.) tree-branch (link) nonlinear elements at the time moment *tj+1*, and the (*m+1*)the iteration; $j_{C,j}$ is the voltage (current) vector of the ideal independent voltage (current) sources from the companion schemes of the linear inductors (capacitors) at the time moment *tj* (or at the previous time steps).

The nonlinear resistor characteristics approximated by piecewise linear continuous curves have, for the time moment *tj+1*, and the (*k+1*)the iteration:

for the v.c. nonlinear resistors:

$$i_{Ru,j+1}^{(m+1)} = \hat{i}_{Ru}(u_{Ru,j+1}^{(m+1)}) = G_{du}\left(V_{j+1}^{(m)}\right)\cdot u_{Ru,j+1}^{(m+1)} + j_{Ru}\left(V_{j+1}^{(m)}\right),$$
$$u_{Ru}^{-}\left(V_{j+1}^{(k)}\right) \le u_{Ru,j+1}^{(m+1)} \le u_{Ru}^{+}\left(V_{j+1}^{(m)}\right) \qquad (2)$$

for the c.c. nonlinear resistors

$$u_{Ri,j+1}^{(m+1)} = \hat{u}_{Ri}(i_{Ri,j+1}^{(m+1)}) = R_{di}\left(V_{j+1}^{m}\right)\cdot i_{Ri,j+1}^{(m+1)} + e_{Ri}\left(V_{j+1}^{(m)}\right),$$
$$i_{Ri}^{-}\left(s_{j+1}^{(k)}\right) \le i_{Ri,j+1}^{(k+1)} \le i_{Ri}^{+}\left(V_{j+1}^{(k)}\right) \qquad (3)$$

The current expression of a v.c. nonlinear capacitor, when its characteristic is approximated by piecewise linear continuous curve, for the time moment t_{j+1}, and the (*k+1*)[th] iteration (using the backward Euler integration algorithm):

$$i_{Cu,j+1}^{(k+1)} = G_{dCu}\left(s_{j+1}^{(k)}\right)\cdot u_{Cu,j+1}^{(k+1)} + \hat{j}_{Cu,j+1} - j_{Cu,j}$$
$$G_{dCu}\left(s_{j+1}^{(k)}\right) = \frac{C_{du}\left(s_{j+1}^{(k)}\right)}{h};\ \hat{j}_{Cu,j+1} = \frac{Q_{Cu}\left(s_{j+1}^{(k)}\right)}{h};\ j_{Cu,j} = \frac{q_{Cu,j}}{h}. \qquad (4)$$

The expression of a c.c. nonlinear coil, when its characteristic is approximated by piecewise linear continuous curve, for the time moment t_{j+1}, and the (*k+1*)[th] iteration (using the backward Euler integration algorithm):

$$u_{Li,j+1}^{(k+1)} = R_{dLi}\left(s_{j+1}^{(k)}\right)\cdot i_{Li,j+1}^{(k+1)} + \hat{e}_{Li,j+1} - e_{Li,j},$$

Where $R_{dLi}\left(s_{j+1}^{(k)}\right) = \frac{L_{di}\left(s_{j+1}^{(k)}\right)}{h};\ \hat{e}_{Li.j+1} = \frac{\Phi_{Li}\left(s_{j+1}^{(k)}\right)}{h};\ e_{Li,j} = \frac{\Phi_{Li,j}}{h};$

$$u_{Li,j+1}^{(k+1)} = \frac{1}{h}\left[L_{di}\left(s_{j+1}^{(k)}\right)\cdot i_{Li,j+1}^{(k+1)} + \Phi_{Li}\left(s_{j+1}^{(k)}\right) - \Phi_{Li,j}\right] \qquad (5)$$

where $s_{j+1}^{(k)}$ is a certain segment at the moment t_{j+1} şi iteration(*k*), and parameters $L_{di}\left(s_{j+1}^{(k)}\right)$ şi $\Phi_{Li}\left(s_{j+1}^{(k)}\right)$ represents the slope, the differential inductance, and the ordinate of origin of the corresponding segment $s_{j+1}^{(k)}$. The range $\left[i_{Li}^{-}\left(s_{j+1}^{(k)}\right), i_{Lci}^{+}\left(s_{j+1}^{(k)}\right)\right]$ represents the definition domain of the segment $s_{j+1}^{(k)}$.

Substituting the linear piecewise characteristics of the nonlinear circuit elements into the equations (1), we obtain:

$$\begin{bmatrix} \mathbf{G}_{dvc}\left(s_{(k)}^{(j+1)}\right) - \mathbf{G}_{vc,vc} & -\mathbf{B}_{vc,cc} \\ -\mathbf{A}_{cc,vc} & \mathbf{R}_{dcc}\left(s_{(k)}^{(j+1)}\right) - \mathbf{R}_{cc,cc} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{vc(k+1)}^{(j+1)} \\ \mathbf{i}_{cc(k+1)}^{(j+1)} \end{bmatrix} = \\ = V^{(j+1)} + V_{LC}^{(j)} + \begin{bmatrix} \hat{\mathbf{j}}_{vc}\left(s_{(k)}^{(j+1)}\right) \\ \hat{\mathbf{e}}_{cc}\left(s_{(k)}^{(j+1)}\right) \end{bmatrix} + \begin{bmatrix} \mathbf{j}_{vc}^{(j)} \\ \mathbf{e}_{cc}^{(j)} \end{bmatrix}. \tag{6}$$

The expression (6) contains $m=m_1+m_2$ independent equations in m_1 unknown voltages – voltage vector $\boldsymbol{v}_{vc(k+1)}^{(j+1)}$, and m_2 unknown currents – current vector $\boldsymbol{i}_{cc(k+1)}^{(j+1)}$, and is called the hybrid equation of the nonlinear circuit *C*.

The formulation algorithm for hybrid equations (1) for nonlinear analog circuits involves the following steps:

P1. Select a normal tree under the conditions mentioned above;

P2. First we replace all the nonlinear elements in voltage controlled (c.u) with equivalent ideal independent voltage sources, and the ones in current controlled (c.i.) with equivalent ideal independent current sources;

P3. Second we replace linear coils (magnetic coupled or not) and linear capacitors with equivalent resistive companion discrete circuits, (at appropriate time = + *h, h* being the integration step), associated with a default integration algorithm. Thereby, we obtain a linear resistive circuit which contain linear resistors (including the ones from the equivalent companion circuits of linear coils and capacitors), independent voltage and current sources (including those which have been substituted the nonlinear elements of circuit, c.u respectively c.i.) and controlled sources;

P4. It examines with a suitable simulation program the resistive circuit obtained in step P3. Thus obtaining the full symbolic expressions or numeric-symbolic of currents vector of nonlinear elements c.u, and the tensions of nonlinear elements c.i depending on: vector electromotive voltage (currents) of the ideal independent voltage source (current), depending on the vector electromotive tension (current) of equivalent companion schemes of linear coil (linear capacitors) and depending on the independent variables. From the expression vectors i_u and u^i is determined the hybrid matrix *H,* vector *Vj+1* source vector at the time determined by independent voltage and current sources and the vector V_{LC} - source vector corresponding to independent sources from discrete and resistive models, associated with a default integration algorithm of linear coils and capacitors;

The algorithm of formulation and solving hybrid equations can be used for a large class of nonlinear analog circuits.

The goal is to obtain a simple, flexible and general algorithm.

In order to calculate the symbolic expressions of the hybrid matrices we use the simulator Maple 15. Maple (developed by Maplesoft) is a math software that is used to analyze, explore, visualize, and solve mathematical problems. Maple has a powerful math engine and covers also aspects of technical computing, including visualization, data analysis, matrix computation, and connectivity. This makes it the ideal tool for both education and research.

Some of the helpful functions in Maple: coeff - extract a coefficient of a polynomial, collect - collection after power coefficients, subs - substitute in an expression subexpression, read File- read a file and executes it.

At step 4, the suitable simulation program that examine the resistive circuit is Asinom.

We use the program Asinom to calculate the solution of the entire circuit (tension and current of all analyzed circuit). The two output files contains one the modified nodal-analysis equations and the other output file has the tensions and currents of all analyzed circuit.

3. Calculation of hybrid equations on analog circuits

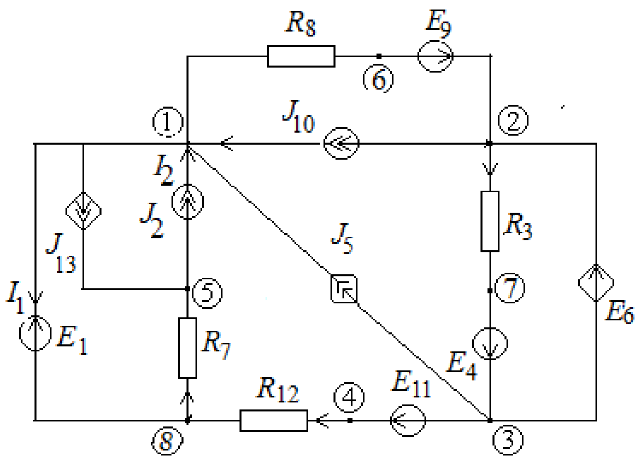

Fig. 1. Initial scheme of the circuit

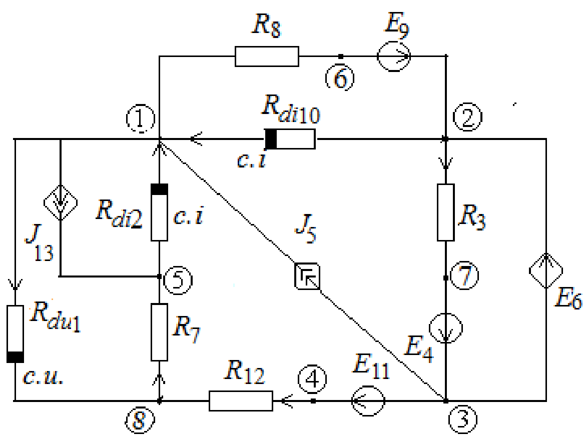

Fig. 2 equivalent circuit diagram of the Fig. 1, where nonlinear resistors c.i or c.u. are replaced with the ideal source equivalent current independent

The input file which describe the circuit in Fig.2:

13	7 3 E4	5 1 J2
8	3 2 e6(u) 8	1 5 j13(i) 3
1 6 R8	3 1 j5(u) 12	3 4 E11
6 2 E9	8 5 R7	4 8 R12
2 7 R3	8 1 U1	2 1 J10

Here are the Kirchhoff's equations for Fig. 2:

+ (+G8)*V1 + (-G5_12)*V4 + (-G8)*V6 + (-1)*I8 + (+B13_3)*I3 = +J2+J10 **(T1K→1)**
+ (-1)*I2 + (-1)*I5 + (+1)*I3 = -J10 **(between nodes 3→4)**
+ (+G5_12)*V4 + (-1)*I4 + (+1)*I11 + (+1)*I5 = 0 **(T1K→3)**
+ (+G12)*V4 + (-1)*I11 = 0 **(T1K→4)**
+ (+G7)*V5 + (-B13_3)*I3 = -J2 **(T1K→5)**
+ (-G8)*V1 + (+G8)*V6 + (+1)*I8 = 0 **(between nodes 1→6)**
+ (+1)*I4 + (-1)*I3 = 0 **(T1K→7)**
+ (-1)*V2 + (+1)*V6 = -E9 **(between nodes 6→2)**
+ (-1)*V3 + (+1)*V7 = -E4 **(between nodes 7→3)**
+ (-1)*V1 = -U1 **(between nodes 1→8)**
+ (+1)*V3 + (-1)*V4 = -E11 **(between nodes 3→4)**
+ (-A6_8)*V1 + (-1)*V2 + (+1)*V3 = 0 **(T2K 3→2)**
+ (+G3)*V2 + (-G3)*V7 + (-1)*I3 = 0 **(between nodes 2→7)**

Where G8=1/R8 , G12=1/R12, G7=1/R7, G5=1/R5, G8=1/R8, G3=1/R3.

In order to calculate the symbolic expressions of the hybrid matrices we use the simulator Maple 15. The program developed below in Maple has a simple, flexible and general algorithm, see Appendix A

Numerical results of currents and voltage: U1=515.99, I1=5.15, U2=-100, I2=5.15, U3=-24.78, I3=-0.12, U4=-20.0, I4=-0.12,U5=44.78, I5=4.71, E5=-44.78, U6=-371.21, I6=-3.8, J6=-3.87, U7=446.82, I7=4.46, U8=22.39, I8=-

5.43, U9=-424.43, I9=4.22, U10=424.43, I10=-0.24, J10=-0.24, U11=-200.00, I11=-0.96, U12=-193.60, I12=-0.96, U13=-415.99, I13=10.00

4. Analysis on Chua's nonlinear circuit

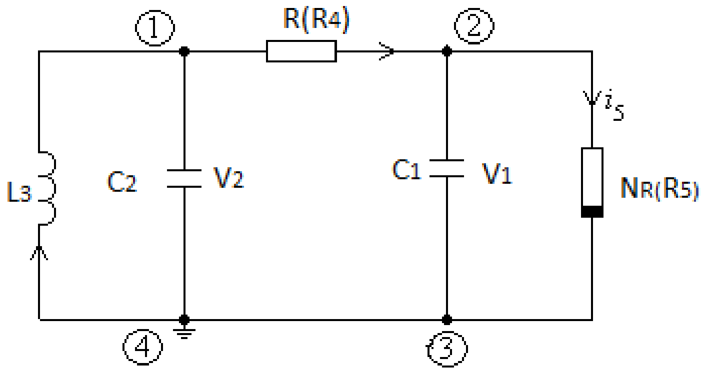

Fig. 3. Chua's nonlinear circuit

Chua's circuit consists of a linear inductor, a linear resistor, two linear capacitors and a single nonlinear resistor *R5*. The non-linear feature is a negative resistor type (all linear segments have a negative slope), which means it can not be implemented through a network that contains only resistors, inductors and capacitors [9-10].

Chua's circuit may be described by three ordinary differential equations

$$\frac{Ldi_L}{dt} = -V_{C1}$$

$$\frac{C1dV_{C1}}{dt} = i_L - \frac{V_{C2} - V_{C1}}{R}$$

$$\frac{C2dV_{C2}}{dt} = \frac{V_{C2} - V_{C1}}{R} - g(V_{C2}) \qquad (7)$$

It is noted that it was considered as state variables i_L, V_{C1} and V_{C2} [4-6].
Interpretations of the three state equations are [6-7]:

1. The voltage on the inductance coil *L* is equal to the voltage on capacitor *C1* considered opposite.

2. The current through capacitor *C1* is equal to the difference between the current through coil and the current through resistor *R4*.

3. The current through capacitor *C2* is equal to the difference between the current through resistor *R4* and current through non-linear element *R5*.

The Fourier analysis of Chua's circuit using Matlab. In order to review the nonlinearity of Chua's circuit, a Fourier analysis was performed in Matlab as presented in the images below:

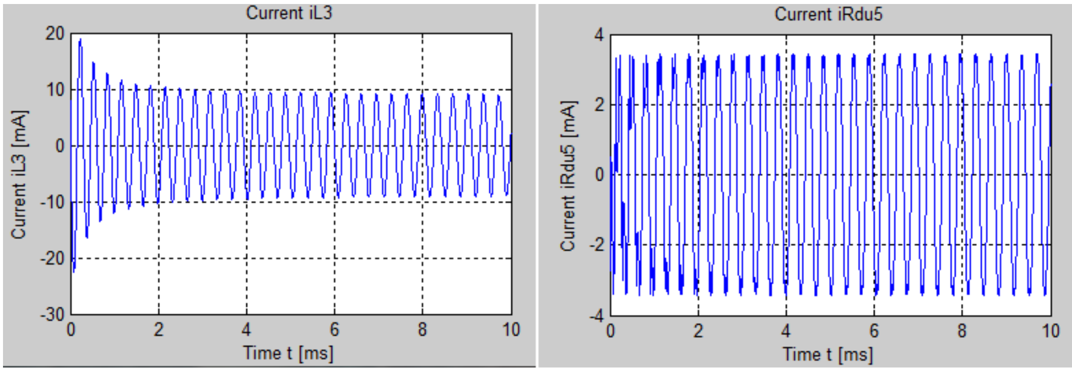

Fig. 4. The current iL_3 and current i_{R5} of Chua's circuit

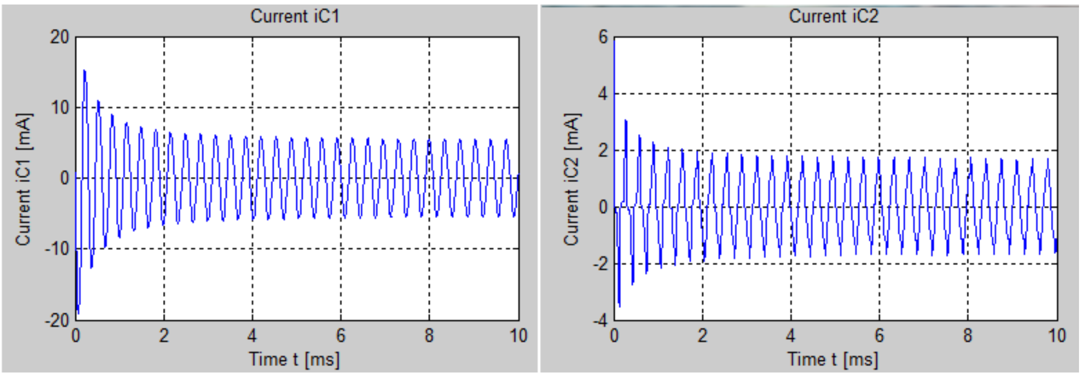

Fig. 5. The current iC_1 and current iC_2 of Chua's circuit

In Fig. 6, it can be observed the behavior associated with a periodic oscillator with an unstable solution, characterized by a trajectory attracted to the limit cycle.

Afterwards, a large outer limit cycle is presented in Fig.7 and Fig.8 corresponding to the outer segments of Chua's diode's characteristics known as the boundary crisis where the active resistor becomes eventually passive.

The simulation results support the fact that the Chua circuit is an oscillator with a complex behavior, characterized by state bifurcations and a tendency towards chaotic behavior.

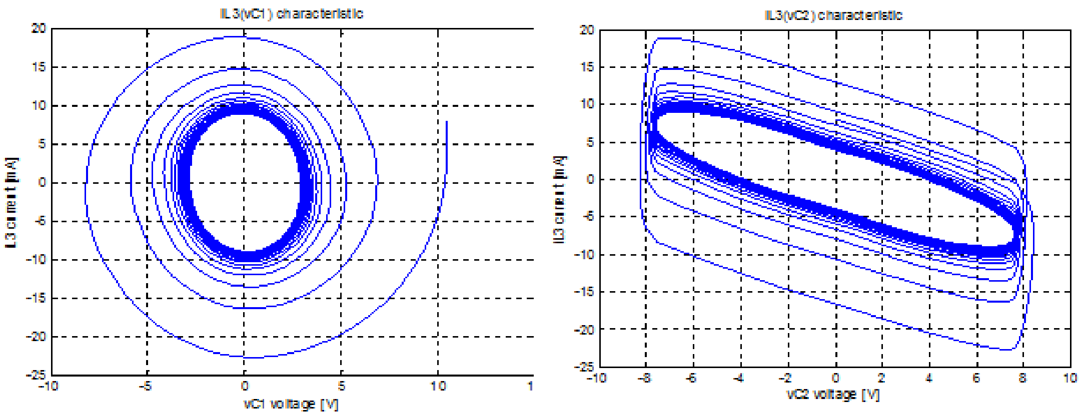

Fig. 6. The iL3(vC1) characteristic and iL3(vC2) characteristic of Chua’s circuit

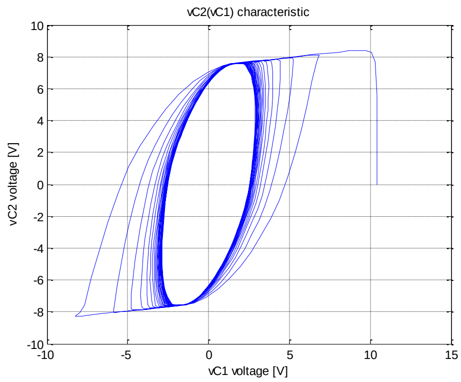

Fig. 7. The vC2(vC1) characteristic of Chua’s circuit

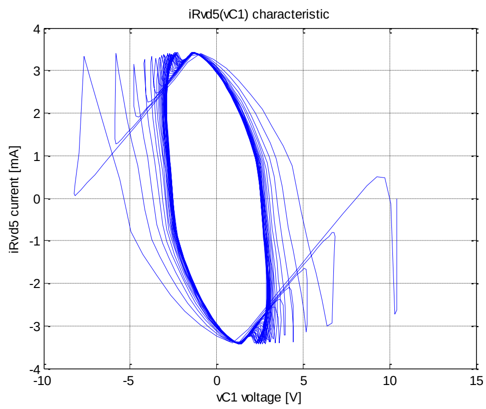

Fig. 8. The iRvd5(vC1) characteristic of Chua’s circuit

5. Conclusions

This paper presents new approaches of analyzing the nonlinear resistive circuit analysis. The hybrid analysis method of the nonlinear analog circuits presents the advantage of separation between the linear and the nonlinear part. This allows the computation only once at the beginning of the iteration process, of those parts of the circuit equations that exclusively depends on the parameters of the linear terms. A significant efficiency in circuit analysis and an improvement of the accuracy in the numerical calculations are obtained by combining this hybrid procedure with a very efficient implicit integration algorithm, in which only the symbols of the parameters corresponding to the nonlinear circuit elements are considered. Also, the dynamic elements are replaced by discrete resistive models associated with an implicit numerical integration algorithm.

Another advantage of the hybrid-method in comparison with nodal-analysis method is flexibility by allowing the nonlinear elements to be either voltage-controlled (c.v.) or current-controlled (c.c.).

Appendix A

Program developed in Maple that calculates the symbolic expressions of the hybrid matrices is showed in Fig. 9.

This algorithm can be used for different circuits and the only thing that must be change is the input file.

- Open the file and read the data from it
- Import the solution from Asinom program
- Compute the covariable vector
- Compute the hybrid matrix *H*
- Compute the hybrid matri**x** V_{j+1} is the source vector corresponding to the independent voltage and current sources at the moment *tj+1*, and V_{LC} represents the source vector corresponding to the companion models of the linear inductors and capacitors at the previous time moment
- Numerical calculation of all the matrix used *H*, *Vej*, V_{LC}
- Calculate the expressions of characteristic of non-linear element parameter, linearized on portions, corresponding to the combination of certain segments *s* and hybrid equations

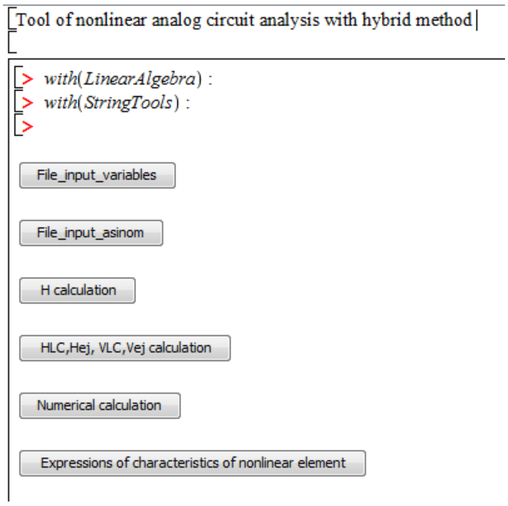

Fig. 9. The program to calculate hybrid matrix in Maple

Compute the covariable vector result:

$$table\Big(\Big[1 = \frac{(A6\_8 + 1 + G5\_12R12 + A6\_8\,G5\_12R12)\,U1}{G5\_12R8R12 + R12 + R8} + \frac{R12\,I10}{G5\_12R8R12 + R12 + R8} + \frac{(G5\_12R12 + 1)\,E9}{G5\_12R8R12 + R12 + R8} + \frac{(G5\_12R12 + 1)\,E11}{G5\_12R8R12 + R12 + R8}, 2 = -1.\,E9, 3 = -\frac{1.\,(-1.\,R3\,G7 - 1.\,B13\_3A6\_8)\,U1}{R3\,G7} + \frac{1.\,I2}{G7} - \frac{1.\,B13\_3\,E4}{R3\,G7}\Big]\Big)$$

Hybrid matrix H

> print(H)

$$table\Big(\Big[1 = table\Big(\Big[1 = \frac{A6\_8 + 1 + G5\_12R12 + A6\_8\,G5\_12R12}{G5\_12R8R12 + R12 + R8}, 2 = 0, 3 = \frac{R12}{G5\_12R8R12 + R12 + R8}\Big]\Big), 2 = table([1 = 0, 2 = 0, 3 = 0]), 3 = table\Big(\Big[1 = -\frac{1.\,(-1.\,R3\,G7 - 1.\,B13\_3A6\_8)}{R3\,G7}, 2 = \frac{1.}{G7}, 3 = 0\Big]\Big)\Big]\Big)$$

Calculate the expressions of characteristic of non-linear element parameter, linearized on portions, corresponding to the combination of certain segments s and hybrid equations:

```
> evalm(B) = evalm(TL)
```

$$[(Gd1(s) + 0.004285714286)\,uu1 - 0.1428571429\,ii10, Rdi2(s)\,ii2, 1.000000000\,uu1 - 100.\,ii2 + Rdi10(s)\,ii10] = \begin{bmatrix} 1.285714286 - ju1(s) \\ -200.0 - ei2(s) \\ -200.0000000 - ei10(s) \end{bmatrix}$$

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>
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