SPREAD OPTION PRICING USING TWO JUMP-DIFFUSION INTEREST RATES

Reyhane mohamadinejad[1], Jafar Biazar[2], Abdolsadeh Neisy[3]

Nowadays, as the financial markets grow larger, to compare with the past, financial investments and their modeling become more complicated. One of the difficulties in these financial modeling is selecting an appropriate model for interest rate. The main reason is that, in real market some data goes under sudden changes in some points so, they may not be in complete harmony with our selected model. As a result many questions come to existence, putting our choice under question mark. Thus, studying and analyzing the interest rate model that has jump-diffusion terms is of great importance. In this paper, we assume the spread options are based on two LIBOR interest rates, while one of them follows a geometric Brownian motion (GBM) with jump-diffusion terms. As the aim of our work, is to find a suitable model for pricing the spread options, first, we attempt to achieve a partial integro-differntial equation (PIDE) which determine the price of the options, regarding initial and boundary conditions. Then a generalized alternating direction implicit (ADI) method with a proper step-size is proposed in order to solve our model. The reader should note, this is due to reason that there is no closed form solution for our model. Eventually, MATLAB software is used to implement the ADI method using data that fits the model. In addition, to illustrate the simplicity and reliability of the proposed approach some examples are provided in the last chapter of this paper.

Keywords: Interest rate, Spread option pricing, Jump-diffusion models, Numerical method, Alternating Direction Implicit.

1. Introduction

Speaking of pricing models, the one with jumps are highly predominant. To put it simply, this is because, some factors including, economic or political issues or natural disaster such as flood and earthquake could have major impact on the price of the options or its underlying asset. Therefore jump-diffusion terms play an important role in pricing model in financial market,

[1]Lecturer, Student of Applied Mathematics, University of Guilan, Iran, e-mail: r.m.math81@gmail.com

[2]Professor, Department of Mathematics, Faculty of Applied Sciences, University of Guilan , Iran

[3]Professor, Department of Mathematics, Faculty of Statistics, Mathematics and Computer, Allameh Tabataba'i University , Iran

and not considering them may lead us to inaccurate outcomes. In other words, sudden price jumps are causing a breakdown in the economy and have great effects on the market and the investors. As previously mentioned, we obtain a pricing model which is a parabolic equation (partial differential-integral equation) and includes some jumps. However, these jumps should be limited by some considerations or one can put some restriction to them in order to control their value while constructing model.

In order for investors in the market to be able to hedge the risk arising from the sharp fluctuations in the price of the underlying asset(s), it is necessary to adopt different positions, such as the long or short position in the market. There are several trading strategies in the market which investors can take according to their expectations from the underlying asset price. Therefore here, we present an efficient strategy that describes how to apply the spread options on different interest rates. For this purpose, first, we determine our interest rate models, and then extract the pricing model for the spread options the interest. Applications of the Spread options are in cases where the Payoff is based on the difference in price between two or more assets with different maturity dates.
Assume that Company's bonds with two different LIBOR interest rates with different models, with maturity T_1, equal to L_1 and maturity T_2 equals to L_2 , respectively. We make the financial basket based on these two interest rates and solve the pricing model using the ADI method with the step length of $\frac{1}{6}$ .After putting, the data obtained from the company (Pi), in our model, if the acquired price of the spread option is higher than the market price, the investor should take short position, but in case, the value is lower than the market price, the investor better to take long position in the market.
ADI method is used for solving the parabolic partial differential equation, and it is considered to be a classic method to solve the two-dimensional diffusion equations. Therefore, in this study, we use this method.

Option contracts have been recognized for centuries. Chicago Dealership Deal was established in 1973. In 1997, Milersen, Sandman, and Sutterman introduced the classic LIBOR market model. In 1999, in relation to interest rate modeling, Bringo and colleagues introduced a book of financial models for pricing and immunizing a wide range of derivatives [2, 3]. In 2010, Suares and C. Vazquez studied the range option pricing using two interest rates in LIBOR, where changes in economic or political factors were not observed at those rates [4]. In this paper, the weak spot caused by not considering these factors is solved by taking jump-diffusion term into account in the model of the interest rate.It should be noted that stochastic interest rate models should be mean-reverting, since the interest rates cannot grow as unconventionally as the stock price or other financial variables.

This paper is organized as follows: In section 2 we describe the model and the partial-integro differential equation.The numerical solution is obtained by using the ADI method and the boundary and initial conditions in section 3. In section 4, we implement our method to the model and represent the numerical results. Eventually, Section 5 includes conclusion and some suggestion for conducting further research.

2. The mathematical model

In this section we obtain a spread option model based on LIBOR interest rates. For this purpose we consider spread option based on two LIBOR interest rates at T_1 and T_2 maturities. Let L_1 and L_2 be two LIBOR interest rates that are presented by the following dynamics [4]:

$$dL_1 = \sigma_1 L_1 dW^1, dL_2 = \alpha_2 dt + \sigma_2 L_2 dW^2.$$

Where $\alpha_2 = -\frac{\delta L_1 L_2 \rho \sigma_1 \sigma_2}{1+\delta L_2}$ in which the variable δ is the step size, W^1, and W^2 are the Standard Brownian Motion satisfying $Corr(W^1, W^2) = \rho$.
Now assume that L_2 faces some jumps due to different situations which are caused by an unknown reason. We assume that the time is divided into smaller and equal intervals namely δ. σ_1 and σ_2 are volatilities.
By considering these assumptions, the dynamic representing the LIBOR interest rate L_2 is changed as follows:

$$dL_2 = \alpha_2 dt + \sigma_2 L_2 dW^2 + d(\sum_{i=1}^{N_t} J_i)$$

Where N_t is a Poisson process with intensity rate of $\lambda > 0$, and the sequence J_i denotes the magnitude of jumps which are independent and identically distributed $(i.i.d)$ and have the following distribution function:

$$f_J(x) = \begin{cases} \lambda e^{-\lambda x} & , x > 0 \\ 0 & , x \leq 0. \end{cases}$$

We also note that processes W and N_t are independent.
After considering π as the spread option price, with L_1 and L_2 values at time t and according to Ito's formula, the dynamic of the option price is computed:

$$d\pi = (\tfrac{\partial \pi}{\partial t} + \alpha_2 L_2 \tfrac{\partial \pi}{\partial L_2} + L_2^2 \sigma_2^2 \tfrac{\partial^2 \pi}{\partial L_2^2} + L_1^2 \sigma_1^2 \tfrac{\partial^2 \pi}{\partial L_1^2} + \rho \sigma_1 \sigma_2 L_1 L_2 \tfrac{\partial^2 \pi}{\partial L_1 \partial L_2})dt$$

$$+ (L_1 \sigma_1 \tfrac{\partial \pi}{\partial L_1} + L_2 \sigma_2 \tfrac{\partial \pi}{\partial L_2})dW + (\pi(t, L_1, L_2 + x) - \pi(t, L_1, L_2))dN_t.$$

Where x is the jump that had been caused by a catastrophe. We define the portfolio P consisting of two spread options π_1 and π_2 as follows [5]:

$$P = x_1\pi_1 + x_2\pi_2.$$

Where x_1 and x_2 are the number of units of the options π_1 and π_2 respectively. By substituting the variations of P in dynamics of π_1 and π_2, we have:

$$\begin{cases} d\pi_1 = \mu_1 dt + \Delta_1 dW + (\pi_1(t, L_1, L_2 + x) - \pi_1(t, L_1, L_2))dN_t \\ d\pi_2 = \mu_2 dt + \Delta_2 dW + (\pi_2(t, L_1, L_2 + x) - \pi_2(t, L_1, L_2))dN_t \end{cases} \quad (1)$$

Where

$$\begin{cases} \mu_1 = \frac{\partial \pi_1}{\partial t} + \sigma_1^2 L_1^2 \frac{\partial^2 \pi}{\partial L_1^2} + \frac{1}{2}\rho\sigma_1\sigma_2 L_1 L_2 \frac{\partial^2 \pi}{\partial L_1 \partial L_2} \\ \mu_2 = \frac{\partial \pi_2}{\partial t} + \alpha_2 L_2 \frac{\partial \pi}{\partial L_2} + L_2^2\sigma_2^2 \frac{\partial^2 \pi}{\partial L_2^2} + \frac{1}{2}\rho\sigma_1\sigma_2 L_1 L_2 \frac{\partial^2 \pi}{\partial L_1 \partial L_2} \end{cases} , \Delta_i = \pi_{L_i} L_i \sigma_i \quad (2)$$

The expected value of P is $rPdt$ since the portfolio is risk-neutral, so:

$$E(x_1\pi_1 + x_2\pi_2) = r(x_1\pi_1 + x_2\pi_2)dt.$$

Let us take $\Delta_1 = x_2$ and $\Delta_2 = -x_1$, so we have the following term:

$$\frac{\mu_1 - (\lambda + r)\pi_1 + E(\pi_1(t, L_1, L_2 + x)}{\Delta_1} = \frac{\mu_2 - (\lambda + r)\pi_2 + E(\pi_2(t, L_1, L_2 + x)}{\Delta_2}. \quad (3)$$

Which is called the market price of the risk and is shown by q. By substituting (1) and (2) in (3), and with respect to the definition of the expectation, we reach the following partial differential-integral equation:

$$\frac{\partial \pi}{\partial t} + (\alpha_2 - q\sigma_2)L_2 \frac{\partial \pi}{\partial L_2} + (-q\sigma_1)L_1 \frac{\partial \pi}{\partial L_1} + \sigma_1^2 L_1^2 \frac{\partial^2 \pi}{\partial L_1^2} + \sigma_2^2 L_2^2 \frac{\partial^2 \pi}{\partial L_2^2} +$$
$$\rho\sigma_1\sigma_2 L_1 L_2 \frac{\partial^2 \pi}{\partial L_1 \partial L_2} - (\lambda + r)\pi + \int_{-\infty}^{+\infty} \pi(t, L_1, L_2 + x) f(x) dx = 0 \quad (4)$$

The initial and boundary conditions can be written as:

$$\frac{\partial^2 \pi}{\partial L_1^2}(0, L_2, \tau) = \frac{\partial^2 \pi}{\partial L_1^2}(L, L_2, \tau) = 0,$$
$$\frac{\partial^2 \pi}{\partial L_2^2}(L_1, 0, \tau) = \frac{\partial^2 \pi}{\partial L_2^2}(L_1, M, \tau) = 0,$$

for $0 \le L_1 \le L$, $0 \le L_2 \le M$, and $0 \le \tau \le T$.

If K represent the strike price, the payoff function of an European put option with its boundary conditions are as follow:

$$\pi(L_1, L_2, 0) = \max\{L_1 - K, L_2 - K, 0\},$$
$$\pi(0, L_2, \tau) = 1, \pi(L_1, 0, \tau) = 1,$$
$$\frac{\partial \pi}{\partial L_1}(L, L_2, \tau) = \frac{\partial \pi}{\partial L_2}(L_1, M, \tau) = 0.$$

3. ADI method with $\frac{1}{6}$ step-size for solving spread option pricing model

Let us consider the following changes of variables for L_1 and L_2 :

$$L_1 = e^x, L_2 = e^y,$$

And also, by changing the variables $t = T - \tau$ then $L_{1x} = e^x, L_{2y} = e^y$. By applying these changes, equation (4) can be rewritten as follows:

$-\frac{\partial\pi}{\partial\tau} + (\alpha - q\sigma_2)\frac{1}{L_2}L_2\frac{\partial\pi}{\partial y} + (-q\sigma_1)\frac{1}{L_1}L_1\frac{\partial\pi}{\partial x} + \sigma_1^2\frac{1}{L_1^2}L_1^2(\frac{\partial^2\pi}{\partial x^2} - \frac{\partial\pi}{\partial x}) +$
$\sigma_2^2\frac{1}{L_2^2}L_2^2(\frac{\partial^2\pi}{\partial y^2} - \frac{\partial\pi}{\partial y}) + \rho\sigma_1\sigma_2\frac{1}{L_1L_2}L_1L_2\frac{\partial^2\pi}{\partial x\partial y} - (r+\lambda)\pi +$
$\int_{-\infty}^{+\infty}\pi(t, e^x, e^y + z)f(z)dz = 0.$

So we have:

$$\frac{\partial\pi}{\partial\tau} = (\alpha - q\sigma_2 - \sigma_2^2)\frac{\partial\pi}{\partial y} + (-q\sigma_1 - \sigma_1^2)\frac{\partial\pi}{\partial x} + \sigma_1^2\frac{\partial^2\pi}{\partial x^2} + \sigma_2^2\frac{\partial^2\pi}{\partial y^2} + \rho\sigma_1\sigma_2\frac{\partial^2\pi}{\partial x\partial y}$$

$$-(r+\lambda)\pi + \int_{-\infty}^{+\infty}\pi(t, e^x, e^y + z)f(z)dz = 0. \qquad (5)$$

For solving the integral part, one can take, $Y = e^y$ and $X = e^x$ and consider the first three terms of the Taylor expansion of the integrand, around $(0,0)$.

$$\pi(t, X, Y + z) = \pi(t, x_i, y_i) + z\frac{\partial\pi}{\partial y} + \frac{z^2}{2}\frac{\partial^2\pi}{\partial y^2}.$$

The integral is computed as follows:
$\int_{-\infty}^{+\infty}\pi(t, X, Y + z)f(z) = \lambda\int_{-\infty}^{+\infty}(\pi + z\frac{\partial\pi}{\partial Y} + \frac{z^2}{2}\frac{\partial^2\pi}{\partial Y^2})e^{-\lambda z}dz$
$= -\pi e^{-\lambda z} - ze^{-\lambda z}\frac{\partial\pi}{\partial Y} - \frac{1}{\lambda}e^{-\lambda z}\frac{\partial\pi}{\partial Y} - \frac{1}{2}z^2e^{-\lambda z}\frac{\partial^2\pi}{\partial Y^2} - \frac{z}{\lambda}e^{-\lambda z}\frac{\partial^2\pi}{\partial Y^2} - \frac{1}{\lambda^2}e^{-\lambda z}\frac{\partial^2\pi}{\partial Y^2}\Big|_0^{+\infty} =$
$\pi + \frac{1}{\lambda}\frac{\partial\pi}{\partial Y} + \frac{1}{\lambda^2}\frac{z^2}{2}\frac{\partial^2\pi}{\partial Y^2},$
These outcomes for the integral part in equation (5) results in.
$\frac{\partial\pi}{\partial\tau} = (\alpha - q\sigma_2 - \sigma_2^2)\frac{\partial\pi}{\partial Y} + (-q\sigma_1 - \sigma_1^2)\frac{\partial\pi}{\partial X} + \sigma_1^2\frac{\partial^2\pi}{\partial X^2} + \sigma_2^2\frac{\partial^2\pi}{\partial Y^2} + \rho\sigma_1\sigma_2\frac{\partial^2\pi}{\partial X\partial Y} - (r +$
$\lambda - 1)\pi + \frac{1}{\lambda}\frac{\partial\pi}{\partial Y} + \frac{1}{\lambda^2}\frac{\partial^2\pi}{\partial Y^2},$
Now we substitute $Y = e^y$ and $X = e^x$ in the last equation

$$\frac{\partial\pi}{\partial\tau} = (\alpha - q\sigma_2 - \sigma_2^2 + \frac{1}{\lambda}e^{-y} - \frac{1}{\lambda^2}e^{-2y})\frac{\partial\pi}{\partial y} + (-q\sigma_1 - \sigma_1^2)\frac{\partial\pi}{\partial x} + \sigma_1^2\frac{\partial^2\pi}{\partial x^2} +$$

$$(\sigma_2^2 + \frac{1}{\lambda^2}e^{-2y})\frac{\partial^2\pi}{\partial y^2} + \rho\sigma_1\sigma_2\frac{\partial^2\pi}{\partial x\partial y} - (r + \lambda - 1)\pi. \qquad (6)$$

We define the $\pounds\pi$ operator as follows:

$\pounds\pi = \pounds^x\pi + \pounds^y\pi + \pounds^{xy}\pi + \pounds^{xx}\pi + \pounds^{yy}\pi + \Phi$ (7)
Where the components of $\pounds\pi$ (7) are defined based on the components of equation (6) :

$$\pounds^x\pi = (-q\sigma_1 - \sigma_1^2)\frac{\partial\pi}{\partial x},$$
$$\pounds^y\pi = (\alpha - q\sigma_2 - \sigma_2^2 + \frac{1}{\lambda}e^{-y} - \frac{1}{\lambda^2}e^{-2y})\frac{\partial\pi}{\partial y} = B\frac{\partial\pi}{\partial y},$$

Where $B = \alpha - q\sigma_2 - \sigma_2^2 + \frac{1}{\lambda}e^{-y} - \frac{1}{\lambda^2}e^{-2y}$.

$$\pounds^{xy}\pi = C\frac{\partial^2\pi}{\partial x\partial y}, whereC = \rho\sigma_1\sigma_2$$
$$\pounds^{xx}\pi = D\frac{\partial^2\pi}{\partial x^2}, D = \sigma_1^2,$$
$$\pounds^{yy}\pi = E\frac{\partial^2\pi}{\partial y^2}, E = \sigma_2^2 + \frac{1}{\lambda^2}e^{-2y},$$
$$\Phi = F\pi, F = -(r + \lambda - 1).$$

We decompose the PIDE for the solution as follows [6] :

$$\begin{aligned}
\pounds^x\pi_{ij}^{n+\frac{1}{6}} &= \frac{\pi_{ij}^{n+\frac{1}{6}} - \pi_{ij}}{\Delta\tau}, \\
\pounds^y\pi_{ij}^{n+\frac{1}{3}} &= \frac{\pi_{ij}^{n+\frac{1}{3}} - \pi_{ij}^{n+\frac{1}{6}}}{\Delta\tau}, \\
\pounds^{xy}\pi_{ij}^{n+\frac{1}{2}} &= \frac{\pi_{ij}^{n+\frac{1}{2}} - \pi_{ij}^{n+\frac{1}{3}}}{\Delta\tau}, \\
\pounds^{xx}\pi_{ij}^{n+\frac{2}{3}} &= \frac{\pi_{ij}^{n+\frac{2}{3}} - \pi_{ij}^{n+\frac{1}{2}}}{\Delta\tau}, \\
\pounds^{yy}\pi_{ij}^{n+\frac{5}{6}} &= \frac{\pi_{ij}^{n+\frac{5}{6}} - \pi_{ij}^{n+\frac{2}{3}}}{\Delta\tau}, \\
\frac{\pi_{ij}^{n+1} - \pi_{ij}^{n+\frac{5}{6}}}{\Delta\tau} &= \Phi.
\end{aligned} \tag{8}$$

By applying Adams-Bashforth formula [7], equation (8) is rewritten as follows [8] :
$\frac{\pi_{ij}^{n+\frac{1}{6}} - \pi_{ij}^{n}}{\Delta\tau} = \pounds^x\pi_{ij}^{n+\frac{1}{6}}$,
$\frac{\pi_{ij}^{n+\frac{1}{3}} - \pi_{ij}^{n+\frac{1}{6}}}{\Delta\tau} = \frac{1}{2}(-\pounds^y\pi_{ij}^{n+\frac{1}{6}} + 3\,\pounds^y\pi_{ij}^{n+\frac{1}{3}})$,

$\frac{\pi_{ij}^{n+\frac{1}{2}} - \pi_{ij}^{n+\frac{1}{3}}}{\Delta\tau} = \frac{1}{12}(5\ £^{xy}\pi_{ij}^{n+\frac{1}{6}} - 16\ £^{xy}\pi_{ij}^{n+\frac{1}{3}} + 23\ £^{xy}\pi_{ij}^{n+\frac{1}{2}}),$

$\frac{\pi_{ij}^{n+\frac{2}{3}} - \pi_{ij}^{n+\frac{1}{2}}}{\Delta\tau} = \frac{1}{24}(-9\ £^{xx}\pi_{ij}^{n+\frac{1}{6}} + 37\ £^{xx}\pi_{ij}^{n+\frac{1}{3}} - 59\ £^{xx}\pi_{ij}^{n+\frac{1}{2}} + 55\ £^{xx}\pi_{ij}^{n+\frac{2}{3}}),$

$\frac{\pi_{ij}^{n+\frac{5}{6}} - \pi_{ij}^{n+\frac{2}{3}}}{\Delta\tau} = \frac{1}{720}(251\ £^{yy}\pi_{ij}^{n+\frac{1}{6}} - 1274\ £^{yy}\pi_{ij}^{n+\frac{1}{3}} + 2616\ £^{yy}\pi_{ij}^{n+\frac{1}{2}} - 2774\ £^{yy}\pi_{ij}^{n+\frac{2}{3}} + 1901\ £^{yy}\pi_{ij}^{n+\frac{5}{6}}),$

$$\frac{\pi_{ij}^{n+1} - \pi_{ij}^{n+\frac{5}{6}}}{\Delta\tau} = \pi_{ij}^{n+1} \tag{10}$$

The following equations are derived from equation (8) and equation (9) [9] :

$\frac{\pi_{ij}^{n+\frac{1}{6}} - \pi_{ij}^{n}}{\Delta\tau} = (-q\sigma_1 - \sigma_1^2)\frac{\pi_{i+1j}^{n+\frac{1}{6}} - \pi_{ij}^{n+\frac{1}{6}}}{h} \Rightarrow \pi_{ij}^{n+\frac{1}{6}} - \pi_{ij}^{n} = A(\pi_{i+1j}^{n+\frac{1}{6}} - \pi_{ij}^{n+\frac{1}{6}})$

$\Rightarrow (1+A)\pi_{ij}^{n+\frac{1}{6}} - A\pi_{i+1j}^{n+\frac{1}{6}} = \pi_{ij}^{n}, (9)$

where $A = \frac{(-q\sigma_1 - \sigma_1^2)\Delta\tau}{h}$. If we substitute $i = 0$ in equation (10) :

$(1+A)\pi_{0j}^{n+\frac{1}{6}} - A\pi_{1j}^{n+\frac{1}{6}} = \pi_{0j}^{n},$

And apply the initial conditions:

$\frac{\pi_{2j}^{n+\frac{1}{6}} - 2\pi_{1j}^{n+\frac{1}{6}} + \pi_{0j}^{n+\frac{1}{6}}}{h^2} = 0 \Rightarrow \pi_{0j}^{n+\frac{1}{6}} = 2\pi_{1j}^{n+\frac{1}{6}} - \pi_{2j}^{n+\frac{1}{6}},$

We derive:

$(1+A)(2\pi_{1j}^{n+\frac{1}{6}} - \pi_{2j}^{n+\frac{1}{6}}) - A\pi_{1j}^{n+\frac{1}{6}} = \pi_{0j}^{n} \Rightarrow (2+A)\pi_{1j}^{n+\frac{1}{6}} - (1+A)\pi_{2j}^{n+\frac{1}{6}} = \pi_{0j}^{n}.$

Therefore, the following system is obtained for $i = 1, 2, ..., n$:

$$\begin{bmatrix} 2+A & -(1+A) & 0 & 0 & \cdots & 0 \\ 1+A & -A & 0 & 0 & \cdots & 0 \\ 0 & 1+A & -A & 0 & \cdots & 0 \\ 0 & 0 & 1+A & -A & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A & 1-A \end{bmatrix} \begin{bmatrix} \pi_{1j}^{n+\frac{1}{6}} \\ \pi_{2j}^{n+\frac{1}{6}} \\ \pi_{3j}^{n+\frac{1}{6}} \\ \vdots \\ \pi_{n-1j}^{n+\frac{1}{6}} \\ \pi_{nj}^{n+\frac{1}{6}} \end{bmatrix} = \begin{bmatrix} \pi_{1j}^{n} \\ \pi_{2j}^{n} \\ \pi_{3j}^{n} \\ \vdots \\ \pi_{n-1j}^{n} \\ \pi_{nj}^{n} \end{bmatrix}$$

Due to the second relation in equation (9), We have:

$$\frac{\pi_{ij}^{n+\frac{1}{3}} - \pi_{ij}^{n+\frac{1}{6}}}{\Delta\tau} = \frac{1}{2}(-B_j\frac{\pi_{ij+1}^{n+\frac{1}{6}} - \pi_{ij}^{n+\frac{1}{6}}}{h} + 3B_j\frac{\pi_{ij+1}^{n+\frac{1}{3}} - \pi_{ij}^{n+\frac{1}{3}}}{h}).$$

For boundary conditions and for $j = 0, 1, ..., m$, the system is derived as:

$$\pi_{ij}^{n+\frac{1}{3}} - \pi_{ij}^{n+\frac{1}{6}} = \frac{\Delta\tau}{2h} B_j(-\pi_{ij+1}^{n+\frac{1}{6}} + \pi_{ij}^{n+\frac{1}{6}} + 3\pi_{ij+1}^{n+\frac{1}{3}} - 3\pi_{ij}^{n+\frac{1}{3}}),$$

$$\begin{bmatrix} -2 & 3(\beta_j+\frac{1}{3}) & 0 & 0 & \cdots & 0 \\ -3(\beta_j+\frac{1}{3}) & 3\beta_j & 0 & 0 & \cdots & 0 \\ 0 & -3(\beta_j+\frac{1}{3}) & 3\beta_j & 0 & \cdots & 0 \\ 0 & 0 & -3(\beta_j+\frac{1}{3}) & 3\beta_j & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -3\beta_j & (3\beta_j - 1) \end{bmatrix} \begin{bmatrix} \pi_{i1}^{n+\frac{1}{3}} \\ \pi_{i2}^{n+\frac{1}{3}} \\ \pi_{i3}^{n+\frac{1}{3}} \\ \vdots \\ \pi_{im-1}^{n+\frac{1}{3}} \\ \pi_{im}^{n+\frac{1}{3}} \end{bmatrix} = \begin{bmatrix} f_{i1} \\ f_{i2} \\ f_{i3} \\ \vdots \\ f_{im-1} \\ f_{im} \end{bmatrix},$$

where $\beta_j = \dfrac{\Delta\tau}{2h} B_j$ and $f_{ij} = -(\beta_j + 1)\pi_{ij}^{n+\frac{1}{6}} + \beta_j \pi_{ij+1}^{n+\frac{1}{6}}$.
By using third relation in equation (9) :

$$\frac{\pi_{ij}^{n+\frac{1}{2}} - \pi_{ij}^{n+\frac{1}{3}}}{\Delta\tau} = \frac{C}{12}((5\frac{\pi_{i+1j+1}^{n+\frac{1}{6}} - \pi_{i+1j}^{n+\frac{1}{6}} - \pi_{ij+1}^{n+\frac{1}{6}} + 2\pi_{ij}^{n+\frac{1}{6}} - \pi_{i-1j}^{n+\frac{1}{6}} - \pi_{ij-1}^{n+\frac{1}{6}} + \pi_{i-1j-1}^{n+\frac{1}{6}}}{h^2})$$
$$-16(\frac{\pi_{i+1j+1}^{n+\frac{1}{3}} - \pi_{i+1j}^{n+\frac{1}{3}} - \pi_{ij+1}^{n+\frac{1}{3}} + 2\pi_{ij}^{n+\frac{1}{3}} - \pi_{i-1j}^{n+\frac{1}{3}} - \pi_{ij-1}^{n+\frac{1}{3}} + \pi_{i-1j-1}^{n+\frac{1}{3}}}{h^2})$$
$$+23(\frac{\pi_{i+1j+1}^{n+\frac{1}{2}} - \pi_{i+1j}^{n+\frac{1}{2}} - \pi_{ij+1}^{n+\frac{1}{2}} + 2\pi_{ij}^{n+\frac{1}{2}} - \pi_{i-1j}^{n+\frac{1}{2}} - \pi_{ij-1}^{n+\frac{1}{2}} + \pi_{i-1j-1}^{n+\frac{1}{2}}}{h^2})),$$

The above relation can be rearranged as:
$\pi_{ij}^{n+\frac{1}{2}} - 23C'\pi_{i+1j+1}^{n+\frac{1}{2}} + 23C'\pi_{i+1j}^{n+\frac{1}{2}} + 23C'\pi_{ij+1}^{n+\frac{1}{2}} - 46C'\pi_{ij}^{n+\frac{1}{2}} + 23C'\pi_{i-1j}^{n+\frac{1}{2}} +$
$23C'\pi_{ij-1}^{n+\frac{1}{2}} - 23C'\pi_{i-1j-1}^{n+\frac{1}{2}} = f_{ij},$
where f_{ij} and C' are defined as follows:
$f_{ij} = C'((5)(\pi_{i+1j+1}^{n+\frac{1}{6}} - \pi_{i+1j}^{n+\frac{1}{6}} - \pi_{ij+1}^{n+\frac{1}{6}} + 2\pi_{ij}^{n+\frac{1}{6}} - \pi_{i-1j}^{n+\frac{1}{6}} - \pi_{ij-1}^{n+\frac{1}{6}} + \pi_{i-1j-1}^{n+\frac{1}{6}})$
$-(16)(\pi_{i+1j+1}^{n+\frac{1}{3}} - \pi_{i+1j}^{n+\frac{1}{3}} - \pi_{ij+1}^{n+\frac{1}{3}} + 2\pi_{ij}^{n+\frac{1}{3}} - \pi_{i-1j}^{n+\frac{1}{3}} - \pi_{ij-1}^{n+\frac{1}{3}} - \pi_{i-1j-1}^{n+\frac{1}{3}} - \frac{1}{16}\pi_{ij}^{n+\frac{1}{3}}), C' =$
$\dfrac{C\Delta\tau}{12h^2}$.

By using the boundary conditions, and for $i = 0, 1, ..., n$ the following system is derived:

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 23C' & 1-46C' & 23C' & \cdots & 0 \\ 0 & 0 & 1-46C' & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \pi_{0j}^{n+\frac{1}{2}} \\ \pi_{1j}^{n+\frac{1}{2}} \\ \pi_{2j}^{n+\frac{1}{2}} \\ \vdots \\ \pi_{n-1j}^{n+\frac{1}{2}} \\ \pi_{nj}^{n+\frac{1}{2}} \end{bmatrix} +$$

$$\begin{bmatrix} -23C' & 23C' & 0 & \cdots & 0 & 0 \\ -23C' & 23C' & 0 & \cdots & 0 & 0 \\ 0 & -23C' & 23C' & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -23C' & 23C' \end{bmatrix} \begin{bmatrix} \pi_{0j-1}^{n+\frac{1}{2}} \\ \pi_{1j-1}^{n+\frac{1}{2}} \\ \pi_{2j-1}^{n+\frac{1}{2}} \\ \vdots \\ \pi_{n-1j-1}^{n+\frac{1}{2}} \\ \pi_{nj-1}^{n+\frac{1}{2}} \end{bmatrix} +$$

$$\begin{bmatrix} 23C' & -23C' & 0 & \cdots & 0 & 0 \\ 0 & 23C' & -23C' & \cdots & 0 & 0 \\ 0 & 0 & 23C' & -23C' & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 23C' & -23C' \end{bmatrix} \begin{bmatrix} \pi_{0j+1}^{n+\frac{1}{2}} \\ \pi_{1j+1}^{n+\frac{1}{2}} \\ \pi_{2j+1}^{n+\frac{1}{2}} \\ \vdots \\ \pi_{n-1j+1}^{n+\frac{1}{2}} \\ \pi_{nj+1}^{n+\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} f_{0j} \\ f_{1j} \\ f_{2j} \\ \vdots \\ f_{n-1j} \\ f_{nj} \end{bmatrix}.$$

In the next step, we obtain:

$$\frac{\pi_{ij}^{n+\frac{2}{3}} - \pi_{ij}^{n+\frac{1}{2}}}{\Delta\tau} = \tfrac{D}{24}(-9\frac{\pi_{i+1j}^{n+\frac{1}{6}} - 2\pi_{ij}^{n+\frac{1}{6}} + \pi_{i-1j}^{n+\frac{1}{6}}}{h^2} + 37\frac{\pi_{i+1j}^{n+\frac{1}{3}} - 2\pi_{ij}^{n+\frac{1}{3}} + \pi_{i-1j}^{n+\frac{1}{3}}}{h^2}$$
$$- 59\frac{\pi_{i+1j}^{n+\frac{1}{2}} - 2\pi_{ij}^{n+\frac{1}{2}} + \pi_{i-1j}^{n+\frac{1}{2}}}{h^2} + 55\frac{\pi_{i+1j}^{n+\frac{2}{3}} - 2\pi_{ij}^{n+\frac{2}{3}} + \pi_{i-1j}^{n+\frac{2}{3}}}{h^2}).$$

Now by moving the $n + \frac{2}{3}$ powers to the left side, we derive:

$$-55D'\pi_{i+1j}^{n+\frac{2}{3}} + (110D' + 1)\pi_{ij}^{n+\frac{2}{3}} - 55D'\pi_{i-1j}^{n+\frac{2}{3}} = f_{ij}.$$

Where f_{ij} and D' are defined as follows:

$f_{ij} = D'((-9)(\pi_{i+1j}^{n+\frac{1}{6}} - 2\pi_{ij}^{n+\frac{1}{6}} + \pi_{i-1j}^{n+\frac{1}{6}}) + (37)(\pi_{i+1j}^{n+\frac{1}{3}} - 2\pi_{ij}^{n+\frac{1}{3}} + \pi_{i-1j}^{n+\frac{1}{3}}) - (59)(\pi_{i+1j}^{n+\frac{1}{2}} - 2\pi_{ij}^{n+\frac{1}{2}} + \pi_{i-1j}^{n+\frac{1}{2}})) + \pi_{ij}^{n+\frac{1}{2}}, D' = \dfrac{D\Delta\tau}{24h^2}.$

For boundary conditions and for $i = 0, 1, ..., n$, we obtain the following system:

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -55D' & 110D'+1 & -55D' & \cdots & 0 & 0 \\ 0 & -55D' & 110D'+1 & -55D' & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_{0j}^{n+\frac{2}{3}} \\ \pi_{1j}^{n+\frac{2}{3}} \\ \pi_{2j}^{n+\frac{2}{3}} \\ \vdots \\ \pi_{n-1j}^{n+\frac{2}{3}} \\ \pi_{nj}^{n+\frac{2}{3}} \end{bmatrix} = \begin{bmatrix} f_{0j} \\ f_{1j} \\ f_{2j} \\ \vdots \\ f_{n-1j} \\ f_{nj} \end{bmatrix}.$$

So in the next step, we have:

$\frac{\pi_{ij}^{n+\frac{5}{6}} - \pi_{ij}^{n+\frac{2}{3}}}{\Delta\tau} = \frac{E}{720h^2}(251(\pi_{ij+1}^{n+\frac{1}{6}} - 2\pi_{ij}^{n+\frac{1}{6}} + \pi_{ij-1}^{n+\frac{1}{6}}) - 1274(\pi_{ij+1}^{n+\frac{1}{3}} - 2\pi_{ij}^{n+\frac{1}{3}} + \pi_{ij-1}^{n+\frac{1}{3}}) + 2616(\pi_{ij+1}^{n+\frac{1}{2}} - 2\pi_{ij}^{n+\frac{1}{2}} + \pi_{ij-1}^{n+\frac{1}{2}}) - 2774(\pi_{ij+1}^{n+\frac{2}{3}} - 2\pi_{ij}^{n+\frac{2}{3}} + \pi_{ij-1}^{n+\frac{2}{3}}) + 1901(\pi_{ij+1}^{n+\frac{5}{6}} - 2\pi_{ij}^{n+\frac{5}{6}} + \pi_{ij-1}^{n+\frac{5}{6}}))$.

By moving the $n + \frac{5}{6}$ powers to the left side, we derive:

$\pi_{ij}^{n+\frac{5}{6}} - 1901E'(\pi_{ij+1}^{n+\frac{5}{6}} - 2\pi_{ij}^{n+\frac{5}{6}} + \pi_{ij-1}^{n+\frac{5}{6}}) = f_{ij}$.

Where f_{ij} and E' are defined as follows:

$f_{ij} = E'(251(\pi_{ij+1}^{n+\frac{1}{6}} - 2\pi_{ij}^{n+\frac{1}{6}} + \pi_{ij-1}^{n+\frac{1}{6}}) - 1274(\pi_{ij+1}^{n+\frac{1}{3}} - 2\pi_{ij}^{n+\frac{1}{3}} + \pi_{ij-1}^{n+\frac{1}{3}}) + 2616(\pi_{ij+1}^{n+\frac{1}{2}} - 2\pi_{ij}^{n+\frac{1}{2}} + \pi_{ij-1}^{n+\frac{1}{2}}) - 2774(\pi_{ij+1}^{n+\frac{2}{3}} - 2\pi_{ij}^{n+\frac{2}{3}} + \pi_{ij-1}^{n+\frac{2}{3}})) + \pi_{ij}^{n+\frac{2}{3}}, E' = \frac{E\Delta\tau}{720h^2}$.

By considering the boundary conditions, and $j = 0, 1, ..., m$, the system is derived as:

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -1901E' & 3802E'+1 & -1901E' & \cdots & 0 & 0 \\ 0 & -1901E' & 3802E'+1 & -1901E' & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_{i0}^{n+\frac{5}{6}} \\ \pi_{i1}^{n+\frac{5}{6}} \\ \pi_{i2}^{n+\frac{5}{6}} \\ \vdots \\ \pi_{im-1}^{n+\frac{5}{6}} \\ \pi_{im}^{n+\frac{5}{6}} \end{bmatrix} = \begin{bmatrix} f_{i0} \\ f_{i1} \\ f_{i2} \\ \vdots \\ f_{im-1} \\ f_{im} \end{bmatrix}.$$

And in the last step, we have:

$$\frac{\pi_{ij}^{n+1} - \pi_{ij}^{n+\frac{5}{6}}}{\Delta\tau} = \Phi$$

Then $\pi_{ij}^{n+1} = \Phi\Delta\tau + \pi_{ij}^{n+\frac{5}{6}} \Rightarrow \pi_{ij}^{n+1} = \pi_{ij}^{n}\Delta\tau + \pi_{ij}^{n+\frac{5}{6}}$. In the following section we obtain the following system for $i = 0, 1, ..., n$, by substituting the above-mentioned values and we observe the respective numerical results.

4. **Numerical Results**

As an illustration, we first consider short-term and long–term interest rates to implement the ADI numerical method with $\frac{1}{6}$ step-size. Second, we obtain the changes in short-term and long–term interest rates for these banks. the parameters of interest rate are $L = 300, M = 300, q = 1, \sigma_1 = 0.3, \sigma_2 = 0.3, T = 0.5, \alpha = 0.7, \lambda = 0.1, \rho = 0.5, r = 0.03$[4].

The following figures demonstrate the fluctuations of L_1, L_2 in terms of t :

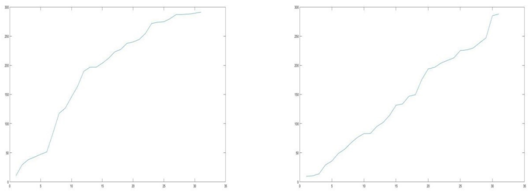

FIGURE 1. Diagram of L_1, L_2 for default parameters

Diagram of the Payoff function is as follows:

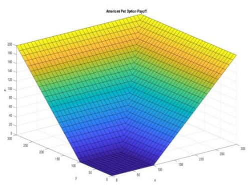

FIGURE 2. Payoff of two-interest rates

Finally we find the spread option price by the model and the proposed solution method. As can be seen from the following figures the price of the spread option varies for different amount of the variable n.

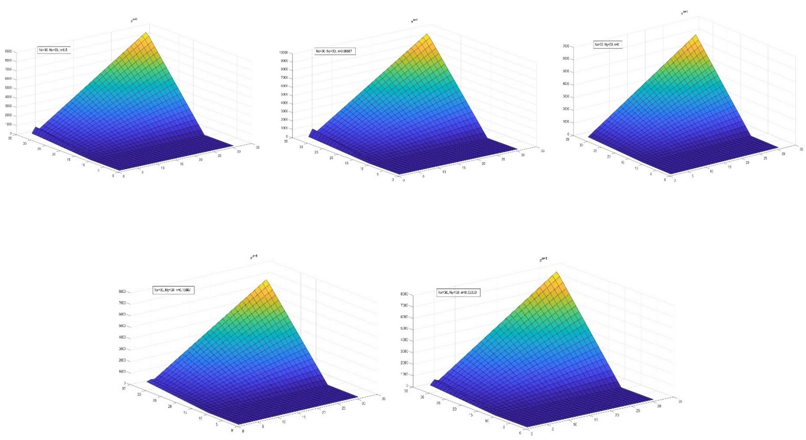

FIGURE 3. Numerical results using the ADI method with a step-size of $\frac{1}{6}$ for n=0, 0.16667, 0.3333, 0.5, 0.66667

5. **Conclusions**

In this paper, we studied the spread option pricing which depends on two Libor interest rates while one of them had jump terms which led us to an obtaining partial intergo-differential equation (PIDE). As we discussed in the earlier chapters, in the real world the price of the options or an underlying asset doesn't change in a continuous way, necessarily. This is because of a jump occured due to any unkonwn reason. However, while representing a model we don't ask about the reason of the jump, but we concentrate on the jump itself. All these discussions made us to take these unexpected jumps into account with hope to achieve more realsitic results. As its already discussed, the numerical solution for this PIDE was alternating directional implicit (ADI) method with a step-size of $\frac{1}{6}$ and its coefficients were obtained by Adams-Bashforth formula. However, it is worth noting that we can apply the Heston model instead of the LIBOR interest rate model or Radial basis function (RBF) as numerical solution as well. All these suggestions can be seen as a good choice for continuing this research in order to find more accurate results.

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