STOCHASTIC FRONTIER MODELS BY COPULAS AND AN APPLICATION

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The main endeavor in this study is to use copulas in the stochastic frontier analysis (SFA) to estimate production frontier and also technical efficiency of DMUs. After presenting theorical discussion using copulas in SFA, we finalize our study by providing an application on real data set. We compare the yielded efficiency scores by using copulas with results of the standard SFA and also CCR and BCC models. We concentrate on several Archimedean families and specially three families which were newly presented to the literature. These new families have trigonometric and hyperbolic generators and they are more flexible in modeling dependence structures.

Keywords: Copulas, hyperbolic functions, MLE, stochastic frontier, technical efficiency

1. Introduction

Measuring efficiency of firms (DMUs) has an important role in economy and managements. Usually Data Envelopment Analysis (DEA), Free Disposal Hull (FDH) and also SFA are the main instruments to obtain the efficiency of firms. DEA and FDH are nonparametric frontier models and so require minimal assumptions respect to structure of production and also they do not impose restrictions on the functional forms relating inputs and outputs. On the other hand they don't include noises in the model and assume that every deviation from the frontier is carried out by the inefficiency. While in the SFA, error term is composed of two types of error ($\varepsilon = u + v$). Common choices for u include the Exponential, the Half-Normal, the Truncated Normal and the

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Gamma distributions, and for v it is typically the Normal distribution (Aigner et al. (1977); Richmond (1974); Amsler et al. (2014)).

In estimating the SFA models researchers assume that error terms u and v are independent. Smith (2008) was one of the first peoples which proposed the potential dependence between u and v. Also he proposed copulas on modeling this dependence and then estimating the SFA models by using copulas. This method followed by others such as Carta and Steel (2012), El Mehdi and Hafner (2013), Amsler et al. (2014) and etc. Of course theorical and also computationally of using copulas in SFA are somewhat complicated and also this subject recently is in challenging.

The main aim of this study is to prepare an overview on the stochastic frontier analysis (SFA), estimating production frontier and technical efficiency of DMUs. Also providing theorical discussion about using copulas in SFA and their advantage in estimating SFA parameters by modeling error terms. With relying on twelve Japanese professional baseball teams data, and using several Archimedean families, technical efficiency of the mentioned teams will be compared with CCR, BCC and also standard SFA models. We concentrate on seven Archimedean families which three families were newly presented to the literature. These new families have trigonometric and hyperbolic generators and they are more flexible in modeling dependence structures.

The rest of the paper is organized as follows. Section 2 reviews stochastic frontier analysis and copulas. Section 3 describes using copulas in the stochastic frontier models. Applications of the discussed models are in Section 4. Finally, conclusions are given in Section 5.

2. Preliminaries

In this section we plan to review stochastic frontier analysis (SFA) and also copulas functions. For interested readers which are looking for deeper results we will refer to several origins.

2.1. Stochastic frontier analysis. Stochastic frontier analysis (SFA) is a method of economic modeling and firstly presented by Aigner et al. (1977) and Meeusen & Van den Broeck (1977). This method can be formulated both in parametric and nonparametric framework. Nevertheless, in the literature and applications parametric SFA is preferred (El Mehdi and Hafner, 2013). The production frontier model without random component can be written as,

$$y_i = f(x_i;\beta).TE_i,\ i = 1, ..., I \qquad (\mathbf{2}.1)$$

where y_i is the observed scalar output of the producer i, x_i is a vector of N inputs used by the producer i, $f(x_i,\beta)$ is the production frontier and β is a vector of technology parameters to be estimated. TE_i denotes the technical efficiency and it is defined as the ratio of observed output to maximum feasible output. Let TE_i is a stochastic variable, so we can write it as $TE_i = exp(-u_i)$, where $u_i \geq 0$. By adding a component of random shocks (which may come from weather changes, economic adversities or plain luck and it is assumed to be as $exp(v_i)$) to (**2**.1) we get

$$y_i = f(x_i; \beta).exp(-u_i).exp(v_i). \qquad (\mathbf{2}.2)$$

Now, if we also assume that $f(x_i; \beta)$ takes the log-linear Cobb-Douglas form, the model (**2**.2) can be written as the following,

$$ln\ y_i = \beta_0 + \sum_n \beta_n \mathtt{ln} x_{ni} + v_i - u_i. \qquad (\mathbf{2}.3)$$

In this study we use the log-linear Cobb-Douglas form. The interested readers are referred to Meeusen & Van Den Broeck (1977) and Aigner et al. (1977).

2.2. **Copulas and their properties.** A copula is a function $C : [0,1]^2 \to [0,1]$ which satisfies:
(a) for every u, v in [0,1], $C(u,0) = 0 = C(0,v)$, and $C(u,1) = u$ and $C(1,v) = v$;
(b) for every u_1, u_2, v_1, v_2 in [0,1] such that $u_1 \leq u_2$ and $v_1 \leq v_2$,

$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0. \qquad (\mathbf{2}.4)$$

Copulas functions are powerful technique in modeling dependence structures. Copulas allow us to combine univariate distributions to obtain a joint distribution with a particular dependence structure, in the famous Sklar Theorem: Let X and Y be random variables with joint distribution function H and marginal distribution functions F and G, respectively. Then there exists a copula C such that, $H(x,y) = C(F(x), G(y))$, for all x, y in $\mathbb{R}$. If F and G are continuous, then C is unique. Otherwise, the copula C is uniquely determined on $Ran(F) \times Ran(G)$. Conversely, if C is a copula and F and G are distribution functions, then the function H is a joint distribution function with margins F and G. As a result of the Sklar Theorem, copulas link joint distribution functions to their one-dimensional margins, see Sklar (1959), Kimberling (1974).

One of important classes of copulas is Archimedean copulas. Archimedean copulas originally appeared in the study of probabilistic metric spaces, where they were studied as part of the development of a probabilistic version of the triangle inequality. The interested readers are referred to Schweizer (1991) and Nelsen (2006). These copulas are very easy to construct, many parametric families belong to this class and have great variety of different dependence structures, hence there are wide efforts on this class of copulas in the literature and applications. For details see, Bacigál et al.(2015a, 2015b), Schweizer (1991), Genest and MacKay (1986a, 1986b) and Nelsen (2006).

Basic properties of AC are presented below and more information could be found in Nelsen (2006). Let φ be a continuous, strictly decreasing function from $[0,1]$ to $[0,\infty]$ such that $\varphi(1)=0$. The pseudo-inverse of φ is the function $\varphi^{[-1]}$ given by

$$\varphi^{[-1]}(t)=\begin{cases}\varphi^{(-1)}(t), & 0\le t\le\varphi(0)\\ 0, & \varphi(0)\le t\le\infty.\end{cases} \tag{2.5}$$

Copulas of the form

$$C(u,v)=\varphi^{[-1]}(\varphi(u)+\varphi(v)), \tag{2.6}$$

for every u,v in $[0,1]$ are called AC and the function φ is called a generator of the copula. If $\varphi(0)=\infty$ we say that φ is a strict generator. In this case,

$$\varphi^{[-1]}=\varphi^{(-1)}, \tag{2.7}$$

and

$$C(u,v)=\varphi^{(-1)}(\varphi(u)+\varphi(v)) \tag{2.8}$$

is said a strict Archimedean copula.

3. Stochastic frontier models and copulas

There are rare studies in stochastic frontier models which are related with copulas in the literature. Smith (2008) was one of the first peoples which proposed copula technique in SFA. Recently Carta and Steel (2012) introduced a new methodology for multi-output production frontiers which is based on copulas, also there are another related studies by El Mehdi and Hafner (2013) and Amsler et al. (2014). In this section we aim to explain main relation between copulas and SFA.

Let consider the traditional stochastic frontier model proposed by Aigner et al. (1977) and Meeusen and Van Den Broeck (1977),

$$\ln y_i=\beta_0+\sum_n\beta_n\ln x_{ni}+\varepsilon_i. \tag{3.1}$$

where $\varepsilon_i=v_i-u_i$ and $i=1,...,I$ denotes firms. We assume that v_i (and u_i) are independent over i, also there is potential dependence between v and u.

Let $u\sim G_1$ and $v\sim G_2$ and H be the joint distribution function of v and u. Then by the Sklar Theorem there is copula C_θ which satisfies in the following relation,

$$H(u,v)=C_\theta(G_1(u),G_2(v)) \tag{3.2}$$

and so its joint density function is as follows,

$$h(u,v) = g_1(u)g_2(v)c_\theta(G_1(u),G_2(v)). \quad (\mathbf{3}.3)$$

As $\varepsilon = v - u$, by the marginal distribution of h we get

$$h(\varepsilon) = \int_0^{+\infty} g_1(u)g_2(u+\varepsilon)c_\theta(G_1(u),G_2(u+\varepsilon))du. \quad (\mathbf{3}.4)$$

Replacing $\varepsilon = \mathtt{ln}y - f(x;\beta)$ in the (**3**.4) gives the density of y. Using the maximum likelihood estimator (MLE) is a way to obtain a more efficient estimator of stochastic frontier models. Clearly, copulas allow to model marginal distributions separately from their dependence structure, so we have a flexible joint distribution function, whose marginals are specified by the researcher.

After estimating stochastic frontier models we desire to calculate technical efficiency of DMUs. This technical efficiency is defined as follows

$$TE = E(\mathtt{exp}\{-u\}|\varepsilon) \quad (\mathbf{3}.5)$$

by using (**3**.3) and (**3**.4) we get

$$TE = \frac{1}{h(\varepsilon)} \int_{\mathbb{R}+} \mathtt{exp}\{-u\}h(u,\varepsilon)du \quad (\mathbf{3}.6)$$

see also Smith (2008), El Mehdi and Hafner (2013). In this study we assume that $u \sim N^+(0,\sigma_u^2)$, $u \geq 0$ and $v \sim N(0,\sigma_v^2)$. Clearly $E(u) = \sigma_u\sqrt{2/\pi}$ and $Var(u) = ((\pi-2)/\pi)\sigma_u^2$. If we assume that MLE of parameters $\vartheta = (\sigma_u,\sigma_v,\theta,\beta)$ in (**3**.4) are $\vartheta_{ML} = (\hat{\sigma}_u,\hat{\sigma}_v,\hat{\theta},\hat{\beta})$ then by replacing these estimates in (**3**.6) we get to TE_{ML} which is the MLE of TE.

Note that $\varepsilon = v - u$ and so

$$Var(\varepsilon) = Var(v) + Var(u) - 2Cov(u,v). \quad (\mathbf{3}.7)$$

This means that a positive correlation between u and v reduces the variance of ε and a negative correlation between u and v increase the variance of ε.

4. Application

The main aim in this section is to provide SFA model for baseball teams. These data consist of twelve Japanese professional baseball teams and are available in Cooper et al. (2006). There are 2 inputs and 1 outputs for this evaluation as follows,

Input 1: average annual salary of managers, including those of coaching.

Input 2: average annual salary of players: the top ranked 9 fielders and 6 pitchers.

Output: "attendance" which is measured as the ratio of total annual attendance vs. annual maximum capacity of the team's home stadium and is expressed as a percentage. Hence this number cannot exceed 100%. Table 1

TABLE 1. Twelve Japanese professional baseball teams

Teams	Manager	Player	Attendance
Swallows	5220	6183	80.87
Dragons	2250	5733	93.76
Giants	6375	8502	100
Tigers	3125	4780	76.53
Bay Stars	3500	4042	79.12
Carp	3125	5623	51.95
Lions	5500	10180	56.37
Fighters	3625	5362	57.44
Blue Wave	2715	4405	58.78
Buffalos	3175	6193	53
Marines	2263	5013	43.47
Hawks	3875	3945	82.78

exhibits these data for 12 teams. Salaries of managers and players, as inputs, are in tens of thousands of Japanese yen.

In calculations, estimating technical efficiency and SFA models, Matlab software had been used. "fminsearchbnd" command in Matlab had an important role in the calculating.

TABLE 2. Details of the selected copula families in this study

Family	Generator	Kendall's tau	λ_L	λ_U	θ interval
Clayton	$\frac{1}{\theta}(\frac{1}{t^\theta}-1)$	$\frac{\theta}{\theta+2}$	$2^{\frac{-1}{\theta}}$	0	$(0,\infty)$
Gumbel	$(-lnt)^\theta$	$\frac{\theta-1}{\theta}$	0	$2-2^{\frac{1}{\theta}}$	$[1,\infty)$
A12	$(\frac{1}{t}-1)^\theta$	$1-\frac{2}{3\theta}$	$2^{\frac{-1}{\theta}}$	$2-2^{\frac{1}{\theta}}$	$[1,\infty)$
cot-copula	$cot^\theta(\frac{\pi t}{2})$	$1-\frac{8}{\pi^2\theta}$	$2^{\frac{-1}{\theta}}$	$2-2^{\frac{1}{\theta}}$	$[1,\infty)$
coth-copula	$coth(\theta t)-coth(\theta)$	$1+\frac{2}{\theta^2}-\frac{2}{\theta}coth(\theta)$	$\frac{1}{2}$	0	$[1,\infty)$
csch-copula	$csch(t^\theta)-csch(1)$	$\frac{\theta}{\theta+2}$	$2^{\frac{-1}{\theta}}$	0	$(0,\infty)$

Note: A12 family is numbered as 4.2.12 in Table 4.1 Nelsen's book [13]

To provide SFA model for the mentioned baseball teams, beside standard SFA model, several copula families also are used in this study. Details of these families are summarized in Table 2. As it is seen, there are three new Archimedean families which are recently presented to the literature. cot-copula family has trigonometric generator and proposed by Pirmoradian and Hamzah (2011). Also csch-copula and coth-copula families have hyperbolic generators and were proposed by Bal and Najjari (2013), Najjari et al. (2014) respectively. Moreover, technical efficiency of CCR and BCC models are provided for these data to compare their results by the other SFA models.

Table 3 exhibits SFA model parameters which are estimated by using copulas and also standard SFA model. In the rest of paper, standard SFA

TABLE 3. The estimated parameters of the SFA models

Family	σ_u	σ_v	θ	β_0	β_1	β_2	τ
Clayton	0.1516	0.0695	8.2052	4.1232	0.283	-0.2042	0.8040
Gumbel	0.1517	0.0257	8.6872	4.2636	0.0951	-0.0528	0.8849
A12	0.1683	0.0669	9.1186	4.2712	0.278	-0.2148	0.9269
cot-copula	0.1563	0.0685	8.5636	4.1924	0.2762	-0.2061	0.9053
coth-copula	0.1881	0.0301	7.7433	4.7314	0.0787	-0.0902	0.7751
csch-copula	0.1418	0.0896	7.9721	3.8015	0.3936	-0.2647	0.7994
Product	0.1517	0.0257	-	4.2636	0.0951	-0.0528	-
Frontier	0.1652	0.0421	-	4.1352	0.0499	-0.0834	-

model is called as "Frontier" in Table 3, Table 4 and Table 5. As is it seen by the Table 3, dependence parameter θ has different values for any families. This value is an evidence on the dependence between u and v. As an example, for the Clayton family, this parameter is $\theta = 8.2052$ and so $\tau = \frac{\theta}{\theta+2} = 0.8040$. Namely there is 80.4% dependence between u and v. Similarly for the Gumbel family this parameter is $\theta = 8.6872$ and $\tau = 0.8849$. The maximum dependence is shown by A12 family as $\tau = 0.9269$ and the minimum dependence belongs to the coth copula, $\tau = 0.7751$. However, last column in the Table 3 shows that all of the mentioned copula families have solidarity on the high dependence between u and v. As discussed by (**3**.7), a positive correlation between u and v reduces the variance of ε and a negative correlation between u and v increase the variance of ε. It means that in the mentioned model by (**3**.1), the minimum variance tends the model to the closest estimation.

Table 4 demonstrates efficiency scores and also ranks of the mentioned baseball teams by the models which their estimated parameters are given in the Table 3. Technical efficiency of CCR and BCC models also are provided for the mentioned data to compare their results by the mentioned SFA models.

Table 5 consists of correlations between results of the mentioned models in Table 4. These correlations are based on ranks of the baseball teams in every model.

Gumbel and Product copulas have resulted same ranks for the baseball teams as correlation between them is one, namely, standard SFA model and SFA by usnig the Gumbel copula have calculated same ranks for baseball teams. Also cot and Clayton copulas have clarified same ranks for the baseball teams. Maximum correlation in results of the CCR model is linked by the Clayton and cot copulas which is 0.804, and the minimum correlation of the CCR model is with the results of coth copula and it is 0.720. Related with BBC model, maximum correlation is with the Clayton and cot copulas which

Table 4. Efficiency scores (ES) and ranks of twelve Japanese professional baseball teams

	CCR	CCR	BCC	BCC	Clayton	Clayton	Gumbel	Gumbel	A12	A12
Teams	ES	Rank	ES	Rank	ES	Rank	ES	Rank	ES	Rank
Swallows	0.6572	6	0.6691	11	0.6903	6	0.7994	4	0.6819	5
Dragons	1	1	1	2	1	1	1	1	0.9829	1
Giants	0.6139	7	1	1	0.8608	2	0.9863	2	0.8542	2
Tigers	0.8699	4	0.9514	7	0.7167	3	0.7836	6	0.7042	3
Bay Stars	0.9753	3	1	6	0.6934	5	0.7944	5	0.6805	6
Carp	0.5239	9	0.8199	8	0.5029	11	0.5365	11	0.495	11
Lions	0.3162	12	0.4612	12	0.5249	9	0.5692	9	0.5215	8
Fighters	0.5765	8	0.8042	9	0.528	8	0.5834	8	0.5199	9
Blue Wave	0.7368	5	1	5	0.5633	7	0.6073	7	0.5527	7
Buffalos	0.4947	11	0.7739	10	0.5209	10	0.5493	10	0.5133	10
Marines	0.5156	10	1	4	0.4504	12	0.4601	12	0.4421	12
Hawks	1	2	1	3	0.7014	4	0.822	3	0.6886	4

	cot	cot	coth	coth	csch	csch	Product	Product	Frontier	Frontier
Teams	ES	Rank	ES	Rank	ES	Rank	ES	Rank	ES	Rank
Swallows	0.6943	6	0.7983	4	0.6267	6	0.7994	4	0.7734	3
Dragons	1	1	0.9822	2	0.9919	1	1	1	0.9993	2
Giants	0.8676	2	1	1	0.7793	2	0.9863	2	1	1
Tigers	0.718	3	0.7685	6	0.678	3	0.7836	6	0.7441	5
Bay Stars	0.695	5	0.7757	5	0.6412	4	0.7944	5	0.7272	6
Carp	0.504	11	0.5294	11	0.4804	11	0.5365	11	0.5247	11
Lions	0.5287	9	0.5796	8	0.4883	10	0.5692	9	0.6012	7
Fighters	0.5297	8	0.5761	9	0.4948	9	0.5834	8	0.5611	9
Blue Wave	0.5638	7	0.5925	7	0.5386	7	0.6073	7	0.5726	8
Buffalos	0.5222	10	0.5441	10	0.4997	8	0.5493	10	0.5463	10
Marines	0.4503	12	0.4497	12	0.4428	12	0.4601	12	0.4486	12
Hawks	0.7035	4	0.8033	3	0.6404	5	0.822	3	0.7450	4

Table 5. Correlation between several models by ranks of the baseball teams

	CCR	BCC	Clayton	Gumbel	A12	cot	coth	csch	Product	Frontier
CCR	1.0000	0.5800	0.8040	0.7900	0.7550	0.8040	0.7200	0.7830	0.7900	0.6364
BCC	0.5800	1.0000	0.5380	0.5100	0.4830	0.5380	0.4970	0.5240	0.5100	0.3846
Clayton	0.8040	0.5380	1.0000	0.9510	0.9860	1.0000	0.9370	0.9720	0.9510	0.9231
Gumbel	0.7900	0.5100	0.9510	1.0000	0.9510	0.9510	0.9860	0.9160	1.0000	0.9580
A12	0.7550	0.4830	0.9860	0.9510	1.0000	0.9860	0.9510	0.9510	0.9510	0.9580
cot	0.8040	0.5380	1.0000	0.9510	0.9860	1.0000	0.9370	0.9720	0.9510	0.9231
coth	0.7200	0.4970	0.9370	0.9860	0.9510	0.9370	1.0000	0.9020	0.9860	0.9790
csch	0.7830	0.5240	0.9720	0.9160	0.9510	0.9720	0.9020	1.0000	0.9160	0.8811
Product	0.7900	0.5100	0.9510	1.0000	0.9510	0.9510	0.9860	0.9160	1.0000	0.9580
Frontier	0.6364	0.3846	0.9231	0.9580	0.9580	0.9231	0.9790	0.8811	0.9580	1.0000

is 0.538, and the minimum correlation is with the results of A12 copula and it is 0.483. It is notable that standard SFA model (Frontier) has the minimum correlation with CCR and also BBC models. Between copulas models, the model which uses coth copula has the highest correlation 0.9790 by the

standard SFA model. Gumbel, cot and Product copulas are in the second rate as they have high correlation 0.9580 with the standard SFA. BCC model has the lowest correlation 0.3846 with the standard SFA in this test.

It is remarkable that in the situation which ranks of DMUs are available, clearly a model is the best which the estimated ranks of DMUs (by the model) are closest to the real ranks of DMUs. Meanwhile we recall that the CCR model evaluates both technical and scale efficiency (so named as overall technical efficiency), combining both measures in a single efficiency score. The BCC model evaluates pure technical efficiency (PTE) of DMUs (so named as local technical efficiency), for details see Sözen and Alp (2009). Based on this information, solutions which have high correlation with the results of CCR model, can be said to be relatively good models. By the Table 5, results of the standard SFA has the minimum correlation with results of the CCR model, but results of the models with copula technique in SFA, have higher correlation with results of the CCR model. Results of SFA models by Clayton and cot copula families have the maximum correlation with results of the CCR model and it is 0.804. For this reason SFA models by Clayton and cot copula families can be said relatively good models in this test.

5. Conclusion

In the aim of satisfying efficiency scores and ranks of the twelve Japanese professional baseball teams, we have relied on copula technique in SFA models. Calculations summarize that Gumbel and Product copulas have resulted same ranks for the baseball teams as correlation between them is one. The same story is about cot and Clayton copulas. It is notable that standard SFA model has the minimum correlation with CCR and also BBC models. Between copulas models, the model which uses coth copula has the highest correlation 0.9790 by the standard SFA model. Gumbel, cot and Product copulas are in the second rate which have high correlation 0.9580 with the standard SFA. BCC model has the lowest correlation 0.3846 with the standard SFA in this test. As the CCR model evaluates both technical and scale efficiency solutions which have high correlation with the results of CCR model, can be said to be relatively good models. By the Table 5, results of SFA models by Clayton and cot copula families have the maximum correlation with results of the CCR model and it is 0.804. For this reason SFA models by Clayton and cot copula families can be said relatively good models in this test.

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