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TARGET PARAMETER ESTIMATION OF MIMO RADAR BASED ON ELECTROMAGNETIC ENVIRONMENT SENSORY IN COLORED NOISE

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A concept of Multiple Inputs Multiple Output radar based on electromagnetic environment sensory is presented. An algorithm for parameter estimation method based on quaternion theory is discussed. based on the study on the forth order cumulant slice, a method of direction of arrival estimation for Multiple Input Multiple Output radar in electromagnetic environment sensory based on the 4th order cumulant slice in colored noise environment is derived. Simulation results show that MUSIC algorithm based on the theory of accumulative amount-quaternion can estimate the parameters of electromagnetic environment sensory Multiple Input Multiple Output radar effectively in view of existing algorithm insufficiency. This paper compares the two mentioned algorithms and demonstrates the estimation technology and application value of the algorithms in the application of electromagnetic environment sensory Multiple Input Multiple Output radar target parameter estimation under the background of colored noise.

Keywords: Bistatic Multiple Input Multiple Output radar, Quaternion, Fourth order cumulant, MUSIC, DOA estimation

1. Introduction

Bistatic EES-MIMO (Electromagnetic environment sensory-Multiple Input Multiple Output) [1] radar has the double advantage of bistatic radar and MIMO (Multiple Input Multiple Output) technology, and it is easier to realize in the project than the statistical MIMO radar, so it has become a hotspot in radar research field [2,3]. Due to the quaternion orthogonal vector has stronger orthogonal constraint condition than complex [4, 5, 6, 7], so the parameters estimation algorithm based on quaternion has better model error tolerance and anti-noise [8]. Therefore, the quadratic theory is used to estimate the target parameter estimation of bistatic EES-MIMO radar.

Bistatic radar with separated transmitter and receivers has some new features, such as anti-jamming, anti-stealth, anti-addition [9-15], therefore bistatic EES-MIMO radar has the advantages of bistatic radar and MIMO technology, it has great development potential. Bistatic EES-MIMO radar has many array forms,

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such as uniform linear array, uniform circular array, L array and planar array, etc. In this paper, a bistatic EES-MIMO radar with uniform linear array is developed.

Around the study of bistatic MIMO radar target parameter estimation, the content and arrangement of the paper is as follows: the second part introduces the bistatic EES-MIMO radar signal model and quaternion model. The third part introduces the MUSIC algorithm based on quaternion theory, the last part is the full text summary.

2. Bistatic EES-MIMO radar structure and quaternion model

2.1 Bistatic EES-MIMO radar structure and signal model

The transmitting and receiving arrays structure of bistatic EES-MIMO radar which are uniform linear array are shown in Fig.1.

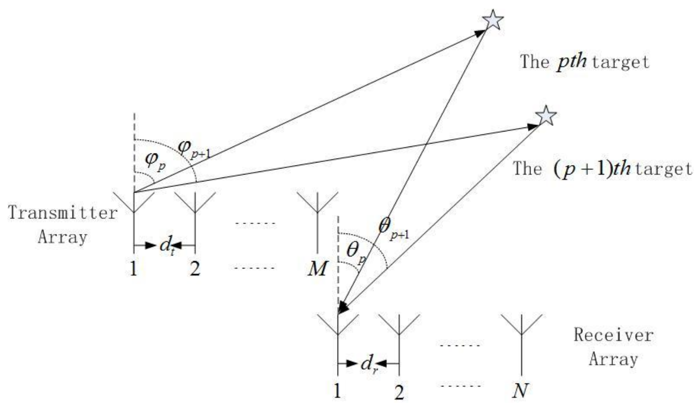


Fig.1 Transceiver array structure model of bistatic EES-MIMO radar

Assuming the radar signal is based on far field narrowband. The number of transmitter and receiver elements is M and N, the distance of transmitter and receiver elements is d_t and d_r; the number of point targets in far field space is P, all targets are in the circle of bistatic EES-MIMO radar range resolution. Assume that there are P uncorrelated targets, and the pth target located at (φ_p, θ_p) where φ_p denotes the direction of the pth target with represents to the transmit array (i.e. DOD) and θ_p denotes the direction of the pth target with represents to the receive array (i.e. DOA). The radial velocity is v_{tp} and v_{rp}.

According to the working principle of bistatic MIMO radar, the signal waveform of each transmitter elements is mutually orthogonal, so that echo signal can be separated in the receiver [16-23].Assume that the transmitted signal is narrowband, carrier wave length of the signal is λ, a pulse signal waveform emitted by mth transmitter element can be expressed as follows:

$$s_m(t) = \left[s_{m1}(t), s_{m2}(t), \cdots s_{mK}(t) \right] \tag{1}$$

The matrix form as follows:

$$S(t) = \left[s_1(t), s_2(t), \cdots, s_M(t) \right]^T \in C^{M \times K} \tag{2}$$

As the transmitted signals are orthogonal to each other,

$$\int s_i(t) s_j^*(t) dt = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}, \quad i, j \in (1, M) \tag{3}$$

2.2 Quaternion model of bistatic EES-MIMO radar

Assume that there is no noise for convenience of discussion. The $((n-1)M + m)$ row of receive data Y in all L pulse echo:

$$y_{m,n} = \sum_{l=1}^{L} \sum_{p=1}^{P} D_p e^{i2\pi f_{dp} Tl} e^{i(m-1)2\pi d_t \sin\varphi_p / \lambda} e^{i(n-1)2\pi d_r \sin\theta_p / \lambda} \tag{4}$$

Where $\alpha_P = 2\pi d_t \sin\varphi_p / \lambda$; $\beta_P = 2\pi d_r \sin\theta_p / \lambda$; $F_{p,l} = e^{i2\pi f_{dp} Tl}$

Also,

$$y_{m,n} = \sum_{l=1}^{L} \sum_{p=1}^{P} D_p F_{p,l} e^{i(m-1)\alpha_p} e^{i(n-1)\beta_p} \tag{5}$$

Take related operations

$$\begin{aligned} r_1(\tau, \zeta) &= E\left[y_{m+\tau, n+\zeta} y_{m,n}^H \right] \\ &= \sum_{l=1}^{L} \sum_{p=1}^{P} D_p F_{p,l} (D_p F_{p,l})^H e^{i(m+\tau-1)\alpha_p} e^{i(n+\zeta-1)\beta_p} e^{i(1-m)\alpha_p} e^{i(1-n)\beta_p} \\ &= \sum_{p=1}^{P} D_{Fp} e^{i\tau\alpha_p} e^{i\zeta\beta_p} \end{aligned} \tag{6}$$

Where, $D_{Fp} = \sum_{l=1}^{L} D_p F_{p,l} (D_p F_{p,l})^H = \sum_{l=1}^{L} D_p D_p^H e^{i2\pi f_{dp} Tl} e^{-i2\pi f_{dp} Tl} = D_p D_p^H$

It follows from (6) that

$$r_1(-\tau, \zeta) = \sum_{p=1}^{P} D_{Fp} e^{-i\tau\alpha_p} e^{i\zeta\beta_p} \tag{7}$$

$$r_1(\tau, -\zeta) = \sum_{p=1}^{P} D_{Fp} e^{i\tau\alpha_p} e^{-i\zeta\beta_p} \tag{8}$$

$$r_1(-\tau, -\zeta) = \sum_{p=1}^{P} D_{Fp} e^{-i\tau\alpha_p} e^{-i\zeta\beta_p} \tag{9}$$

It follows from (6) (7) (8) (9) that:

$$\begin{aligned} f(\tau,\zeta) &= r_1(\tau,\zeta)+r_1(-\tau,\zeta)+r_1(\tau,-\zeta)+r_1(-\tau,-\zeta) \\ &= \sum_{p=1}^{P} D_{Fp}(e^{i\tau\alpha_p}+e^{-i\tau\alpha_p})(e^{i\zeta\beta_p}+e^{-i\zeta\beta_p}) \\ &= 4\sum_{p=1}^{P} D_{Fp}\cos(\tau\alpha_p)\cos(\zeta\beta_p) \end{aligned} \tag{10}$$

The partial Hilbert transform, and the general Hilbert transform of $f(\tau,\zeta)$ are as follows:

$$f^{\tau}(\tau,\zeta) = 4\sum_{p=1}^{P} D_{Fp}\sin(\tau\alpha_p)\cos(\zeta\beta_p) \tag{11}$$

$$f^{\zeta}(\tau,\zeta) = 4\sum_{p=1}^{P} D_{Fp}\cos(\tau\alpha_p)\sin(\zeta\beta_p) \tag{12}$$

$$f^{T}(\tau,\zeta) = 4\sum_{p=1}^{P} D_{Fp}\sin(\tau\alpha_p)\sin(\zeta\beta_p) \tag{13}$$

Where $f(\tau,\zeta)$, $f^{\tau}(\tau,\zeta)$, $f^{\zeta}(\tau,\zeta)$, $f^{T}(\tau,\zeta)$ are pairwise orthogonal. $f^{\tau}(\tau,\zeta)$; $f^{\zeta}(\tau,\zeta)$; $f^{T}(\tau,\zeta)$ can be calculated by formula (10).

$$\begin{aligned} f^{\tau}(\tau,\zeta) &= 4\sum_{p=1}^{P} D_{Fp}\sin(\tau\alpha_p)\cos(\zeta\beta_p) \\ &= \frac{1}{i}\left[r_1(\tau,\zeta)-r_1(-\tau,\zeta)+r_1(\tau,-\zeta)-r_1(-\tau,-\zeta)\right] \end{aligned} \tag{14}$$

$$\begin{aligned} f^{\zeta}(\tau,\zeta) &= 4\sum_{p=1}^{P} D_{Fp}\cos(\tau\alpha_p)\sin(\zeta\beta_p) \\ &= \frac{1}{i}\left[r_1(\tau,\zeta)+r_1(-\tau,\zeta)-r_1(\tau,-\zeta)-r_1(-\tau,-\zeta)\right] \end{aligned} \tag{15}$$

$$\begin{aligned} f^{T}(\tau,\zeta) &= 4\sum_{p=1}^{P} D_{Fp}\sin(\tau\alpha_p)\sin(\zeta\beta_p) \\ &= -\left[r_1(\tau,\zeta)-r_1(-\tau,\zeta)-r_1(\tau,-\zeta)+r_1(-\tau,-\zeta)\right] \end{aligned} \tag{16}$$

Four pairwise orthogonal data can be got by the above calculations, which are used as the 4-dimensional orthogonal component of the quaternion, and then the bistatic EES-MIMO radar based on quaternion model, is constructed.

In conclusion, the definition of bistatic EES-MIMO radar model based on quaternion as follows:

$$\begin{aligned} r(\tau,\zeta) &= f(\tau,\zeta)+if^{\tau}(\tau,\zeta)+jf^{\zeta}(\tau,\zeta)+kf^{T}(\tau,\zeta) \\ &= 4\sum_{p=1}^{P} D_{Fp}\begin{bmatrix}\cos(\tau\alpha_p)\cos(\zeta\beta_p)+i\sin(\tau\alpha_p)\cos(\zeta\beta_p)+ \\ j\cos(\tau\alpha_p)\sin(\zeta\beta_p)+k\sin(\tau\alpha_p)\sin(\zeta\beta_p)\end{bmatrix} \\ &= 4\sum_{p=1}^{P} D_{Fp}\left[e^{i\tau\alpha_p}\cos(\zeta\beta_p)+j\sin(\zeta\beta_p)e^{i\tau\alpha_p}\right] \\ &= 4\sum_{p=1}^{P} D_{Fp}\left[e^{i\tau\alpha_p}e^{j\zeta\beta_p}\right] \\ &= 4\sum_{p=1}^{P} D_{Fp}\left[e^{i\tau 2\pi d_t\sin\varphi_p/\lambda}e^{j\zeta 2\pi d_r\sin\theta_p/\lambda}\right] \end{aligned} \tag{17}$$

3. Quaternion MUSIC algorithm under colored noise background

In this section, the definition of fourth-order cumulant and the derivation of fourth-order cumulant matrix are analyzed at first, then the model of cumulant -quaternion is derived. Finally, we propose two kinds of MUSIC algorithm based on the cumulative -quaternion model.

3.1 Definition of cumulant and cumulant -quaternion model

The signal model should be expressed as

$$x(m,n)=4\sum_{p=1}^{P}D_{Fp}\left[e^{\mathrm{im}2\pi d_t\sin\varphi_p/\lambda}e^{jn2\pi d_r\sin\theta_p/\lambda}\right]+v(m,n) \tag{18}$$

Where D_{Fp} is scattering coefficient $v(m,n)$ is colored noise, noise and signal are independent. First of all, the following formula is defined according to the signal model and the independence in signal and noise.

$$\begin{aligned}&\left[\exp(im2\pi d_t\sin\varphi_p/\lambda)\exp(jn2\pi d_r\sin\theta_p/\lambda)\right]^*=\\&\exp(-im2\pi d_t\sin\varphi_p/\lambda)\exp(-jn2\pi d_r\sin\theta_p/\lambda)\end{aligned} \tag{19}$$

The following formula can be got based on operation rules of Ma quaternion:

$$\begin{aligned}&\left[\exp(im2\pi d_t\sin\varphi_p/\lambda)\exp(jn2\pi d_r\sin\theta_p/\lambda)\right]\\&\left[\exp(im2\pi d_t\sin\varphi_p/\lambda)\exp(jn2\pi d_r\sin\theta_p/\lambda)\right]^*=1\end{aligned} \tag{20}$$

Operation rules of Ma quaternion [25] is:

$$j=jj=kk=-1,ijj=ji=k,$$
$$jk=kj=-i,ki=ik=-j$$

Ma quaternion satisfies commutative law of multiplication and $\exp(ity)=\cos ty+i\sin ty$ .In addition:

$$\begin{aligned}\cos(\omega_{1l}p)\cos(\omega_{2l}q)=[&\exp(i\omega_{1l}p)\exp(j\omega_{2l}q)+\exp(-j\omega_{2l}q)\exp(-i\omega_{1l}p)+\\&\exp(j\omega_{2l}q)\exp(-i\omega_{1l}p)+\exp(-j\omega_{2l}q)\exp(i\omega_{1l}p)]/4\end{aligned} \tag{21}$$

$$\begin{aligned}\sin(\omega_{1l}p)\sin(\omega_{2l}q)=[&\exp(i\omega_{1l}p)\exp(j\omega_{2l}q)+\exp(-j\omega_{2l}q)\exp(-i\omega_{1l}p)-\\&\exp(j\omega_{2l}q)\exp(-i\omega_{1l}p)-\exp(-j\omega_{2l}q)\exp(i\omega_{1l}p)]/4k\end{aligned} \tag{22}$$

$$\begin{aligned}\sin(\omega_{1l}p)\cos(\omega_{2l}q)=[&\exp(i\omega_{1l}p)\exp(j\omega_{2l}q)-\exp(-j\omega_{2l}q)\exp(-i\omega_{1l}p)-\\&\exp(j\omega_{2l}q)\exp(-i\omega_{1l}p)+\exp(-j\omega_{2l}q)\exp(i\omega_{1l}p)]/4i\end{aligned} \tag{23}$$

$$\begin{aligned}\cos(\omega_{1l}p)\sin(\omega_{2l}q)=[&\exp(i\omega_{1l}p)\exp(j\omega_{2l}q)-\exp(-j\omega_{2l}q)\exp(-i\omega_{1l}p)+\\&\exp(j\omega_{2l}q)\exp(-i\omega_{1l}p)-\exp(-j\omega_{2l}q)\exp(i\omega_{1l}p)]/4j\end{aligned} \tag{24}$$

Fourth order cumulant of (25) could be expressed as follows:

$$-E\left[x^*(m,n)x(m+\tau_1,n+\tau_2)\right]E\left[x^*(m+\tau_3,n+\tau_4)x(m+\tau_5,n+\tau_6)\right]$$

Because of the noise is assumed to be Gaussian color noise in signal modeling, the fourth order cumulant of the color noise is zero in literature [24], which proves that the fourth order cumulant can effectively suppress the influence of spatial Gaussian color noise.

Take a two-dimensional diagonal slice as follows:

$$\tau_3=\tau_4=\tau_5=\tau_6=0$$

$$\begin{aligned} C_{4X}(\tau_1,\tau_2)&=C_{4X}(\tau_1,\tau_2,0,0,0,0) \\ &=-\sum_{p=1}^{P} D^4{}_{Fp} e^{i\tau_1 2\pi d_t \sin\varphi_p/\lambda} e^{i\tau_2 2\pi d_r \sin\theta_p/\lambda} \end{aligned} \tag{25}$$

Formula (25) is defined as the cumulative -quantity model.

3.2 MUSIC algorithm based on the singularity decomposition of the cumulant - quaternion matrix (CQ-MUSIC)

For the cumulative-quantity model of EES-MIMO radar in (17), let $\gamma_p = -D^4{}_{Fp}$, then $r(\tau,\zeta)=\sum_{p=1}^{P}\gamma_p\left[e^{i\tau\alpha_p}e^{j\zeta\beta_p}\right]$,where $\alpha_P = 2\pi d_t \sin\varphi_p/\lambda$, $\beta_P = 2\pi d_r \sin\theta_p/\lambda$, for the cumulant-quaternion model of EES-MIMO radar in (17), the data matrix R can be given by：

$$\begin{aligned} R&=\begin{bmatrix} r(0,0) & r(0,1) & \cdots & r(0,N-1) \\ r(1,0) & r(1,1) & \cdots & r(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ r(M-1,0) & r(M-1,1) & \cdots & r(M-1,N-1) \end{bmatrix} \\ &=\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{i\alpha_1} & e^{i\alpha_2} & \cdots & e^{i\alpha_P} \\ \vdots & \vdots & \ddots & \vdots \\ e^{i(M-1)\alpha_1} & e^{i(M-1)\alpha_2} & \cdots & e^{i(M-1)\alpha_P} \end{bmatrix}\begin{bmatrix} \gamma_1 & & & \\ & \gamma_2 & & \\ & & \ddots & \\ & & & \gamma_P \end{bmatrix} \\ &\quad\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j\beta_1} & e^{-j\beta_2} & \cdots & e^{-j\beta_P} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j(N-1)\beta_1} & e^{-j(N-1)\beta_2} & \cdots & e^{-j(N-1)\beta_P} \end{bmatrix} = A\sum B^H \end{aligned} \tag{26}$$

Decomposing cumulant-quaternion mat -rix based on SVD and sorting them from big to small order:

$$R = A\sum B^H = USV^H \tag{27}$$

Where $U \in Q^{M\times M}$ is the left singular value vector matrix of cumulant-quarter nion, the first P column of U is the left signal subspace, which represents to transmit steering matrix $U_S \in Q^{M\times P}$, where $U_S \in Q^{M\times P}$ and $U_S = span(\mathrm{A})$, the $M-P$ column of is the left noise subspace, which represents to receive steering matrix $U_N \in Q^{M\times(M-P)}$, $V \in Q^{N\times N}$ is the right singular value vector matrix of cumulant-quaternion, the first P column of V is the right signal subspace, which represents to receive steering matrix $V_S \in Q^{P\times N}$ and $V_S = span(\mathrm{B})$, $N-P$ column of V is right noise subspace $V_N \in Q^{N\times(N-P)}$. $S \in R^{M\times N}$ represents the singular value matrix of R.

With repect to (27), we form the following matrix:

$$R = A\Sigma B^H = [U_S \quad U_N]\begin{bmatrix}\Sigma' & 0\\ 0 & 0\end{bmatrix}[V_S \quad V_N]^H \tag{28}$$

Where $\sum' \in R^{P\times P}$ represents real diagonal matrix consisted of P large singular values, where

$$U_S \perp U_N,\ A \perp U_N,\ V_S \perp V_N,\ B \perp V_N \tag{29}$$

$$U_N^H u_p = 0,\ \ V_N^H v_p = 0,\ \ (p = 1, 2, \cdots, P) \tag{30}$$

According to the orthogonality relations of left noise singular vector and left signal singular vector, we construct the MUSIC spatial spectrum function in this form

$$f(\varphi) = \frac{1}{u^H(\varphi)U_N U_N^H u(\varphi)} \tag{31}$$

Where
$$u(\varphi) = [1, e^{i\alpha}, e^{i2\alpha}, \cdots, e^{i(M-1)\alpha}]^T$$

$$\alpha = 2\pi d_t \sin\varphi / \lambda;$$

$$v(\theta) = [1, e^{-j\beta}, e^{-j2\beta}, \cdots, e^{-j(N-1)\beta}]^T;$$

$$\beta = 2\pi d_r \sin\theta / \lambda.$$

Simultaneously, we construct the MUSIC spatial spectrum function as right noise singular vector and right signal singular vector are mutually orthogonal.

$$f(\theta) = \frac{1}{v^H(\theta)V_N V_N^H v(\theta)} \tag{32}$$

Maximum likelihood is used to match transceiver angle, we constuct the maxi- mum likelihood cost function

$$\Theta_{ML} = \log\left[Y_l^H P_{\hat{E}}^{\perp} Y_l\right] \tag{33}$$

Where
$$Y_l = \left[Y_{1,1,l}, Y_{1,2,l}, \cdots, Y_{1,M,l}, \cdots, Y_{N,M,l}\right]^T \in C^{MN\times 1}$$

$\hat{E}=\left[b(\hat{\theta}_1)\otimes \mathrm{a}(\hat{\varphi}_1),b(\hat{\theta}_2)\otimes \mathrm{a}(\hat{\varphi}_2),\cdots,b(\hat{\theta}_P)\otimes \mathrm{a}(\hat{\varphi}_P)\right]$, $\hat{E}$ is the direction matrix which is composed of estimated value of DOD and DOA, where $P_{\hat{E}}^{\perp}=I_{MN}-\hat{E}\hat{E}^{+}$.

Matching the estimated DOD and DOA with arbitrary combination, there are P^2 combination modes when there are P targets, the cost function value can be attained via substituting the angle of the P^2 combination into (33), the angle corresponds to the minimum cost function is the correct transceiver angle[23].

3.3 MUSIC algorithm based on the SVD of cumulant-quaternion derived matrix (CQD-MUSIC)

$R^{\sigma}\in C^{2M\times 2N}$ denotes derived matrix of R (complex representation matrix), Decomposing R^{σ} based on SVD and sorting them from big to small order:

$$R^{\sigma}=\begin{bmatrix}U_S^{\sigma} & U_N^{\sigma}\end{bmatrix}\begin{bmatrix}\Sigma_S & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}V_S^{\sigma} & V_N^{\sigma}\end{bmatrix}^H \tag{34}$$

Where $\Sigma_S\in R^{2P\times 2P}$ is real diagonal matrix, there are 5 large singular values in diagonal, where

$$U_S^{\sigma}\perp U_N^{\sigma},\ V_S^{\sigma}\perp V_N^{\sigma} \tag{35}$$

U_S^{σ} 、 V_S^{σ} can be denoted according to the relation between the matrix of quaternion and its derived matrix.

$$U_S^{\sigma}=\begin{bmatrix}1 & 0 & \cdots & 1 & 0\\ e^{i\alpha_1} & 0 & \cdots & e^{i\alpha_P} & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ e^{i(M-1)\alpha_1} & 0 & \cdots & e^{i(M-1)\alpha_P} & 0\\ 0 & 1 & \cdots & 0 & 1\\ 0 & e^{-i\alpha_1} & \cdots & 0 & e^{-i\alpha_P}\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & e^{-i(M-1)\alpha_1} & \cdots & 0 & e^{-i(M-1)\alpha_P}\end{bmatrix}T_1=U_{\alpha}T_1 \tag{36}$$

Where T_1 is full rank matrix with $2P\times 2P$.

$$V_S^{\sigma}=\begin{bmatrix}1 & 0 & \cdots & 1 & 0\\ \cos\beta_1 & -\sin\beta_1 & \cdots & \cos\beta_P & -\sin\beta_P\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ G_1 & -H_1 & \cdots & G_P & -H_P\\ 0 & 1 & \cdots & 0 & 1\\ \sin\beta_1 & \cos\beta_1 & \cdots & \sin\beta_P & \cos\beta_P\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ H_1 & G_1 & \cdots & H_P & G_P\end{bmatrix}T_2=V_{\beta}T_2 \tag{37}$$

Where $G_1=\cos[(N-1)\beta_1]$, $H_1=\sin[(N-1)\beta_1]$, $G_P=\cos[(N-1)\beta_P]$, $H_P=\sin[(N-1)\beta_P]$, T_2 is full rank matrix with $2P\times 2P$.

$$U_{\alpha}^H U_N^{\sigma}=0 \tag{38}$$

$$V_{\beta}^{H} V_{N}^{\sigma} = 0 \tag{39}$$

Where $u(\varphi) = \left[1, e^{i\alpha}, e^{i2\alpha}, \cdots, e^{i(M-1)\alpha}, 0, 0, \cdots, 0\right]^T$

According to the orthogonality relations between left noise singular vector and left signal singular vector, we construct the MUSIC spatial spectrum function in this form

$$f(\varphi) = \frac{1}{u(\varphi)^H U_N^{\sigma} (U_N^{\sigma})^H u(\varphi)} \tag{40}$$

The DOD is estimated by searching the spectrum peak. Similarly, according to the orthogonality relations between right noises singular vector and right signal singular vector, we construct the MUSIC spatial spectrum function in this form

$$f(\theta) = \frac{1}{v(\theta)^H V_N^{\sigma} (V_N^{\sigma})^H v(\theta)} \tag{41}$$

Where $v(\theta) = \left[1, \cos\beta, \cos(2\beta), \cdots, \cos((M-1)\beta), 0, \sin\beta, \cdots, \sin((N-1)\beta)\right]$

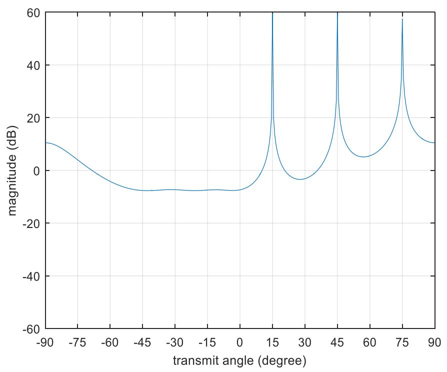
The DOA is estimated by searching the spectrum peak.

The DOD and DOA are estimated separately and needs to be matched with the method of maximum likelihood match.

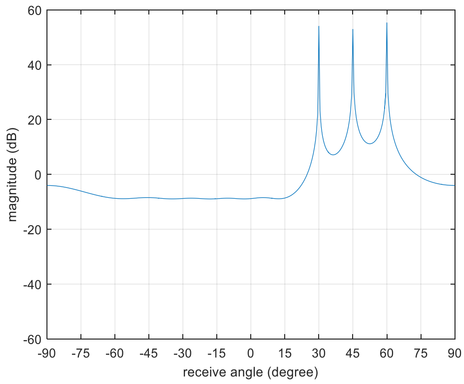
4. Simulation results

Experiment 1: Multi parameter estimation experiment

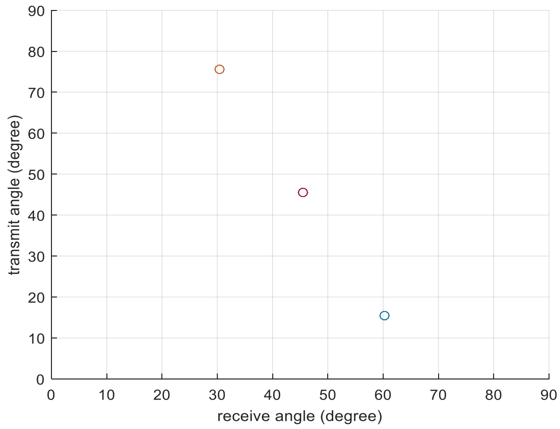
Experimental conditions: We adopt the bistatic EES-MIMO radar system with $M = 6, N = 8$, where three non-coherent sources are assumed to be at the location of angles $(\varphi_1, \theta_1) = (15^\circ, 60^\circ)$, $(\varphi_2, \theta_2) = (45^\circ, 45^\circ)$, $(\varphi_3, \theta_3) = (75^\circ, 30^\circ)$ in algorithm of CQ-MUSIC and $(\varphi_1, \theta_1) = (15^\circ, 15^\circ)$, $(\varphi_2, \theta_2) = (30^\circ, 45^\circ)$, $(\varphi_3, \theta_3) = (75^\circ, 60^\circ)$ in CQD-MUSIC algorithm. The distance between the transceiver elements is $d_t = d_r = \lambda/2$; The orthogonal *Hadamard* coded signal is transmitted by each transmit element, and the number of phase encoding in every repetition interval is $L = 256$; The number of snapshot is $Q = 100$; Background noise is gaussian color noise. The simulation results are shown in Fig. 2 and 3.



(a) DOD spectral peak search

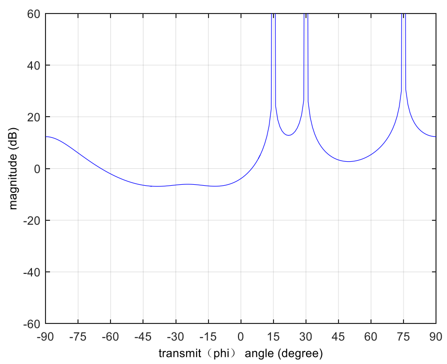


(b) DOA spectral peak search

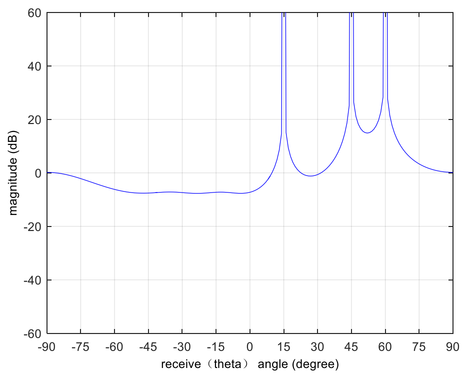


(c) Target location constellation

Fig.2 Multi parameter estimation experiment of CQ-MUSIC



(a) DOD spectral peak search



(b) D spectral peak search

Fig.3 Multi parameter estimation experiment of CQD-MUSIC

One-dimensional spectrum peak search is shown in Fig. 2 (a)、2(b) and

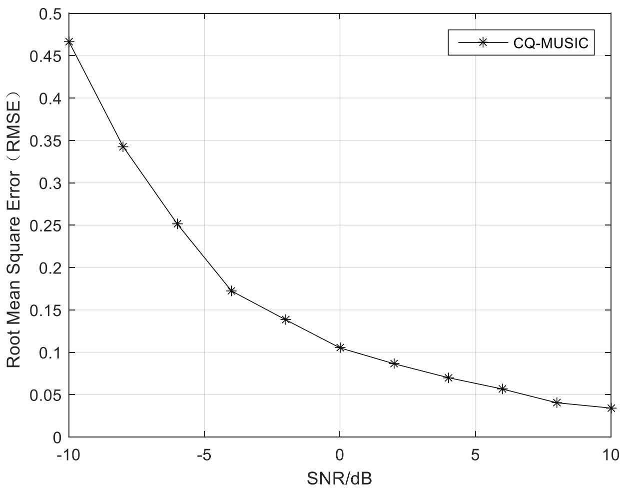
Fig. 3, the location constellation of three targets is shown in Fig. 2(c). It is obviously shown that the algorithm of CQ-MUSIC and CQD-MUSIC are gradually successed in estimating the DOD and DOA, the angle of transceiver are matched correctly, and the target position is estimated accurately.

Experiment 2: The experiment of multi target parameter estimation under different SNR conditions.

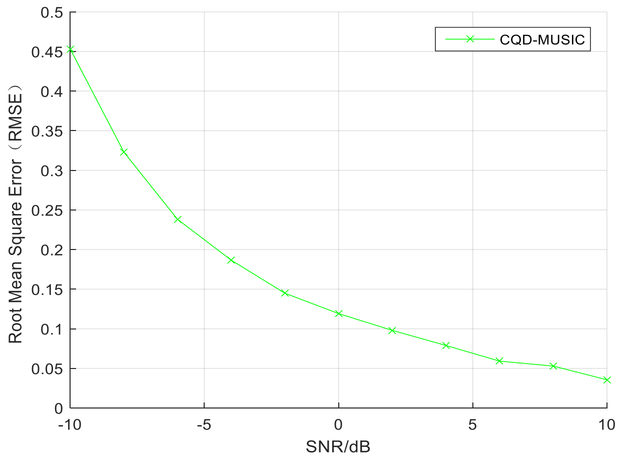
Experimental conditions: The experimental conditions are the same as the experiment 1; SNR is from $-10dB$ to $10dB$ with interval of $5dB$.

Both of the CQ-MUSIC and CQD-MUSIC algorithm are used for 100 independent Monte Carlo experiments at each SNR. The joint root mean square error is used as the evaluation criterion; the root mean square error formula is as follows:

$$RMSE = \frac{1}{P}\sum_{p=1}^{P}\sqrt{\frac{1}{K}\sum_{k=1}^{K}[(\hat{\varphi}_{p,k}-\varphi_p)^2+(\hat{\theta}_{p,k}-\theta_p)^2]} \tag{42}$$



(a) Root mean square error (RMSE) of CQ-MUSIC



(b)Root mean square error (RMSE) of CQD-MUSIC

Fig.4 Root mean square error (RMSE)

As can be seen from Fig.4, the RMSE decreases with the increase of SNR, and the algorithm still has a high accuracy of parameter estimation at low SNR.

Experiment 3: The performance of multi-objective parameter estimation comparisons between CQ-MUSIC and CQD-MUSIC.

Experimental conditions: The experimental conditions are the same as the experiment 1; The CQ-MUSIC and CQD-MUSIC algorithm are used for 100 independent Monte Carlo experiments at each SNR. The experimental conditions are the same as the experiment 1, SNR is from $-10dB$ to $10dB$ with interval of $5dB$.The joint root mean square error is used as the evaluation criterion; the root mean square error formula is as (42).

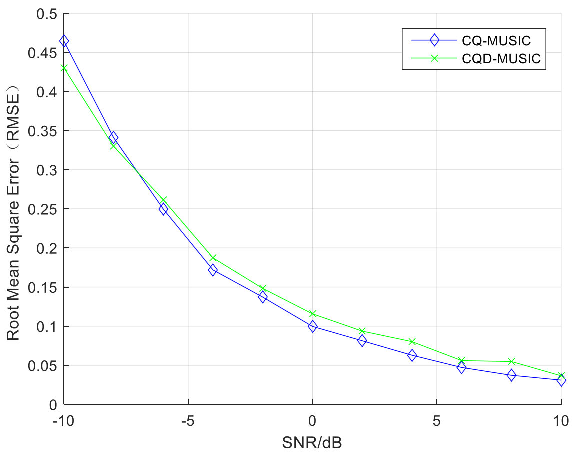


Fig.5 The performance comparation for CQ-MUSIC and CQD-MUSIC

It can be seen from Fig.5 that CQ-MUSIC algorithm and CQD-MUSIC algorithm have almost the same performance, but CQ-MUSIC algorithm needs to decompose the quaternion matrix directly, which leads to the higher computational complexity than CQD-MUSIC algorithm. Making a comprehensive comparison between the two algorithms, CQD is better.

5. Conclusions

In this paper, we present two kinds of MUSIC algorithm based on quaternion model for bistatic EES-MIMO radar in the background of colored noise. The quaternion data matrix of EES-MIMO radar is constructed by the use of quaternion model and then two algorithms that CQ-MUSIC and CQD-MUSIC are proposed; DOD and DOA are estimated separately by the use of above algorithms, the maximum likelihood method is used to match it. Simulation results verify the effectiveness of the two algorithms, and the performances of all algorithms are increased with the growth of number of snapshots. Simultaneously, we also make a performance comparation of the two algorithms, results show that the two algorithms almost have the same performance, but the CQD-MUSIC has less computational complexity.

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