BROADBAND PIEZOELECTRIC VIBRATION ENERGY HARVETING USING ELASTIC ROPES

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Vibration energy harvesting is a promising technique for powering low-consumption electronic devices. A novel design of a frequency up-converted two-degree-of-freedom piezoelectric energy harvester is herein proposed, in which piecewise-linear characteristics are introduced using elastic ropes. This work systematically investigates the influence of various factors on global dynamic performance, such as the position, the number, the damping, and the margin of elastic ropes. The simulation results suggest that such a design could yield a significant performance in terms of the broad frequency bandwidth of energy harvesting and high power output, providing a theoretical reference for practical application.

Keywords: piezoelectric energy harvester; elastic rope; frequency up-conversion; piecewise-linear feature; broadband working bandwidth

1. Introduction

With the rapid development of Internet-of-Things and MEMS technologies, miniaturized and low-consumption electronic devices have been widely used in various production and living environments, such as mechanical engineering, structural health monitoring, and many other fields [1-3]. The traditional power supply mode of using chemical batteries yields many problems such as environmental pollution and high comprehensive cost, while a potential solution is to adopt the renewable and green technology of piezoelectric vibration energy harvesting.

Linear piezoelectric energy harvesting could generate a high energy supply at its natural frequency, while the harvesting capability deteriorates significantly when excited at other frequencies [4]. To solve this problem, efforts have been

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conducted to broaden the effective bandwidth of energy harvesting, which could be simply categorized as multi-mode and nonlinear techniques.

The multi-mode techniques consist in introducing multiple vibration modes into the energy harvesting device, which could be either composed of multiple oscillators of different natural frequencies or a single oscillator of multiple degrees of freedom. Liu et al. [5] studied the energy harnessing feature of a L-form piezoelectric energy harvester composed of a vertical beam and a horizontal beam. By properly designing the mechanical parameters, the two resonant areas are close enough so as to generate a wide-band area of energy harvesting. Zhou et al. [6] proposed a zigzag structure of five degrees of freedom, which could harness low-frequency and low-amplitude vibration energy from multi-directional environmental vibration sources. In general, such kind of techniques are simple in design and effective; however, the structural complexity is unfavorable to the device miniaturization and lowers the density of power captured.

The nonlinear techniques aim at introducing mechanical or electrical nonlinearity into the electromechanical system, including the use of nonlinear magnetic force and the piecewise-linear mechanism. The introduction of nonlinear magnetic force will change the equilibrium position of the energy harvester and could be categorized as monostable, bistable, or multi-stable energy harvester according to the number of equilibrium positions presented in the system [7-9]. Masana et al. [10] investigated the dynamics of fixed-fixed bistable buckled beam, and remarked that the shallower potential well depth is more beneficial since it allows large-orbit inter-well oscillation thereby leading to high output voltage. Meanwhile, the piecewise-linear mechanism consists in perturbating the motion of the energy harvester by using stiff or elastic stoppers, conducting to the piecewise-linearity of mechanical parameters, such as stiffness, damping, or mass [11-12]. Soliman et al. [13] proposed to set a stiff stopper at a specific position around the vibrating cantilever beam, whose stiffness would then suddenly increase when it impacts the stopper. Zhou et al. [14] employed symmetric elastic stoppers and investigated the influence of stopper stiffness on the frequency bandwidth of energy harvesting. However, it should be noted that the impact-type mechanism introduces the piecewise-linear characteristics at the sacrifice of large operation noise, significant energy loss, limited output voltage, and reduced endurability [15].

This work proposes a novel piezoelectric energy harvester integrating the techniques of multiple modes and the piecewise-linear mechanism, aiming at combining their advantages whilst avoiding their drawbacks. In Section 2, the proposed energy harvesting device is first mathematically modeled. Its dynamics will be numerically simulated and the influence of different factors will be discussed in Section 3. Finally, Section 4 draws the main conclusions of this study.

2. Mathematical modeling of proposed piezoelectric vibration energy harvester with elastic ropes

As shown in Fig. 1, the proposed piezoelectric energy harvester is composed of two cantilever beams, in which beam 1 is fixed on the vibrating foundation, while beam 2 is fixed on the tip end of beam 1. A piezoelectric layer is patched on the single side and near the fixed end of beam 1, and it can generate energy when experiencing mechanical deformation. The two-degree-of-freedom piezoelectric energy harvester could generate energy in the proximity of two resonant areas, unlike the single-degree-of-freedom harvest can only capture power near a single vibration mode. Besides, elastic ropes are employed to connect the free ends of both beams to the foundation, and one may easily adjust the margins of different elastic ropes to fulfill specific design objectives. Instead of adopting classic mechanical stoppers at both sides of the cantilever beam, the use of elastic ropes is to introduce piecewise-linear characteristics (including stiffness and damping) into the global dynamics of the electromechanical system and is expected to demonstrate frequency up-conversion broadband energy harvesting performance while maintaining lower energy losses and generating fewer noises.

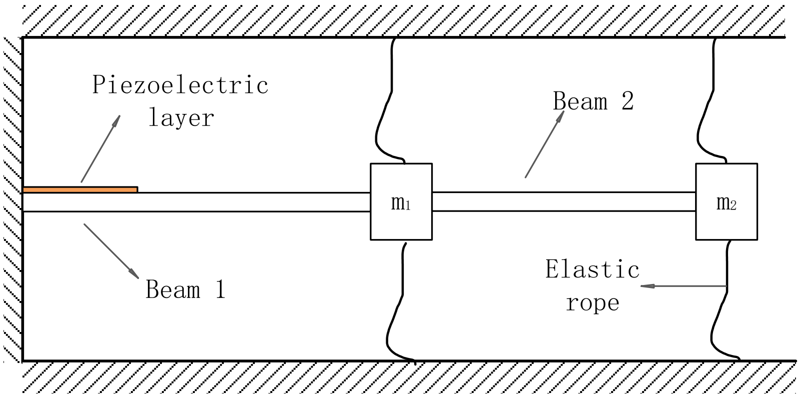


Fig. 1. Schematic of proposed two-degree-of-freedom piezoelectric energy harvester

To investigate systematically the dynamics of the proposed device, the piezoelectric energy harvester can be modeled as a two-degree-of-freedom lumped-mass vibration system as shown in Fig. 2. The equations of motion relevant to such electromechanical system are given by:

$$\begin{aligned} m_1\ddot{x} + c_1\dot{x} + k_1 x + c_2(\dot{x}-\dot{y}) + k_2(x-y) + \theta V - F_1 &= -m_1\ddot{z} \\ m_2\ddot{y} - c_2(\dot{x}-\dot{y}) - k_2(x-y) - F_2 &= -m_2\ddot{z} \\ C_p\dot{V} + \frac{V}{R} &= \theta\dot{x} \end{aligned} \tag{1}$$

where m_1 and m_2 stand for the equivalent lumped mass of cantilever beam 1 and 2, respectively; c_1 and c_2 denote the viscous damping coefficient of each beam, respectively; k_1 and k_2 are the equivalent stiffness of each beam, respectively.

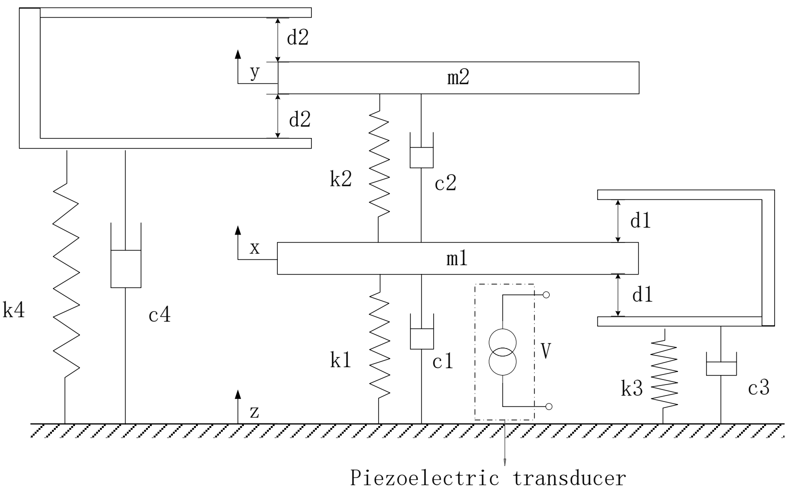


Fig. 2. Lumped-mass model of proposed piezoelectric energy harvester

Besides, C_p is the capacitance of piezoelectric transducer, θ is the electromechanical coupling coefficient characterizing the energy conversion efficiency between mechanical and electrical domains, R is the resistance used in the external harvesting circuit (an extremely large resistance is herein employed to represent the open-circuit circumstance). z is the absolute displacement of the vibrating foundation, while x and y are the relative displacement of equivalent lumped mass m_1 and m_2 with respect to the foundation, respectively. The dot above symbols denotes differentiation with respect to the time t. Therefore, the base acceleration of foundation could be described as:

$$\ddot{z} = A \cdot \sin(2\pi \cdot f \cdot t) \tag{2}$$

where A is the acceleration amplitude of the foundation, f stands for the excitation frequency. Finally, F_1 and F_2 represent the forces introduced by the elastic ropes on the cantilever beams. In the case where elastic ropes are symmetrically placed between cantilever beam and the foundation, the introduced forces could be then described as:

$$F_1 = \begin{cases} -k_3(x-d_1) - c_3\dot{x}, x > d_1 \\ 0, -d_1 \le x \le d_1 \\ -k_3(x+d_1) - c_3\dot{x}, x < -d_1 \end{cases} \tag{3}$$

and

$$F_2 = \begin{cases} -k_4(y-d_2)-c_4\dot{y}, y>d_2 \\ 0, -d_2 \le y \le d_2 \\ -k_4(y+d_2)-c_4\dot{y}, y<-d_2 \end{cases} \tag{4}$$

where k_3 and k_4 stand for the stiffness of elastic rope linked to beam 1 and 2, respectively, while c_3 and c_4 denote the damping effect due to the interaction between the ropes and cantilever beams. Obviously, a larger damping coefficient will lead to a larger energy loss during the energy conversion, thereby deteriorating the energy harvesting performance of the proposed device. This is the reason why the elastic ropes are used in this paper instead of a classic impact-type stopper to introduce piecewise-linear characteristics into the global dynamics, so as to achieve the frequency up-conversion energy harvesting capability. Finally, d_1 and d_2 represent the margin of elastic rope. If the value of the margin is large enough, then the corresponding ropes will not be engaged during the vibration. For example, the entire system could be simplified as sketched in Fig. 3, when d_2 is set as 1000mm.

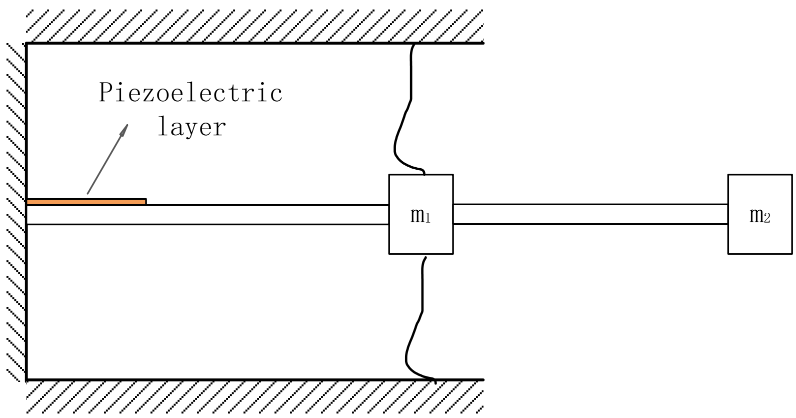


Fig. 3. Equivalent schematic of proposed piezoelectric energy harvester when the margin of the rope at free end of beam 2 is large enough.

It is worth mentioning that if only a single elastic rope is used on a single cantilever beam, then the formulation of the introduced force should be modified accordingly. For example, if only the free end of cantilever beam 2 is linked to the lower foundation, then the introduced force F_2 should be rewritten as:

$$F_2 = \begin{cases} -k_4(y-d_2)-c_4\dot{y}, y>d_2 \\ 0, y \le d_2 \end{cases} \tag{5}$$

Finally, two dimensionless parameters are herein introduced, which represent the damping effect conducted by the elastic ropes during the transition from linear to piecewise-linear state, i.e.:

$$\xi_3 = \frac{c_3}{2\sqrt{k_3 m_1}}, \ \xi_4 = \frac{c_4}{2\sqrt{k_4 m_2}}. \tag{6}$$

3. Numerical simulation of dynamics response of proposed harvester

This study investigates the nonlinear frequency responses of the proposed piezoelectric energy harvester. To this end, the Runge-Kutta integration method implemented in MATLAB® (i.e., the ODE solver) is adopted to numerically solve the ordinary differential Eq. (1) for a given acceleration amplitude A and excitation frequency f, and to achieve the frequency responses by sweeping the excitation frequency f unidirectionally, i.e., from low to high frequency or inversely. The mechanical and electrical parameters of the proposed harvester used for simulation are listed in Table 1.

Table 1

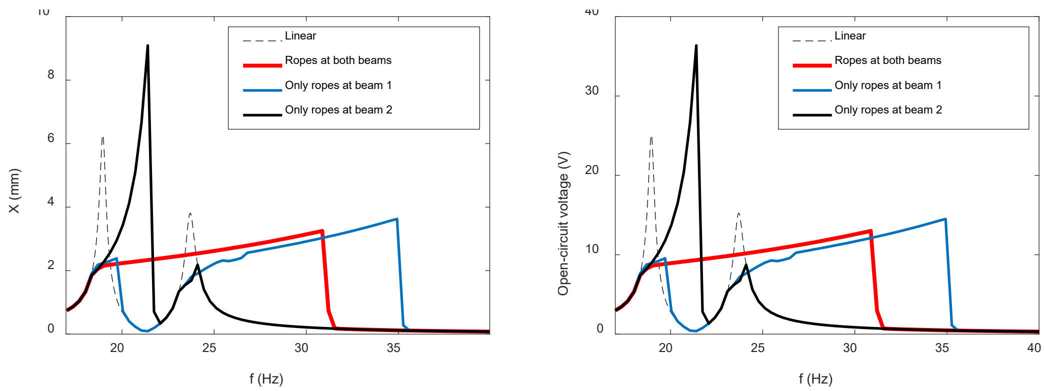
Mechanical and electrical parameters of electromechanical system

Parameter	Symbol	Value	Unit
Equivalent mass of beam 1	m_1	0.2	Kg
Equivalent stiffness of beam 1	k_1	3700	N/m
Equivalent viscous damping coefficient of beam 1	c_1	0.5441	N·s/m
Equivalent mass of beam 2	m_2	0.01	Kg
Equivalent stiffness of beam 2	k_2	170	N/m
Equivalent viscous damping coefficient of beam 2	c_2	0.0261	N·s/m
Capacitance of piezoelectric layer	C_p	50	nF
Electromechanical coupling coefficient	θ	200	μN/V
Resistance of external harvesting circuit	R	10^{30}	Ω

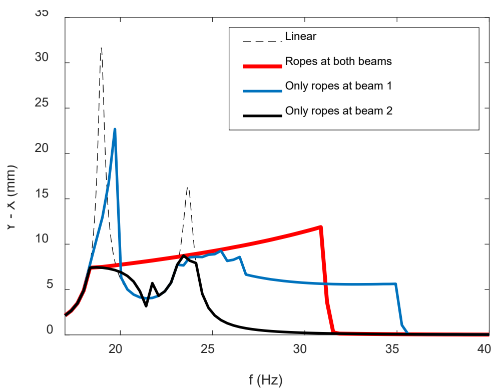
3.1 Influence of position of elastic ropes

We first investigate the influence of the position of elastic ropes on the global behavior of the proposed energy harvester. The base acceleration is set as: $A = 0.5g$ with g being the gravitational constant, and the damping ratios of ropes are fixed at: $\xi_3 = \xi_4 = 0.5\%$.

Fig. 4a depicts the frequency amplitude responses of tip displacement of beam 1 under different boundary conditions. One may find that when only elastic ropes are used at the free end of beam 1 (i.e., the blue solid line), the frequency



(a) Tip displacement of beam 1



(c) Relative displacement between free ends of beam 1 and 2

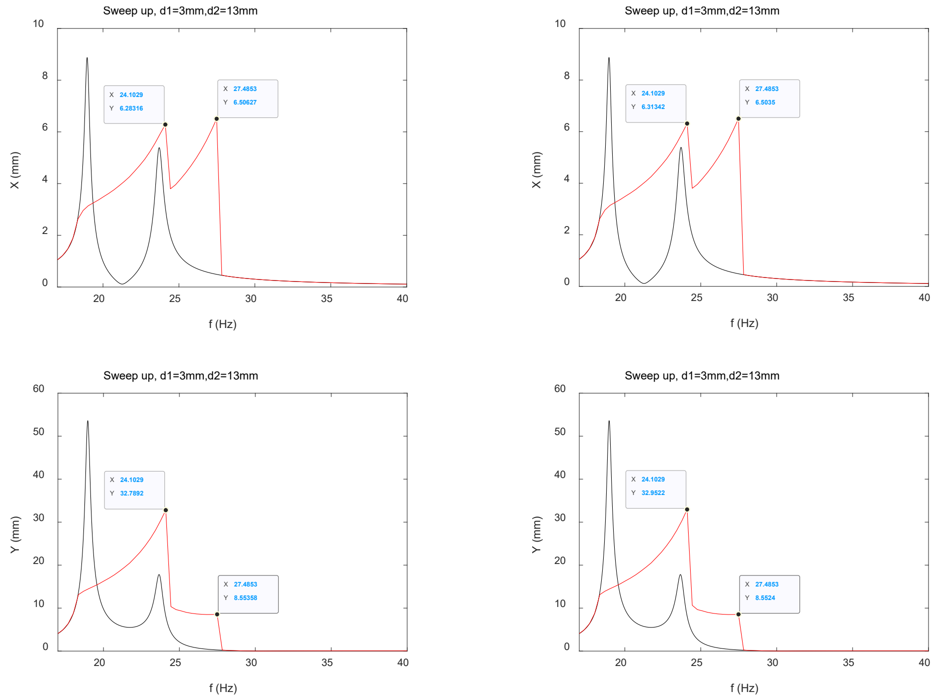
Fig. 4. The frequency amplitude responses. Dashed black line: linear system without any rope; solid red line: with ropes at both beams (d_1=3mm, d_2=13mm); solid blue line: with only ropes set at beam 1 (d_1=3mm, d_2=1000mm); solid black line: with only ropes set at beam 2 (d_1=1000mm, d_2=13mm).

response curve bends to the right at two natural frequencies, while the frequency up-conversion phenomenon is quite significant near the second vibration mode.

When one employs ropes at the free end of beam 2 solely (i.e., the black solid line), only a single important peak is observed between two natural frequencies. In these two cases, a remarkable valley exists in the frequency responses between two natural frequencies. By inspecting the dynamics response relevant to the case with elastic ropes at both beams (marked by a red thick line), we can find that introducing a pair of piecewise-linear mechanisms into the linear system could result in a magnificent energy harvesting performance with an extremely broadband frequency bandwidth and high energy output, while there exists no more valley in the frequency response. Fig. 4b plots the frequency responses of voltage. Since the harvesting shunt is open-circuit, the tendency of voltage response is quite the same as that of tip displacement of beam 1. Fig. 4c shows the frequency amplitude responses of relative displacement between beams 1 and 2. It is hinted that placing elastic ropes at both beams (indicated in a red thick line) could also generate an interesting long-range relative displacement of high amplitude, suggesting that adding piezoelectric patches near the fixed end of beam 2 is also beneficial in terms of global energy harvesting performance.

3.2 Influence of number of elastic ropes

Fig. 5 demonstrates the frequency responses of tip displacement of both beams relative to the foundation for d_1 =3mm, d_2 =13mm. The black solid lines correspond to the case of a linear two-degree-of-freedom energy harvester, while the red solid lines are relevant to the up-sweep frequency responses of the proposed piecewise-linear energy harvester. Meanwhile, Fig. 5a shows the case where only the upward movement of beam 1 and the downward movement of beam 2 are limited by the elastic ropes, Fig. 5b depicts the scenario when only the downward movement of beam 1 and the upward movement of beam 2 are limited by the elastic ropes. By inspecting Fig. 5, one can come to a conclusion that if only a single direction of movement is constrained by rope at the free end of each beam, placing the ropes at either the upper or lower side will not influence the global dynamics of the energy harvester.

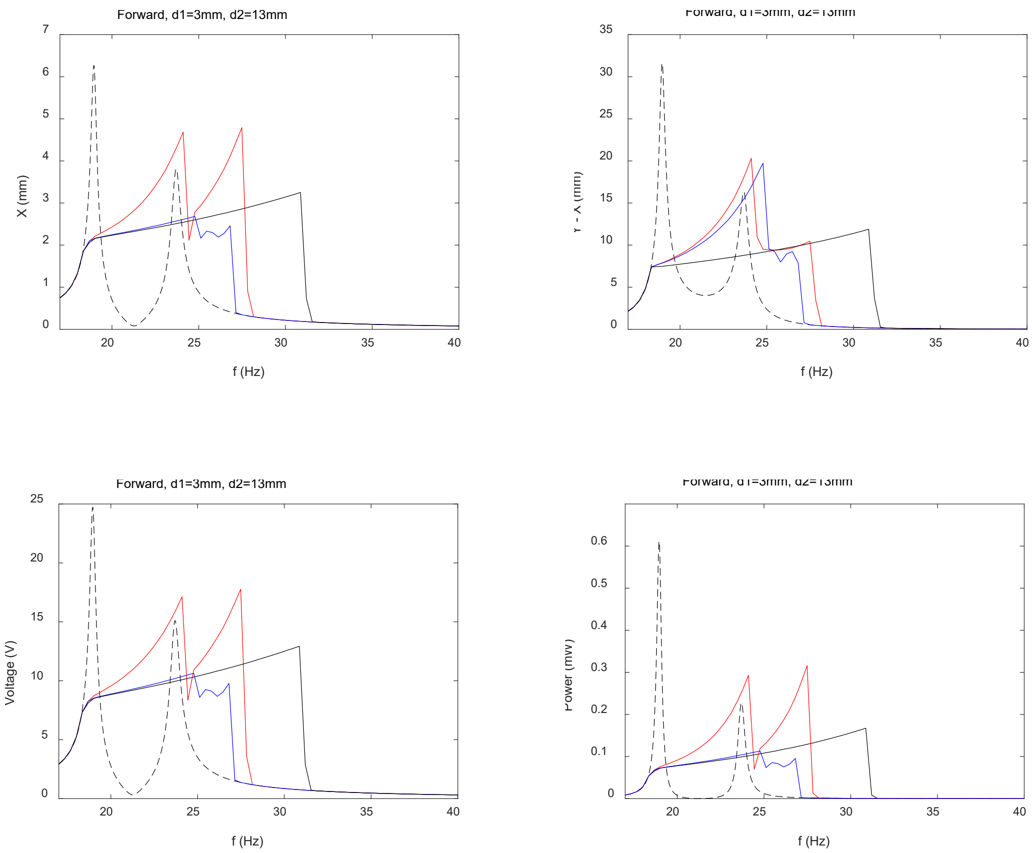


(a) Only the upward movement of beam 1 and the downward movement of beam 2 are limited by the elastic ropes.

) Only the downward movement of beam 1 and the upward movement of beam 2 are limited by the elastic ropes.

Fig. 5. The frequency amplitude responses of tip displacements of both beams for d_1=3mm, d_2=13mm.

Now, let us consider the case where an external resistance of 1MΩ is used, thus the harvesting circuit cannot be considered open-circuit. Fig. 6 depicts the frequency amplitude responses of beam 1 for different conditions, in which Fig. 6a shows the tip displacements of beam 1, Fig. 6b demonstrates the relative displacements between beams 1 and 2, Fig. 6c depicts the voltage across the external resistance, and Fig. 6d is related to the power scavenged by the external harvesting circuit (equal to the voltage squared divided by the resistance). The red curve corresponds to the case where only upward movements of beam 1 and 2 are limited; the blue curve is achieved when both upward and downward movements of beam 1 are limited, while only upward movement of beam 2 is limited; the black curve is obtained when both upward and downward movements of beam 1 and 2 are limited. It is noticeable from Fig. 6 that the voltage across the external resistance and the power harvested have the same tendency as the tip displacement of beam 1. Furthermore, it is hinted from Fig. 6a that by increasing the number of elastic ropes, the maximum displacement amplitude of beam 1 will be reduced, while it can maintain a relatively high vibration amplitude over a broader frequency bandwidth, thus leading to the performance of broadband energy harvesting. Finally, the relative displacement between beams 1 and 2 also maintains a relatively high magnitude over a broad frequency bandwidth, suggesting that one may also harvest energy from the vibration of beam 2.



(d) Harvested power

Fig. 6. The frequency amplitude responses for d_1=3mm, d_2=13mm. Red line: only upward movements of beam 1 and 2 are limited; blue line: both upward and downward movements of beam 1 are limited, while only upward movement of beam 2 is limited; black line: both upward and downward movements of beam 1 and 2 are limited.

3.3 Influence of damping of elastic ropes

Fig. 7 shows the frequency amplitude responses of tip displacement of beam 1 and the open-circuit voltage for different pairs of mechanical damping ratios. One could observe that the ropes with higher damping ratios will lead to a lower peak vibration amplitude and also narrow the effective frequency bandwidth of energy harvesting. Thus, a strict constraint on the introduced damping by ropes should be exerted in order to reduce the energy losses and to guarantee the broadband energy harvesting capability.

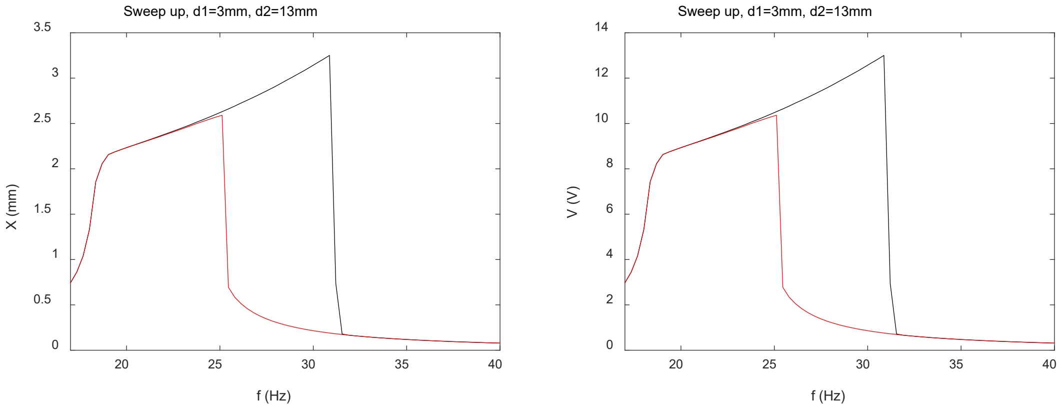


Fig. 7. The frequency amplitude responses of tip displacement of beam 1 (left) and open-circuit voltage (right) for d_1=3mm, d_2=13mm with different mechanical damping ratios. Black line: $\xi_3 = \xi_4 = 0.5\%$; red line: $\xi_3 = \xi_4 = 3\%$.

4. Conclusions

This paper systematically investigates the performance of a novel frequency up-converted piezoelectric energy harvester with the use of low-cost elastic ropes. The key conclusions drawn from this study are listed as follows: (1) placing a pair of elastic ropes at the tip ends of both beams could effectively broaden the frequency bandwidth of energy harvesting while maintaining a relatively high power output; (2) if a single rope is employed for both beams, placing it either on the upper or lower side of each beam will not influence the global harvesting performance; (3) the low-damping nature of elastic rope results in higher power output than classic frequency up-converted generators of impact type which waste significant power during the impact process.

The design of the proposed energy harvester is simple and easy to realize. However, this study only conducted parametric numerical analyses of the proposed energy harvester, while the performance of the harvester can further be improved by adding piezoelectric patches on beam 2, and by optimizing the device parameters (e.g., the stiffness and the margin of elastic ropes). Future work could be dedicated to its compact design and its implementation in applications with spacing restrictions.

Acknowledgments

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