CONDUCTIVE-TYPE BEHAVIORS IN THE DYNAMICS OF COMPLEX SYSTEMS THROUGH SCALE RELATIVITY THEORY

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Conductive-type dynamics in complex systems in the framework of Scale Relativity Theory are analyzed. Using the Madelung scenario in the description of complex system dynamics through continuous and nondifferentiable curves (fractal/multifractal curves), three types of conductivity are highlighted – differentiable conductivity, nondifferentiable conductivity, and global conductivity. These are reciprocally conditional, implying synchronous and nonsynchronous mechanisms in the conductive-type behaviors.

Keywords: fractal object, scales space, scale relativity theory, multifractal curves, differentiable conductivity

1. Introduction

The main purpose of the presented model is to be an alternative to the classical models of complex system dynamics, which analyze the phenomenon only using differentiable mathematical procedures [1-3]. The starting point of our theoretical model is the Scale Relativity Theory that in the last years has been used

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successfully in describing the dynamics of complex systems [4-13]. The hypothesis underlying this theory is that the entities of any complex system move on continuous and non-differential curves, named fractal/multifractal curves, i.e., three dimensional fractured/multifractal lines, whose non-linearity is dependent and proportional with the number of interactions within the system. In this context, the fractalization/multifractalization degree will be defined as a measure of system complexity and physical quantities, characterizing the system evolution, will be fractal/multifractal functions dependent both on spatio-temporal coordinates and resolution scales. Furthermore, the complex system will be considered as a medium without interaction between its components [14,15].

One will use these hypotheses in analyzing the studied complex system and its dynamics which lead to conductive-type behaviors. Such an approach resulted in an analysis through complex system dynamics in the Mandelung fractal/multifractal hydrodynamic scenario.

2. Mathematical model

To further understand the behaviors of the differentiable and nondifferentiable conductivity, a theoretical model has been developed. In the description of complex system dynamics through a hydrodynamic multifractal scenario (Mandelung scenario) [14,15], it is possible to find the involvement of the specific multifractal impulse conservation law:

$$\partial_t v^i + v^l \partial_l v^i = -\partial^i Q, i = 1,2,3 \tag{1}$$

and that of the conservation law of the multifractal states density:

$$\partial_t \rho + \partial^l (\rho v^l) = 0 \tag{2}$$

where:

$$\partial_t = \frac{\partial}{\partial t}, \partial_l = \frac{\partial}{\partial x^l} \tag{3a}$$

$$v^i = 2\lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial^i s, u^i = \lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial^i \ln \rho \tag{3b}$$

$$\rho = \psi \bar{\psi}, \psi = \sqrt{\rho} e^{is} \tag{3c}$$

$$Q = 2\lambda^2 (dt)^{\left[\frac{4}{f(\alpha)}\right]-2} \frac{\partial_l \partial^l \sqrt{\rho}}{\sqrt{\rho}} = \frac{u_i u^i}{2} + \lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial^l u_l \tag{3d}$$

In the above relations the given measures have the following physical meanings:

- t is nonmultifractal time, an affine parameter of movement curves of the entities found in the complex system;
- x^l is the multifractal spatial coordinate;
- v^i is the velocity field at a differentiable scale resolution;
- u^i is the velocity field at a nondifferentiable scale resolution;
- dt is the scale resolution;
- λ is a constant coefficient associated to the multifractal-nonmultifractal scale transition;
- ρ is the state density;
- ψ is the state function with the amplitude $\sqrt{\rho}$ and phase s;
- Q is the scalar specific multifractal potential which quantifies the multifractalization degree of the movement curves in the complex system;
- $f(\alpha)$ is the singularity spectrum of order $\alpha = \alpha(D_F)$ where D_F is the fractal dimension of movement curves of the complex system entities.

This spectrum allows the identification of universality classes in the complex system dynamics [16], even when attractors have different aspects, and it also allows the identification of areas in which the dynamics can be characterized by a specific fractal dimension.

Because of its nonlinearity, the Eqs. (1) and (2) admit analytical solutions only in special, particular cases. Such a case is dictated by one-dimensional dynamics of the complex system entities through the following:

$$\partial_t v + v\partial_x v = 2\lambda^2 (dt)^{\left[\frac{4}{f(\alpha)}\right]-2} \frac{\partial_{xx}\sqrt{\rho}}{\sqrt{\rho}} \tag{4a}$$

$$\partial_t \rho + \partial_x(\rho v) = 0 \tag{4b}$$

with the initial and boundary constraints:

$$v(x, t = 0) = v_0, \rho(x, t = 0) = \rho_0 e^{-\left(\frac{x}{a}\right)^2} \tag{5a}$$

$$v(x = ct, t) = v_0, \rho(x = -\infty, t) = \rho(x = +\infty, t) = 0 \tag{5b}$$

The following solution is found:

$$v = \frac{v_0 a^2 + \left[\frac{\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}}{a}\right]^2 xt}{a^2 + \left[\frac{\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}}{a} t\right]^2} \tag{6}$$

$$\rho = \frac{\pi^{-\frac{1}{2}}}{\left\{a^2 + \left[\frac{\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}}{a} t\right]^2\right\}^{\frac{1}{2}}} \cdot e^{\left\{-\frac{(x-v_0)^2}{a^2+\left[\frac{\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}}{a} t\right]^2}\right\}} \tag{7}$$

This solution, through the nondimensional variables:

$$\frac{v}{v_0} = \bar{v}, \rho\sqrt{\pi}a = \bar{\rho}, \frac{x}{v_0\tau} = \xi, \frac{t}{\tau} = \eta \tag{8}$$

and through the nondimensional parameters:

$$\theta = \frac{\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}\tau}{a^2}, \mu = \frac{v_0\tau}{a} \tag{9}$$

can be rewritten as:

$$\bar{v} = \frac{1 + \theta^2\xi\eta}{1 + \theta^2\eta^2} \tag{10}$$

$$\bar{\rho} = \frac{1}{\sqrt{1 + \theta^2\eta^2}} \cdot e^{\left[-\mu^2\frac{(\xi-\eta)^2}{1+\theta^2\eta^2}\right]} \tag{11}$$

Through Eqs. (3), the solutions in Eqs. (6) and (7) allows us to construct the following set of variables:

- The velocity field at a nondifferentiable scale:

$$u = 2\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \cdot \frac{(x - v_0 t)}{a^2 + \left[\frac{\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}}{a} t\right]^2} \tag{12}$$

- The specific multifractal force field:

$$f = -\partial_x Q = 2\lambda(dt)^{\left[\frac{4}{f(\alpha)}\right]-2} \cdot \frac{(x - v_0 t)}{\left\{a^2 + \left[\frac{\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}}{a} t\right]^2\right\}^2} \tag{13}$$

This set of variables employs the notations:

$$\frac{u}{2v_0} = \bar{u}, \frac{f\tau}{2v_0} = \bar{f} \tag{14}$$

Considering the equations (8) and (9) they become:

$$\bar{u} = \theta \frac{\xi - \eta}{1 + \theta^2 \eta^2} \tag{15}$$

respectively:

$$\bar{f} = \theta^2 \frac{\xi - \eta}{(1 + \theta^2 \eta^2)^2} \tag{16}$$

Then, let us assume the functionality, in nondimensional coordinates, of a relation of the form:

$$\bar{J} = \bar{\sigma} \bar{f} \tag{17}$$

where $\bar{J}$ is a mass current density, $\bar{f}$ is the nondimensional specific multifractal force field, and $\bar{\sigma}$ is a mass conductivity, which then allows us to define the following conductivity types:

- Conductivity at differentiable scale resolutions:

$$\overline{\sigma_D} = \frac{\bar{\rho}\bar{v}}{\bar{f}} = \sqrt{1 + \theta^2 \eta^2} \frac{1 + \theta^2 \xi \eta}{\theta^2 (\xi - \eta)} e^{\left[-\mu^2 \frac{(\xi - \eta)^2}{1 + \theta^2 \eta^2}\right]} \tag{18}$$

- Conductivity at nondifferentiable scale resolutions:

$$\overline{\sigma_F} = \frac{\bar{\rho}\bar{u}}{\bar{f}} = \sqrt{1 + \theta^2 \eta^2} \left(\frac{\mu}{\theta}\right)^2 e^{\left[-\mu^2 \frac{(\xi - \eta)^2}{1 + \theta^2 \eta^2}\right]} \tag{19}$$

- Conductivity at global scale resolutions:

$$\bar{\sigma} = \frac{\bar{\rho}(\bar{v} + i\bar{u})}{f} = \overline{\sigma_D} + i\overline{\sigma_F}$$
$$= \sqrt{1+\theta^2\eta^2}\left[\frac{1+\theta^2\xi\eta}{\theta^2(\xi-\eta)} + i\left(\frac{\mu}{\theta}\right)^2\right]e^{\left[-\mu^2\frac{(\xi-\eta)^2}{1+\theta^2\eta^2}\right]} \tag{20}$$

3. Results and discussion

In this context, since the θ parameter is a measure of the multifractality degree, then $\varepsilon = \frac{1}{\theta}$ will function as a measure of an ordering degree. Then the conductivity species in Eqs. (18-20) change as:

- Conductivity at differentiable scale resolutions:

$$\overline{\sigma_D} = \sqrt{\varepsilon^2+\eta^2}\,\frac{\varepsilon^2+\xi\eta}{\varepsilon(\xi-\eta)}\,e^{\left[-(\mu\varepsilon)^2\frac{(\xi-\eta)^2}{\varepsilon^2+\eta^2}\right]} \tag{21}$$

- Conductivity at nondifferentiable scale resolutions:

$$\overline{\sigma_F} = \sqrt{\varepsilon^2+\eta^2}\,\varepsilon\mu^2 e^{\left[-(\mu\varepsilon)^2\frac{(\xi-\eta)^2}{\varepsilon^2+\eta^2}\right]} \tag{22}$$

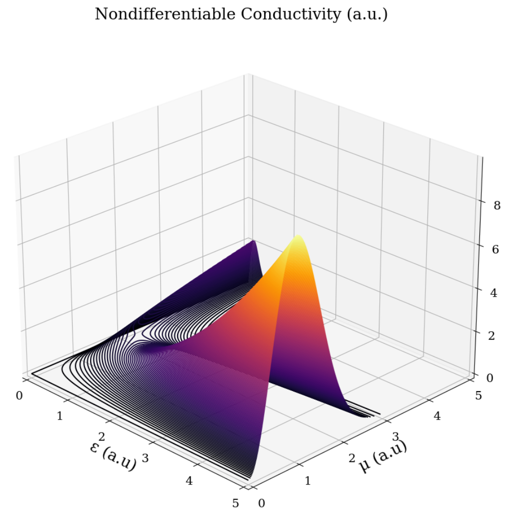
- Conductivity at global scale resolutions:

$$\bar{\sigma} = \sqrt{\varepsilon^2+\eta^2}\left[\frac{\varepsilon^2+\xi\eta}{\varepsilon(\xi-\eta)} + i\varepsilon\mu^2\right]e^{\left[-(\mu\varepsilon)^2\frac{(\xi-\eta)^2}{\varepsilon^2+\eta^2}\right]} \tag{23}$$

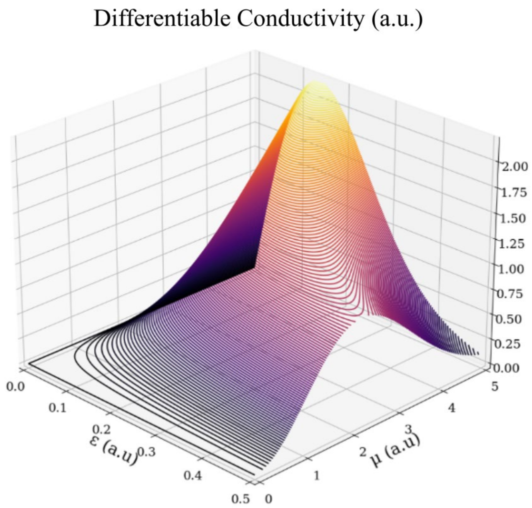
We present in Figs. 1a-c the theoretical dependencies of $\overline{\sigma_F}(\varepsilon)$, $\overline{\sigma_D}(\varepsilon)$ and $\bar{\sigma}(\varepsilon)$ for $\xi, \eta = const.$, and the restriction $\xi \neq \eta$. By presented selecting clear resolution scales for particular types of conductivity, it is possible to address both various interaction scales and fractalization/multifractalization degrees.

Conduction in complex systems is performed through specific mechanisms dependent on scale resolution. As a consequence, we make the distinction between differentiable conduction $\overline{\sigma_D}$, nondifferentiable conduction $\overline{\sigma_F}$ and global conduction $\bar{\sigma}$. Conduction mechanisms at the two types of scale resolutions are simultaneous and reciprocally conditional. Thus, the values of $\overline{\sigma_D}$ and $\overline{\sigma_F}$ increase along with the increase of the ordering degree (synchronous type conductions) and with the increase of the multifractalization degree $\overline{\sigma_D}$ values increase and $\overline{\sigma_F}$ values decrease (asynchronous type conductions). We also notice that higher degrees of fractalization/multifractalization are seen as a higher mismatch in long scale dynamics of the complex system. In the framework of the model, it reads as losses in the inflection point of the trajectory.

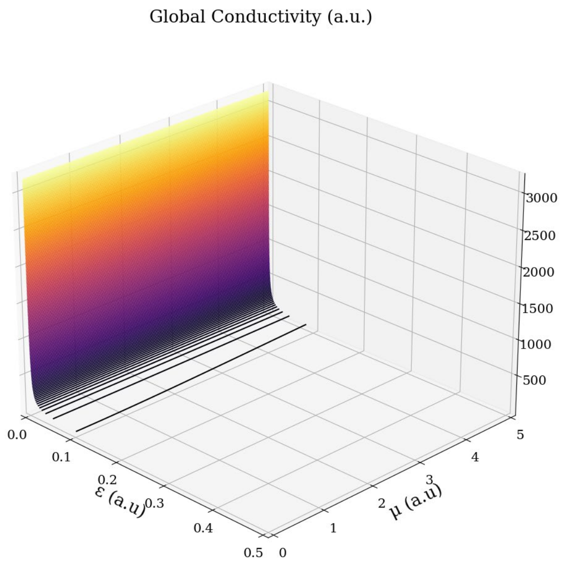
The conductivity of the complex system in the fractal/multifractal interpretations is seen as a measure of the available entity of fractal/multifractal fluid to be transferred in different points of the material [17, 18]. The flow of the current is well characterized by the fractal/multifractal hydrodynamic model, thus in each inflection point of the entity trajectories losses can appear and thus lead to a lower conductivity. It is also seen that there is an optimum where we can obtain a relatively higher conductivity, this point is an unstable one as the system is overcome by losses and the conductivity decreases again. with the decrease of the fractalization/multifractalization degree we observe an exponential-type increase in conductivity.



a) $$\overline{\sigma_F} = \sqrt{\varepsilon^2+\eta^2}\,\varepsilon\mu^2 e^{\left[-(\mu\varepsilon)^2\frac{(\xi-\eta)^2}{\varepsilon^2+\eta^2}\right]}$$



b) $$\overline{\sigma_D} = \sqrt{\varepsilon^2+\eta^2}\,\frac{\varepsilon^2+\xi\eta}{\varepsilon(\xi-\eta)}\,e^{\left[-(\mu\varepsilon)^2\frac{(\xi-\eta)^2}{\varepsilon^2+\eta^2}\right]}$$



c) $$\bar{\sigma} = \sqrt{\varepsilon^2 + \eta^2}\left[\frac{\varepsilon^2+\xi\eta}{\varepsilon(\xi-\eta)} + i\varepsilon\mu^2\right] e^{\left[-(\mu\varepsilon)^2\frac{(\xi-\eta)^2}{\varepsilon^2+\eta^2}\right]}$$

Fig. 1. 3D representation of the three types of conductivities derived from the multifractal model. a) nondifferentiable conductivity; b) differentiable conductivity; c) global conductivity.

In figure 1 all three types of derived conductivities from the multifractal model are presented. These are shown in the order in which they appear, the so-called nondifferentiable conductivity, differentiable conductivity and global conductivity, respectively.

As a justification of the present theory, we can cite reference papers in the field of plasma plume characterization and laser ablation studies [4, 7, 8]. The same results were obtained in the medical field, when investigating by fractal analysis the images obtained with CT (computed tomography) and MRI (magnetic resonance imaging), on the human brain and on the lungs [19-21]. The advantage of interpreting the pictures is to establish a pixel topology and, depending on the calculation of the fractal dimension and the lacunarity, to determine the diseases that affect these vital organs, as well as their temporal evolution.

4. Conclusions

The base conclusions of the present paper refer to the widely portrayed model and will be depicted underneath.

Thus, in the framework of Scale Relativity Theory, the Mandelung scenario of complex system dynamics description is given. This scenario implies the fractal/multifractal hydrodynamic equation, i.e., the momentum conservation laws and the density conservation law. In the 1-dimensional case, the solution of fractal/multifractal hydrodynamic equations, with initial and boundary specific conditions, are given in such an integrator context. Admitting a local Ohm-type conduction law, three conductivity types are highlighted, such as differentiable conduction, nondifferentiable conduction, and global conduction. These are reciprocally-conditioning such that synchronous and asynchronous dynamics can be explained.

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