CO-ROTATIONAL MAXWELL FLUID ANALYSIS IN HELICAL SCREW RHEOMETER USING ADOMIAN DECOMPOSITION METHOD

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This paper considers a theoretical study on steady incompressible flow of co-rotational Maxwell fluid in helical screw rheometer (HSR). The rheological constitutive equation for co-rotational Maxwell fluid model gives the second order nonlinear coupled differential equations which could not be solved explicitly. An iterative procedure, Adomian decomposition method (ADM) is used to obtain the analytical solution. Expressions for velocity components in θ *and* $z-$*direction are obtained. The volume flow rates are calculated for the azimuthal and axial components of velocity field by introducing the effect of flights. The results have been discussed with the help of graphs as well. We observe that the velocity profiles are strongly depend on non-dimensional parameter* $\tilde{\alpha}$*, with the increase in* $\tilde{\alpha}$*, progressive increase seen in the flow profiles. We also noted that the parabolicity of flow profiles increase with increase in the magnitude of pressure gradients. Thus the profound conclusion is that extrusion process depends on the involved non-dimensional parameters.*

Keywords: Helical Screw Rheometer; Adomian decomposition method; Second order nonlinear coupled differential equations.
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1. Introduction

The Helical Screw Rheometer (HSR) is used for rheological measurements of fluid food suspensions. It contains a helical screw in a tight fitting cylinder, with the inlet and outlet ports closed the inner screw. Rotation of screw creates a pressure gradient along the axis of the screw. The geometry of an HSR matches to a single screw extruder [1]. Extrusion process is widely used in food processing i.e., cookie dough, sevai, pastas, breakfast cereals, french fries, baby food, ready to eat snacks and dry pet food. Extrusion process also include fluids like multi-grade oils, liquid detergents, paints, polymer solutions and polymer melts, the injection molding process for polymeric materials, the production of pharmaceutical products and processing of plastics [2, 3, 4].

During processing physical and chemical changes can occur so it is desirable to monitor the process to achieve excellent output and quality control

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[5]. Bird et al.,[6] presented an asymptotic solution and arbitrary values of the flow behavior index for the Power-Law fluid in a very thin annulus.
Booy [7] studied the influence of channel curvature on flow, pressure distribution, and power requirements of screw pumps and melt extruders considering Newtonian fluid. Tamura et al.,[1] also investigated the flow of Newtonian fluid in Helical Screw Rheometer.

The classical Navier-Stokes equations have been proved inadequate to describe complete characteristics of non-Newtonian fluids. To study these fluids different new theories have been developed [8], and different models are proposed [6, 8]. In the present work an attempt has been made to study co-rotational Maxwell fluid in Helical Screw Rheometer (HSR). We have choosen the cylindrical coordinate system (r,θ,z) which seems to be a more natural choice due to the geometry of HSR. The expressions for the v and $w-$component of velocity profiles are obtained from the solution of developed second order nonlinear coupled differential equations by using Adomian decomposition method with new modification suggested by Wazwaz [9]. Volume flow rates are calculated by introducing the effect of flights. The behavior of the velocity profiles are presented through graphs and discussed.

The paper is organized as follows. Section 2 contains the governing equations of the fluid model. In Section 3 the problem under consideration is formulated and, the governing equation of the problem is solved using Adomian decomposition method. In Section 4 discussion about the behavior of the velocity profiles is given. Section 5 contains conclusion.

2 Basic Equations

The basic equations, governing the motion of an isothermal, homogeneous and incompressible co-rotational Maxwell fluid are:

$$div\mathbf{V} = 0, \tag{1}$$

$$\rho\frac{D\mathbf{V}}{Dt} = \rho\mathbf{f} - \nabla P + \nabla.\mathbf{S}, \tag{2}$$

where ρ is the constant fluid density, $\mathbf{V}$ is the velocity vector, $\mathbf{f}$ is the body force per unit mass, P denotes the dynamic pressure and the operator $\frac{D}{Dt}$ denotes the material time derivative defined as, $\frac{D(*)}{Dt} = \frac{\partial}{\partial t}(*) + (V\cdot\nabla)(*),$

$\mathbf{S}$ is the extra stress tensor which for co-rotational Maxwell fluid model is defined as

$$\mathbf{S} + \lambda_1\overset{\nabla}{\mathbf{S}} + \frac{1}{2}\lambda_1(\mathbf{A}_1\mathbf{S} + \mathbf{S}\mathbf{A}_1) = \eta_0\mathbf{A}_1, \tag{3}$$

here η_0 and λ_1 are zero shear viscosity and relaxation time, respectively. The upper contravariant convected derivative designed by ∇ over $\mathbf{S}$ is defined as

$$\overset{\nabla}{\mathbf{S}} = \frac{D\mathbf{S}}{Dt} - \left\{(grad\ \mathbf{V})^T \mathbf{S} + \mathbf{S}(grad\ \mathbf{V})\right\},$$

and

$$\mathbf{A}_1 = (grad\ \mathbf{V}) + (grad\ \mathbf{V})^T,$$

is the first Rivlin-Ericksen tensor.

3 Problem Formulation

Steady, laminar, isothermal, flow of an incompressible co-rotational Maxwell fluid is considered in Helical screw rheometer (HSR). The screw channel is assumed to be bounded by the barrel and screw root surfaces and by the two sides of a helical flight as shown in Fig.1. The geometry is approximated as a shallow infinite channel, by assuming the width B of the channel large compared with the depth h i.e., $\frac{h}{B} << 1$. Further more side effects can be ignored. Here we choose the cylindrical coordinate system (r, θ, z) which is more suitable choice for the flow analysis in HSR. A congruent velocity distribution is assumed at parallel cross sections through the channel. The viscosity of the fluid is assumed to be constant. The outer barrel of radius r_2 is assumed to be stationary and the screw root of radius r_1 rotates with angular velocity Ω [7, 10].

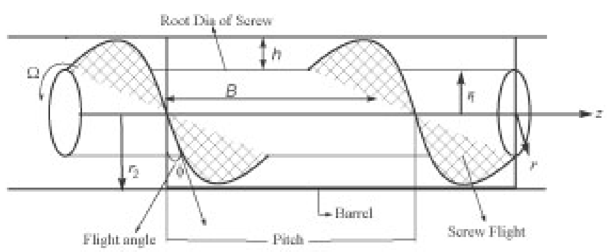

Fig.1 Geometry of helical screw rheometer

The boundary conditions are

$$\begin{aligned} v &= \Omega r_1, \quad w = 0, \quad at \quad r = r_1, \\ v &= 0, \quad w = 0, \quad at \quad r = r_2. \end{aligned} \tag{4}$$

The flow is assumed fully developed in the θ and the $z-$directions so that,

$$\mathbf{V} = [0, v(r), w(r)] \qquad \mathbf{S} = \mathbf{S}(r), \tag{5}$$

where u, v and w are radial, azimuthal and axial velocity components, respectively. On substituting these assumptions in equation (3), we obtain non-zero components of extra stress tensor, $\mathbf{S}$ as,

$$S_{rr} = -\eta_0\lambda_1\left\{\left(\frac{dv}{dr}-\frac{v}{r}\right)^2+\left(\frac{dw}{dr}\right)^2\right\}M, \tag{6}$$

$$S_{r\theta} = S_{\theta r} = \eta_0\left(\frac{dv}{dr}-\frac{v}{r}\right)M, \qquad S_{rz} = S_{zr} = \eta_0\frac{dw}{dr}M, \tag{7}$$

$$S_{\theta\theta} = \eta_0\lambda_1\left(\frac{dv}{dr}-\frac{v}{r}\right)^2 M, \qquad S_{zz} = \eta_0\lambda_1\left(\frac{dw}{dr}\right)^2 M, \tag{8}$$

$$S_{\theta z} = S_{z\theta} = \eta_0\lambda_1\frac{dw}{dr}\left(\frac{dv}{dr}-\frac{v}{r}\right)M, \tag{9}$$

where

$$M = \frac{1}{1+\lambda_1^2\left\{\left(\frac{dv}{dr}-\frac{v}{r}\right)^2+\left(\frac{dw}{dr}\right)^2\right\}}. \tag{10}$$

For highly viscous fluids the effect of acceleration of fluid and body forces can be ignored [12]. Equation (2) for creeping flow of an incompressible fluid can be written as,

$$0 = -gradP + div\ \mathbf{S}. \tag{11}$$

A number of simplifications became possible by assuming infinite aspect ratio $\frac{h}{B} << 1$ and the congruent velocity distributions at parallel cross sections. Velocities are tangential to cylindrical surfaces in the limiting case of infinite aspect ratio. The pressure in the channel does not change with radius for very shallow channels, therefore $\frac{\partial P}{\partial r} = 0$ [12].

In view of our assumptions equation (1) is satisfied identically and substitution of the calculated components of extra tress tensor given in equations (6)-(9) in the component form of equation (11) give

$$\frac{1}{r}\frac{d}{dr}\left[r\left\{\frac{\left(\frac{dv}{dr}-\frac{v}{r}\right)^2+\left(\frac{dw}{dr}\right)^2}{1+\lambda_1^2\left\{\left(\frac{dv}{dr}-\frac{v}{r}\right)^2+\left(\frac{dw}{dr}\right)^2\right\}}\right\}\right]-\frac{1}{r}\left[\frac{\left(\frac{dv}{dr}-\frac{v}{r}\right)^2}{1+\lambda_1^2\left\{\left(\frac{dv}{dr}-\frac{v}{r}\right)^2+\left(\frac{dw}{dr}\right)^2\right\}}\right] = 0, \tag{12}$$

$$\eta_0\frac{1}{r^2}\frac{d}{dr}\left[r^2\left\{\frac{\left(\frac{dv}{dr}-\frac{v}{r}\right)}{1+\lambda_1^2\left\{\left(\frac{dv}{dr}-\frac{v}{r}\right)^2+\left(\frac{dw}{dr}\right)^2\right\}}\right\}\right] = \frac{1}{r}\frac{\partial P}{\partial\theta}, \tag{13}$$

$$\eta_0 \frac{1}{r}\frac{d}{dr}\left[r\left\{ \frac{\dfrac{dw}{dr}}{1+\lambda_1^2\left\{\left(\dfrac{dv}{dr}-\dfrac{v}{r}\right)^2+\left(\dfrac{dw}{dr}\right)^2\right\}} \right\}\right] = \frac{\partial P}{\partial z}. \quad (14)$$

The equations (13) and (14), imply that $P = P(\theta, z)$, since the left hand sides of equations (13) and (14) are functions of r alone and $P \neq P(r)$, this implies $\frac{\partial P}{\partial \theta}$ =*constant* and $\frac{\partial P}{\partial z}$ =*constant*. Our concentration is on torsional and axial flow, so we will consider only equations (13) and (14).

Introducing non-dimensional parameters,

$$r^* = \frac{r}{r_1}, \quad z^* = \frac{z}{r_1}, \quad v^* = \frac{v}{\Omega r_1}, \quad w^* = \frac{w}{\Omega r_1}, \quad P^* = \frac{P}{\eta_0 \Omega},$$

in equations (13) and (14), after dropping "*" and the integrating with respect to ``r'', we get

$$\frac{d}{dr}\left(\frac{v}{r}\right) = \left(\frac{K_1}{r}+\frac{C_1}{r^3}\right)\left[1+\tilde{\alpha}\left\{r^2\left(\frac{d}{dr}\left(\frac{v}{r}\right)\right)^2+\left(\frac{dw}{dr}\right)^2\right\}\right], \quad (15)$$

$$\frac{dw}{dr} = \left(K_2 r+\frac{C_2}{r}\right)\left[1+\tilde{\alpha}\left\{r^2\left(\frac{d}{dr}\left(\frac{v}{r}\right)\right)^2+\left(\frac{dw}{dr}\right)^2\right\}\right]. \quad (16)$$

where C_1 and C_2 are constant of integration and $\tilde{\alpha} = (Wi)^2$, where $Wi = \lambda_1 \Omega$ is the Weissenberg number raised in non-dimensionlization and assume that $K_1 = \frac{1}{2}\frac{\partial P}{\partial \theta}$ and $K_2 = \frac{1}{2}\frac{\partial P}{\partial z}$.

The associated non-dimensional boundary conditions are

$$\begin{aligned} v &= 1, \quad w = 0, \quad at \quad r = 1, \\ v &= 0, \quad w = 0, \quad at \quad r = \delta, \end{aligned} \quad (17)$$

where $\delta = \frac{r_2}{r_1} > 1$.

The resultant equations (15) and (16) are coupled first order nonlinear ordinary differential equations, the exact solution seems to be difficult. In the next section we use Adomian decomposition method (ADM) to obtain the approximate solution with the help of symbolic computation software Wolfram Mathematica 7.

According to the ADM the nonlinear ordinary differential equation can be written in operator form as $Lu + Ru + Nu = g$,

where L is the higher order linear differential operator which is assumed to be invertible and R is a linear linear differential operator of order less than L, N is nonlinear operator and g is a source term. In our problem $L=\frac{d}{dr}$, $L^{-1}=\int ()dr$, $R=0$,

$$Nu=\tilde{\alpha}\left(K_1 r+\frac{C_1}{r}\right)\left(r^2\left(\frac{d}{dr}\left(\frac{v}{r}\right)\right)^2+\left(\frac{dw}{dr}\right)^2\right),$$

$$Nu=\tilde{\alpha}\left(K_2 r+\frac{C_2}{r}\right)\left(r^2\left(\frac{d}{dr}\left(\frac{v}{r}\right)\right)^2+\left(\frac{dw}{dr}\right)^2\right)$$ and

$$g=\left(K_1 r+\frac{C_1}{r}\right),\ g=\left(K_2 r+\frac{C_2}{r}\right).$$

ADM suggest that the unkown functions v and w be expressed as $v=\sum_{n=0}^{\infty}v_n$, $w=\sum_{n=0}^{\infty}w_n$ and the nonlinear term Nu can be explore in the form of ADM polynomials A_n 's as $N\left(\sum_{n=0}^{\infty}u_n\right)=\sum_{n=0}^{\infty}A_n$.

Adomian decomposition method simplifies the developed equations (15) and (16) in components form along with the boundary conditions in the following way.

3.1 Zeroth component Solution

Zeroth component problem

$$v_0=C_3 r+rL^{-1}\left(\frac{K_1}{r}+\frac{C_{1,0}}{r^3}\right), \tag{18}$$

$$w_0=C_4+L^{-1}\left(K_2 r+\frac{C_{2,0}}{r}\right), \tag{19}$$

along with the boundary conditions

$$\begin{aligned} v_0 &= 1, \quad w_0=0, \quad at \quad r=1,\\ v_0 &= 0, \quad w_0=0, \quad at \quad r=\delta, \end{aligned} \tag{20}$$

give

$$v_0=K_1 r\ln(r)-\frac{N_1}{r}+N_2 r, \tag{21}$$

$$w_0=M_1(r^2-1)+M_2\ln(r), \tag{22}$$

Equations (21) and (22) are the solutions for linearly viscous fluid [12].

3.2 First component Solution

First component problem

$$v_1 = rL^{-1}\left(\frac{C_{1,1}}{r^3}\right) + \tilde{\alpha} r L^{-1}(A_0), \tag{23}$$

$$w_1 = L^{-1}\left(\frac{C_{2,1}}{r}\right) + \tilde{\alpha} L^{-1}(B_0), \tag{24}$$

together with boundary conditions

$$\begin{aligned} v_1 &= 0, \quad w_1 = 0, \quad at \quad r = 1, \\ v_1 &= 0, \quad w_1 = 0, \quad at \quad r = \delta, \end{aligned} \tag{25}$$

where A_0 and B_0 are Adomian polynomials in v_0 and w_0, given as

$$A_0 = \left(\frac{K_1}{r} + \frac{C_{1,0}}{r^3}\right)\left\{ r^2\left(\frac{d}{dr}\left(\frac{v_0}{r}\right)\right)^2 + \left(\frac{dw_0}{dr}\right)^2 \right\}, \tag{26}$$

$$B_0 = \left(K_2 r + \frac{C_{2,0}}{r}\right)\left\{ r^2\left(\frac{d}{dr}\left(\frac{v_0}{r}\right)\right)^2 + \left(\frac{dw_0}{dr}\right)^2 \right\}. \tag{27}$$

Equations (23) and (24) result in the solution

$$v_1 = \tilde{\alpha}\left(-\frac{N_3}{6r^5} - \frac{N_4}{4r^3} + (N_8 - \frac{N_5}{2})\frac{1}{r} + N_6 r \ln(r) + \frac{N_7}{2} r^3 - N_9 r \right), \tag{28}$$

$$w_1 = \tilde{\alpha}\left(-\frac{M_3}{4r^4} - \frac{M_4}{2r^2} + (M_8 + M_5)\ln(r) + \frac{M_6}{2} r^2 + \frac{M_7}{4} r^4 + M_9 \right), \tag{29}$$

3.3 Second Order Solution

The second component problem

$$v_2 = rL^{-1}\left(\frac{C_{1,2}}{r^3}\right) + \tilde{\alpha} r L^{-1}(A_1), \tag{30}$$

$$w_2 = L^{-1}\left(\frac{C_{2,2}}{r}\right) + \tilde{\alpha} L^{-1}(B_1), \tag{31}$$

with the boundary conditions

$$\begin{aligned} v_2 &= 0, \quad w_2 = 0, \quad at \quad r = 1, \\ v_2 &= 0, \quad w_2 = 0, \quad at \quad r = \delta. \end{aligned} \tag{32}$$

containing Adomian polynomials A_1 and B_1 in v_0, v_1, w_0 and w_1, given as

$$A_1 = \left(\frac{K_1}{r} + \frac{C_{1,0}}{r^3}\right)\left\{ 2r^2 \frac{d}{dr}\left(\frac{v_0}{r}\right)\frac{d}{dr}\left(\frac{v_1}{r}\right) + 2\frac{dw_0}{dr}\frac{dw_1}{dr} \right\}$$

$$+\frac{C_{1,1}}{r^3}\left\{r^2\left(\frac{d}{dr}\left(\frac{v_0}{r}\right)\right)^2+\left(\frac{dw_0}{dr}\right)^2\right\}, \tag{33}$$

$$B_1=\left(K_2r+\frac{C_{2,0}}{r}\right)\left\{2r^2\frac{d}{dr}\left(\frac{v_0}{r}\right)\frac{d}{dr}\left(\frac{v_1}{r}\right)+2\frac{dw_0}{dr}\frac{dw_1}{dr}\right\}$$

$$+\frac{C_{2,1}}{r}\left\{r^2\left(\frac{d}{dr}\left(\frac{v_0}{r}\right)\right)^2+\left(\frac{dw_0}{dr}\right)^2\right\}, \tag{34}$$

results in the solution

$$v_2=\tilde{\alpha}^2\left(-\frac{N_{10}}{10r^9}-\frac{N_{11}}{8r^7}-\frac{N_{12}}{6r^5}-\frac{N_{13}}{4r^3}-\frac{N_{14}}{2r}+N_{15}r\ln r+\frac{N_{16}r^3}{2}\right.$$

$$\left.+\frac{N_{17}r^5}{4}+\frac{N_{18}}{2}\left(r-\frac{1}{r}\right)+N_{19}r\right), \tag{35}$$

$$w_2=\tilde{\alpha}^2\left(-\frac{M_{10}}{8r^8}-\frac{M_{11}}{6r^6}-\frac{M_{12}}{4r^4}-\frac{M_{13}}{2r^2}+M_{14}\ln r+\frac{M_{15}r^2}{2}+\frac{M_{16}r^4}{4}\right.$$

$$\left.+\frac{M_{17}r^6}{6}-M_{18}\ln r+M_{19}\right), \tag{36}$$

where $N_i,\ M_j,\ i=1,\cdots,19,\ j=1,\cdots,19$ are constant coefficients.

Since the ADM solution can be accumulated as

$$v=\sum_{n=0}^{\infty}v_n,\qquad and\qquad w=\sum_{n=0}^{\infty}w_n. \tag{37}$$

Thus the ADM solution for the azimuthal and axial velocity components up to second iteration, after substituting in (37) become

$$v=K_1r\ln(r)-\frac{N_1}{r}+N_2r+\tilde{\alpha}\left(-\frac{N_3}{6r^5}-\frac{N_4}{4r^3}+(N_8-\frac{N_5}{2})\frac{1}{r}+N_6r\ln(r)\right.$$

$$\left.+\frac{N_7}{2}r^3-N_9r\right)+\tilde{\alpha}^2\left(-\frac{N_{10}}{10r^9}-\frac{N_{11}}{8r^7}-\frac{N_{12}}{6r^5}-\frac{N_{13}}{4r^3}-\frac{N_{14}}{2r}\right.$$

$$\left.+N_{15}r\ln r+\frac{N_{16}r^3}{2}+\frac{N_{17}r^5}{4}+\frac{N_{18}}{2}\left(r-\frac{1}{r}\right)+N_{19}r\right), \tag{38}$$

$$w=M_1(r^2-1)+M_2\ln(r)+\tilde{\alpha}\left(-\frac{M_3}{4r^4}-\frac{M_4}{2r^2}+(M_8+M_5)\ln(r)+\frac{M_6}{2}r^2\right.$$

$$\left.+\frac{M_7}{4}r^4+M_9\right)+\tilde{\alpha}^2\left(-\frac{M_{10}}{8r^8}-\frac{M_{11}}{6r^6}-\frac{M_{12}}{4r^4}-\frac{M_{13}}{2r^2}+M_{14}\ln r\right.$$

$$+\frac{M_{15}r^2}{2}+\frac{M_{16}r^4}{4}+\frac{M_{17}r^6}{6}-M_{18}\ln r+M_{19}\Bigg). \quad (39)$$

3.4 Volume flow rate in θ-direction

The volume flow rate in $\theta-$direction as given in [12] is

$$Q_\theta = 2\pi(r_2\tan\phi)\int_{r=r_1}^{r=r_2} v dr. \quad (40)$$

Volume flow rate (40) in non-dimensional form is

$$Q_\theta^* = 2\pi\delta\tan\phi\int_1^\delta v dr, \quad (41)$$

where $Q_\theta^* = \frac{Q_\theta}{\Omega r_1^3}$. Dropping "*" we get

$$Q_\theta = 2\pi\delta\tan\phi\Bigg[\left(\frac{K_1}{4}+\frac{\tilde{\alpha}N_6}{4}+\frac{\tilde{\alpha}^2N_{15}}{4}\right)\left(1-\delta^2+2\delta^2\ln\delta\right)-\frac{1}{2}\left(2N_1+\tilde{\alpha}N_5-2\tilde{\alpha}N_8\right.$$

$$\left.+\tilde{\alpha}^2N_{14}\right)\ln\delta+\frac{1}{2}\left(N_2-\tilde{\alpha}N_9+\tilde{\alpha}^2N_{19}\right)\left(\delta^2-1\right)+\left(\frac{\tilde{\alpha}N_3}{24}+\frac{\tilde{\alpha}^2N_{12}}{24}\right)\left(\frac{1}{\delta^4}-1\right)$$

$$+\left(\frac{\tilde{\alpha}N_4}{8}+\frac{\tilde{\alpha}^2N_{13}}{8}\right)\left(\frac{1}{\delta^2}-1\right)+\left(\frac{\tilde{\alpha}N_7}{8}+\frac{\tilde{\alpha}^2N_{16}}{8}\right)\left(\delta^4-1\right)$$

$$+\tilde{\alpha}^2\left\{\frac{N_{10}}{80}\left(\frac{1}{\delta^8}-1\right)+\frac{N_{11}}{48}\left(\frac{1}{\delta^6}-1\right)\right.$$

$$\left.+\frac{N_{17}}{24}\left(\delta^6-1\right)-\frac{N_{18}}{4}\left(1-\delta^2+2\ln\delta\right)\right\}\Bigg]. \quad (42)$$

3.5 Volume flow rate in z-direction

The volume flow rate in z-direction can be calculated as

$$Q_z = 2\pi\int_{r=r_1}^{r=r_2} wr dr. \quad (43)$$

The dimensionless form of equation(43) can be written as

$$Q_z^* = 2\pi\int_1^\delta wr dr, \quad (44)$$

where $Q_z^* = \frac{Q_z}{\Omega r_1^3}$. Now, dropping "*" we

$$Q_z = 2\pi\left\{\frac{M_1}{4}\left(1-2\delta^2+\delta^4\right)+\frac{1}{4}\left(M_2+\tilde{\alpha}\left(M_5+M_8\right)+\tilde{\alpha}^2(M_{14}-M_{18})\right)\left(1-\delta^2+2\delta^2\ln\delta\right)\right.$$

$$+\frac{1}{8}\left(\tilde{\alpha}M_3+\tilde{\alpha}^2M_{12}\right)\left(\frac{1}{\delta^2}-1\right)-\frac{1}{2}\ln\delta\left(\tilde{\alpha}M_4+\tilde{\alpha}^2M_{13}\right)+\frac{1}{8}\left(\tilde{\alpha}M_6+\tilde{\alpha}^2M_{15}\right)\left(\delta^4-1\right)$$

$$+\frac{1}{24}\left(\tilde{\alpha}M_7+\tilde{\alpha}^2M_{16}\right)\left(\delta^6-1\right)-\frac{1}{2}\left(\tilde{\alpha}M_9+\tilde{\alpha}^2M_{19}\right)\left(1-\delta^2\right)$$
$$\left.+\tilde{\alpha}^2\left(\frac{M_{10}}{48}\left(\frac{1}{\delta^6}-1\right)+\frac{M_{11}}{24}\left(\frac{1}{\delta^4}-1\right)+\frac{M_{17}}{48}\left(\delta^8-1\right)\right)\right\}. \qquad (45)$$

4 Results and Discussion

In this work we have considered steady flow of an incompressible co-rotational Maxwell fluid through HSR. We obtained coupled second order nonlinear ODEs. Its exact solutions seems to be difficult, approximate analytical solutions are obtained with the help of ADM as this method gives convergent solution in finite domain. Expressions for azimuthal $v(r)$ and axial velocity w(r) are calculated. The volume flow rates in θ and $z-$directions are also derived. Here we discussed the effect of involved flow parameters on the velocity profiles with the help of graphs. Figure 2(a) is plotted for the velocity v for different values of fluid parameter $\tilde{\alpha}$, steadily increase observed in the velocity from screw toward barrel and the velocity attains maximum values in between the channel which show shear thinning due to increases in the value of $\tilde{\alpha}$. Figure 2(b) is sketched for the velocity profile w for different values of $\tilde{\alpha}$, the velocity profile is seem to be parabolic in nature, and also attains maximum values at some points in the channel. The behavior of velocity w shows that it is responsible to take the fluid toward the exit. Figures 3(a) and 3(b) are shown for the velocity v for different values of pressure gradients $P_{,\theta}$ and $P_{,z}$ respectively, it can be seen that velocity v increases with the increase in pressure gradients. It is noticed that $P_{,z}$ resist the velocity v as graphs show the smaller magnitude of v for $P_{,z}$. Similarly figures 4(a) and 4(b) are plotted for the velocity w for different values of $P_{,\theta}$ and $P_{,z}$. With the increase in the value of $P_{,\theta}$ and $P_{,z}$, increase in the w is observed, however the effect of $P_{,\theta}$ is observed less on w which show $P_{,\theta}$ try to resist the flow in axial direction.

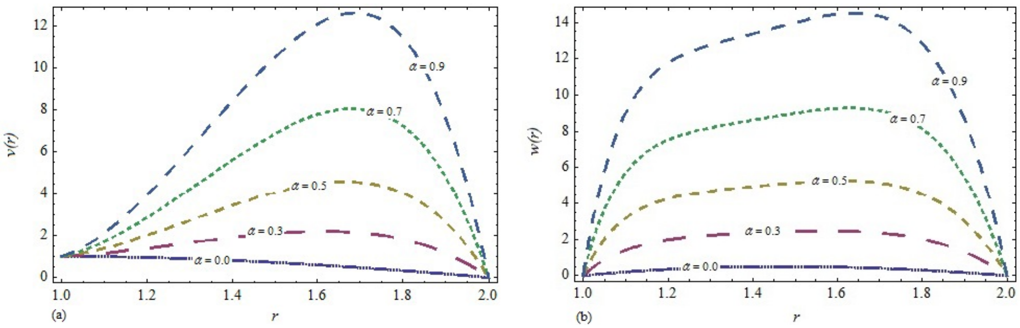

Fig. 2: (a) $v(r)$ for different values of $\tilde{\alpha}$, keeping $P_{,\theta} = -2.0$, $P_{,z} = -2.0$ and $\delta = 2$. (b) $w(r)$ for different values of $\tilde{\alpha}$, keeping $P_{,\theta} = -2.0$, $P_{,z} = -2.0$ and $\delta = 2$.

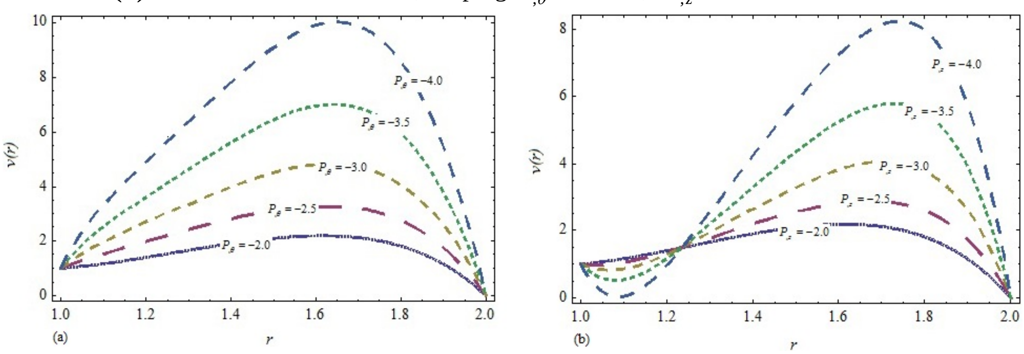

Fig. 3: (a) $v(r)$ for different values of $P_{,\theta}$, keeping $\tilde{\alpha} = 0.3$, $P_{,z} = -2.0$ and $\delta = 2$. (b) $v(r)$ for different values of $P_{,z}$, keeping $\tilde{\alpha} = 0.3$, $P_{,\theta} = -2.0$ and $\delta = 2$.

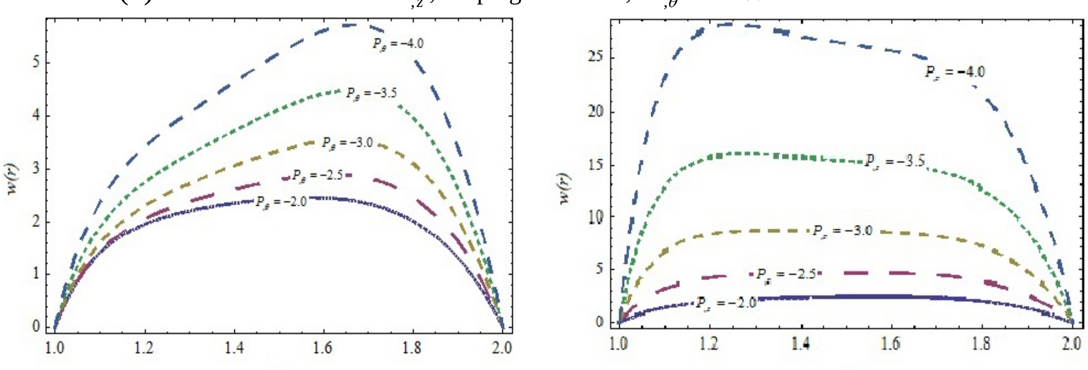

Fig. 4: (a) $w(r)$ for different values of $P_{,\theta}$, keeping $\tilde{\alpha} = 0.3$, $P_{,z} = -2.0$ and $\delta = 2$. (b) $w(r)$ for different values of $P_{,z}$, keeping $\tilde{\alpha} = 0.3$, $P_{,\theta} = -2.0$ and $\delta = 2$.

It is mensioned here the involved coeffeicients are not function of any parameters like $\tilde{\alpha}$, these coefficients are just constants depending on constant ratio

$\delta = \frac{r_2}{r_1} > 1$ and constant pressure gradients i.e., $K_1 = \frac{1}{2}\frac{\partial P}{\partial \theta}$ and $K_2 = \frac{1}{2}\frac{\partial P}{\partial z}$. It is also mensioned here the ADM results in the convergent solution in the finite domain. The problem given in (15) – (17) has finite domain from 0 to 1 so the solutions with respect to ADM are convergent.

5 Conclusion

This paper considers a theoretical study on steady incompressible flow of co-rotational Maxwell fluid in helical screw rheometer (HSR). The rheological constitutive equation for co-rotational Maxwell fluid model gives the second order nonlinear coupled differential equations which could not be solved explicitly. An iterative procedure, Adomian decomposition method (ADM) is used to obtain the analytical solution. It is to be mentioned here that the ADM gives convergent solution in finite domain. Expressions for velocity components in $\theta-$ and $z-$direction are obtained. The volume flow rates are calculated for the azimuthal and axial components of velocity field by introducing the effect of flights. The results have been discussed with the help of graphs as well. We observe that the velocity profiles are strongly depend on non-dimensional parameter and pressure gradients. Thus the profound conclusion is that extrusion process depends on the involved non-dimensional parameters along with the other factors involved in food processing.

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