U.P.B. Sci. Bull., Series D, Vol. 85, Iss. 2, 2023 ISSN 1454-2358

FRACTIONAL ORDER OF TEAGER KAISER ENERGY OPERATOR FOR FAULT DIAGNOSIS OF ROTATING MACHINES

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In recent-day, rotating machinery play a vital role in industries for their direct influence on the finished products. Therefore, minimizing their malfunctions and failures is important to achieve the required product quality and production rates. Indeed, many studies have focused on the diagnosis of rotating machinery, and the monitoring systems have, as a main task, the detection of precise defaults of machines. For this aim, several methods are used, such as spectral analysis, cepstral analysis, envelope analysis, time-frequency analysis and so on. This work presents a modified method based on the Teager Kaiser Energy Operator (TKEO) technique. It makes a use of the notion of fractional calculations in derivatives part of TKEO. The obtained results show the efficiency of the fractional calculus in monitoring rotating machinery domain.

Keywords: defects; fractional calculus; rotating machines; process control.

1. Introduction

Rotating machines occupy an important part in industry, especially in movement transmission systems [1-5]. In these machines, many defects are caused by rolling bearings. Hence, the extraction of their characteristics and the detection of their defects is the main issue in the fault diagnosis domain [6, 7].

For instance, the monitoring of the measured vibration on rotating machines is even more important in maintenance planning. The vibration analysis has been proved the important approach to increase the convenience of machines. It allows controlling the real state of rotating machines to avoid unplanned stops due to breakdowns. Indeed, bearings, which are one of the most important components in rotating machines, are considered as the major cause of damage [8-12]. Such bearing faults are then detected using this powerful tool, which is the vibration analysis [13]. Whereas, many other studies routinely monitor and track

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the condition of this component and use various methods [14-16]. Xiange et al., in [17] propose the detection of bearing fault based on bispectrum of modulated, that allows deleting two types of noise (the static random noise and the discrete aperiodic noise). Huang et al., in [18], analyzed the defects of several rotors using the modulated signal bispectrum and the performance of the conventional bispectrum methods. They demonstrated that the first method performs better than the second one. As for some other studies, Guo et al., in [19], applied bispectral analysis to find segmental features of diesel cylinder piston rings, and then found default information through artificial neural networks, Shaeboub et al., in [20], dealt with the effectiveness of routine diagnostic features in processing current and voltage signals, and Chasalevris et al., in [21], used the bispectral analysis to detect bearing defects.

The most efficient method for diagnosis is vibration analysis, in this area, the Teager Kaiser energy operator TKEO is a popular technique [22, 23], in 1990 Kaiser use this operator for tracking and analysis the signal energy, the Teager Kaiser energy operator TKEO [23] of which the signal, is based on two parameters, amplitude and frequency. This technique has been used for the first time in signal speech analysis [24], and the TKEO operator has been applied successfully to detect and diagnose bearing faults [25-28].

In the time domain, the continuous form TKEO of a signal $x(t)$ is formulated as follows:

$$\psi_c\left(x(t)\right) = [\dot{x}(t)]^2 - x(t)\ddot{x}(t)$$

With : $\dot{x}(t) = \frac{dx}{dt}$ and $\ddot{x}(t) = \frac{d\left(\dot{x}(t)\right)}{dt}$

And the discrete form TKEO is defined as:

$$\psi_d\left(x(n)\right) = [x(n)]^2 - x(n+1)x(n-1).$$

Where $x(n)$ is the amplitude on discrete time "n", the $x(n-1)$ and $x(n+1)$ are the preceding and succeeding samples. TKEO can estimate both the amplitude and frequency of the signal. For a signal $x(t) = A * \cos(\omega t + j)$, the instantaneous energy is $TKEO\left(x(t)\right) = A^2\omega^2$.

The vibration signal produced by a defective bearing produces a different signal than that produced by a healthy bearing. The peaks present in the periodic signal correspond to a defect, at multiple frequencies of the rotational frequency. In this study, we introduced fractional calculus in the Teager-Kaiser energy operator to make us more precise in fault detection. We used the Grünwald-Letnikov method [29] to obtain the fractional linear differential equation of TKEO. The purpose of this study is to investigate a new method FTKEO based on fractional derivatives instead of classical derivatives.

2. Representation of fractional linear differential equation

A fractional linear system is a system described by a fractional differential equation of the form [29]:

$$\begin{aligned} & a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \ldots + a_0 D^{\alpha_0} y(t) \\ & = b_m D^{\beta_m} e(t) + b_{m-1} D^{\beta_{m-1}} e(t) + \ldots + b_0 D^{\beta_0} e(t) \end{aligned} \tag{1}$$

where e(t) and y(t) are, respectively, the input and the output of the fractional linear system, the derived orders $\alpha_i (0 \le i \le n)$ and $\beta_i (0 \le j \le m)$ are real numbers such as $\alpha_n > \alpha_{n-1} > \ldots > \alpha_0$, $\beta_m > \beta_{m-1} > \ldots > \beta_0$ et $\alpha_n > \beta_m$ and the coefficients $a_i (i = 0, 1, ..., n)$ and $b_i (j = 0, 1, ..., m)$ are real numbers. When the derived orders $\alpha_i (0 \le i \le n)$ and $\beta_i (0 \le j \le n)$ are all multiple of the same real number $\alpha\ (0 < \alpha < 1)$, therefore we have $\alpha_i = i.\alpha\ (0 \le i \le n)$ and $\beta_j = j.\alpha\ (0 \le j \le m)$ and $m \le n$; the fractional linear system is known as commensurable fractional linear system. Then, the fractional differential equation of equation (1) becomes:

$$\sum_{i=0}^{n} a_i D^{i\alpha} y(t) = \sum_{j=0}^{m} b_j D^{j\alpha} e(t) \tag{2}$$

3. Resolution method of the fractional linear differential equations

The solution of the fractional differential equation (1) is obtained by Gründwald-Leitnikov technique, this solution is represented by the following form:

$$y(t) = \frac{1}{\sum_{i=1}^{n} \frac{a_i}{h^{\alpha_i}}} \left[\sum_{l=0}^{m} \left(\frac{b_l}{h^{\beta_l}} \right) \sum_{j=0}^{N} w_j^{\beta_i} e(t - jh) - \sum_{i=1}^{n} \left(\frac{a_i}{h^{\alpha_i}} \right) \sum_{j=1}^{N} w_j^{\alpha_i} y(t - jh) \right] \tag{3}$$

With $w_0^m = 1$ and $w_j^m = \left(1 - \frac{m+1}{j}\right) w_{j-1}^m$, h is the sampling period (the calculation step) supposed very small and N = left integer $\left(\frac{t_0}{h}\right)$ where t_0 is the calculation horizon. While being based on the relation of the equation (3), the solution of the fractional linear differential equation using MATLAB is given by the function code in [29]:

3.1. Case 1

In this first case, we have a fractional linear differential equation as following:

$$\frac{d^m y(t)}{dt^m} + y(t) = e(t) \tag{4}$$

Using Matlab code for an input e(t) the level unit, the solution of the fractional linear differential equation is given by the following program:

```
h = 0.001; t=0:h:20; e=ones(size(t));
b=[1]; nb=[0]; a=[1 , 1]; na=[α , 0];
y=fode_sol(a,na,b,nb,e,t);
plot(t,y)
```

The step response of the fractional system represented by the fractional linear differential equation for different values of the parameter m is given in the figure 1.

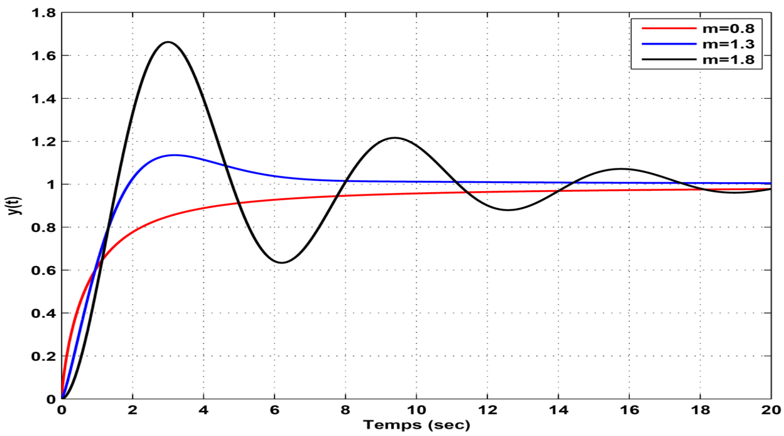

Fig. 1. Step responses of the equation (4) for different values of m parameter

From figure 1, we can easily see that the step response behavior of the general fractional linear differential equation has several forms for different values of the parameters m.

3.2. Case 2

In this second case we have a fractional linear differential equation as follows:

$$\frac{d^{2.53}y(t)}{dt^{2.53}}+3\frac{d^{1.47}y(t)}{dt^{1.47}}+4\frac{d^{0.29}y(t)}{dt^{0.29}}+2y(t) = \frac{d^{1.72}e(t)}{dt^{1.72}}+2\frac{d^{0.54}e(t)}{dt^{0.54}}+e(t) \quad (5)$$

Using the Matlab code for an input e(t) the level unit, the solution of the fractional linear differential equation is given by the following program:

```
h = 0.001; t=0:h:20; e=ones(size(t));
b=[1,2,1]; nb=[1.72,0.54,0]; a=[1,3,4,2];
na=[2.53,1.47,0.29,0];
y=fode_sol(a,na,b,nb,e,t);
plot(t,y)
```

The step response of the fractional system represented by the fractional linear differential equation is given in the figure 2.

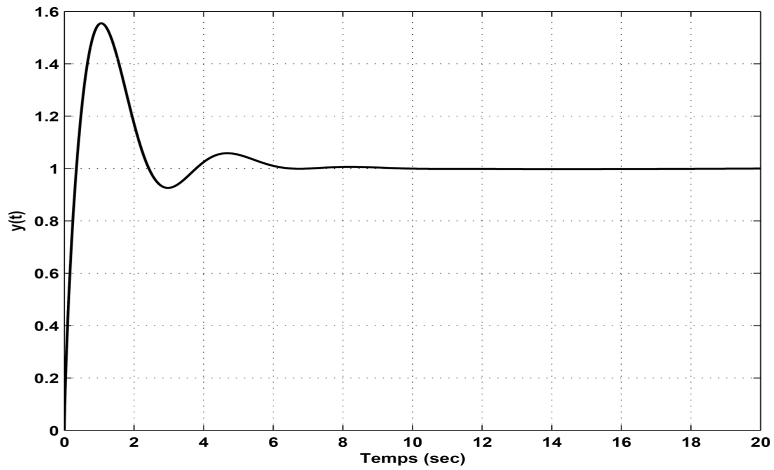

Fig. 2. Step responses of the equation (5)

4. Results and discussion

In the application, the TKEO method are applied to detect the defects in rotating machines. We focus this study of the bearing's part because it is the more exposed to the defects.

The frequencies of defectives bearing are f_B=140Hz (B: Ball fault), f_{IR}= 164 Hz (IR: Inner Race fault), f_{OR}= 107 Hz (OR: Outer Race fault).

A continuous approximation form of the energy operator can be represented as in [23].

$$\psi(x(t)) = \left(\frac{dx(t)}{dt}\right)^2 - x(t)\frac{d^2x(t)}{dt^2} \tag{6}$$

A continuous approximation of our proposed method FTKEO can be represented as follows:

$$\psi(x(t)) = \left(\frac{d^\alpha x(t)}{dt}\right)^2 - x(t)\frac{d^\beta x(t)}{dt^2} \tag{7}$$

A with ($0 < \alpha < 1$ *and* $1 < \beta < 2$).

Measurements of healthy and defective bearings are available on the website of the “Bearing Data Center” of the University of Western Reserve [30].

Figure 3 shows the temporal vibration signals of bearing without defects, i.e. the healthy state.

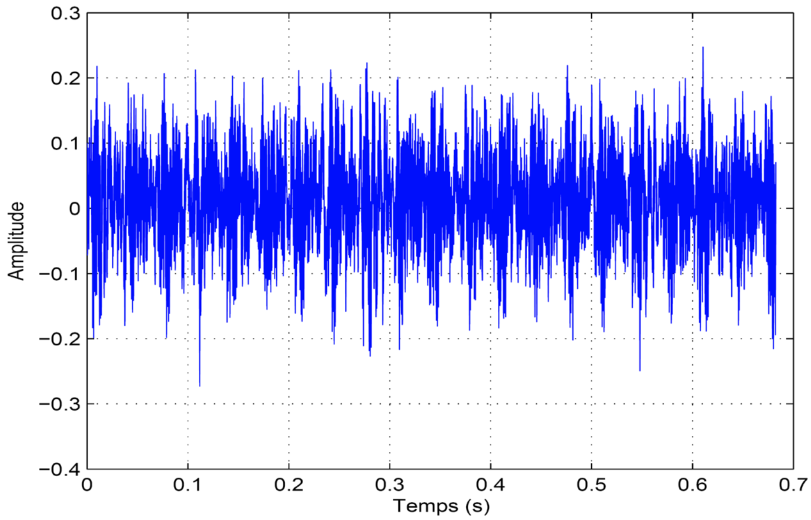

Fig. 3. Temporal signal without defect

The spectral study made it possible to better define the signals in order to appreciate their quality in different free-defect and defective situations using the classical method and the proposed approach.

In practice, in the spectrum of a free-defect bearing, the signal does not show any characteristic frequency of defects. Only two peaks of low amplitude are detected and correspond to the rotation frequency and its harmonic (30 and 60 Hz). As for the spectrum of the defective bearing signal, it clearly shows characteristic frequencies of the different defects and their harmonics.

The different frequencies present on the spectra are:

- *fr, 2*fr*,… (*fr*: rotational frequency)
- *fr, fr+fb*,… (*fb*: ball frequency)

- *fr, fir-fr, fir,fir+fr,…* (*fir*: inner ring frequency)
- *fr, for, 2*for,…* (*for*: outer ring frequency)

The bearing fault frequencies are regrouped in the following table:

Table 1

Bearing fault frequencies

Faults	Inner ring (IR)	Outer ring (OR)	Ball
Frequencies (Hz)	164	107	140

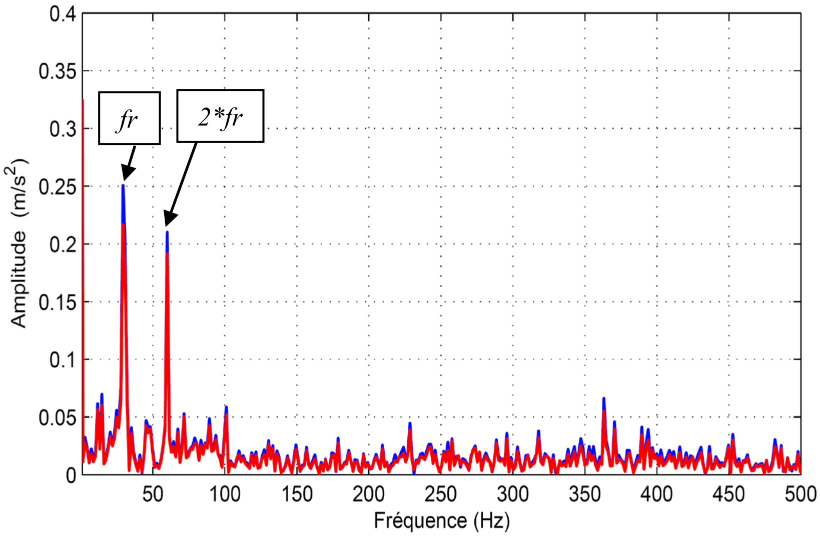

Fig. 4. Spectra of temporal signal without defect using TKEO (red) and FTKEO (blue) methods

Figure 4 shows the spectra of temporal signal without defect using TKEO an FTKEO methods. It has been noted that the peaks in spectra based on the FTKEO method have a higher amplitude than the peaks in spectra based on the TKEO method.

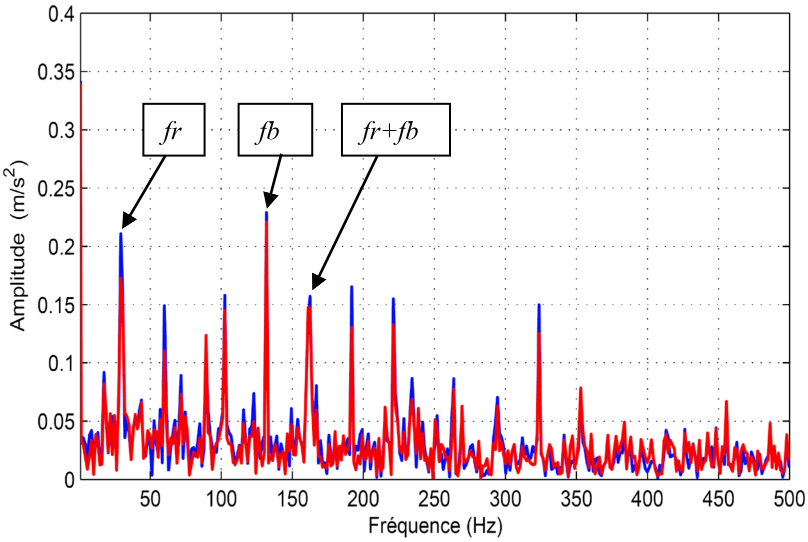

Fig. 5. Spectra of temporal signal with ball defect using TKEO (red) and FTKEO (blue) methods.

The figure 5, above, shows the spectra of temporal signal with ball defect using TKEO and FTKEO methods.

From this figure, we note that the peaks in spectra based on the FTKEO method have a higher amplitude than the peaks in spectra based on the TKEO method.

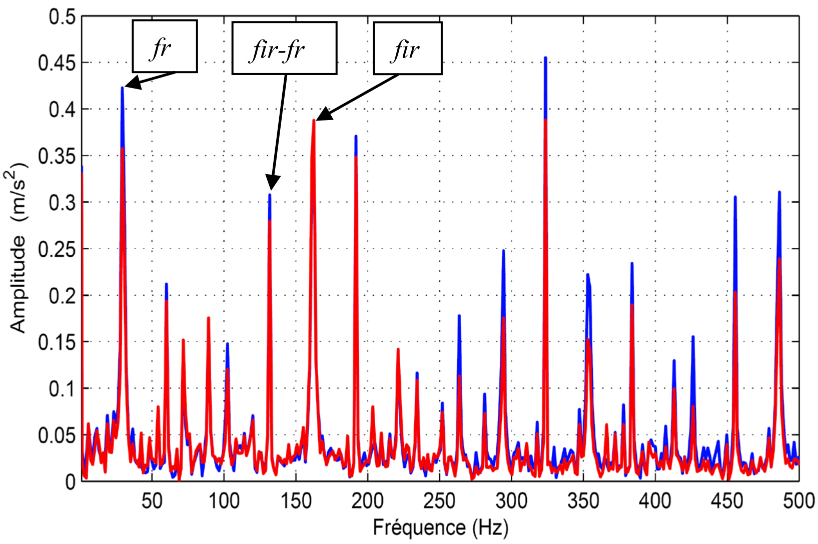

Fig. 6. Spectra of temporal signal with inner race defect using TKEO (red) and FTKEO (blue) methods.

Figure 6 shows the spectra of temporal signal with inner race defect using TKEO an FTKEO methods.

From this figure, we note that the peaks in spectra based on the FTKEO method have a higher amplitude than the peaks in spectra based on the TKEO method.

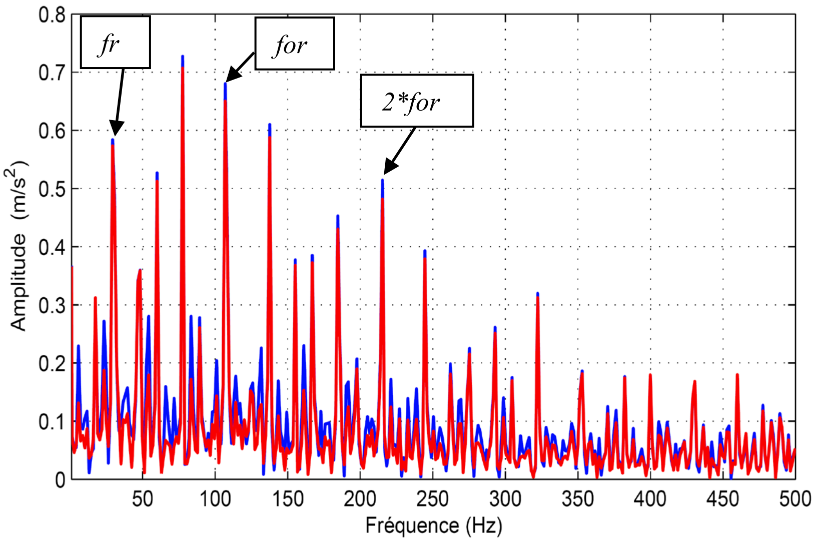

Fig. 7. Spectra of temporal signal with outer race defect using TKEO (red) and FTKEO (blue) methods.

Figure 7 shows the spectra of temporal signal with outer race defect using TKEO an FTKEO methods.

From this figure, we note that the peaks of the frequency signal based on the FTKEO method have a higher amplitude than the frequency signal based on the TKEO.

From these figures, we note that obtained spectrum signals of bearing without defect and the ones with defective bearing presents the higher peaks in the FTKEO case than in the TKEO one. We notice that the peaks of the FTKEO method are higher than the peaks of the TKEO method. This allows a good visualization of the peaks of defects.

The fractional orders α and β were obtained by their variations in the Matlab code by choosing the values which give the good results with comparison between FTKEO and TKEO methods.

The obtained values of fractional derivative orders α and β are as follows:

$\alpha = 0.73,\ \beta = 1.83$ for figures (4,5)

$\alpha = 0.6,\ \beta = 1.8$ for figure (6)

$\alpha = 0.3,\ \beta = 1.75$ for figure (7)

5. Conclusions

The aim of the proposed method is to validate the efficiency of fractional calculation in the defect detection domain of the rolling bearings. It is based on Teager Kaiser Energy Operator approach (TKEO), in which a fractional order derivative has been introduced. The proposed method consists of replacing the two integer; first and second derivatives, by two fractional order derivatives α and β with $0 < \alpha < 1$ and $1 < \beta < 2$ respectively. The variation ranges of the fractional orders α and β make it possible to choose the values which give good results by comparison between FTKEO and TKEO methods. Through the obtained fractional derivative orders (α, β) and the values of the detected peaks, the diagnosis results show that our proposed method FTKEO outperforms the classical TKEO.

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