U.P.B. Sci. Bull., Series D, Vol. 78, Iss. 1, 2016 ISSN 1454-2358

INVERSE DYNAMICS ANALYSIS AND SIMULATION OF A CLASS OF UNDER- CONSTRAINED CABLE-DRIVEN PARALLEL SYSTEM

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For the under-actuated parallel system that 3 robots cooperatively hoist an object of 6 DOF by cables, Newton-Euler method is used to establish the dynamic equation of the system. It is considered that the end effector of each robot has three degrees of freedom in this paper. Therefore, when the position and pose of the object were known, there are 15 unknown parameters and 9 equations by the inverse dynamics analysis. Since that 9 equations can't be resolved to 15 unknown parameters, some methods for 3 cables towing the object with six degrees of freedom will be discussed in this paper. It can be solved by adding constraints to reduce the number of unknown quantities, which aims at achieving the purpose that one solution or a few amount of solutions. Then this paper is divided into two types of variable cable length and fixed cable length to discuss and the Matlab is used for simulation, which verifies the feasibility of the method and shows that the dynamics modeling is reasonable. It lays the foundation for further study.

Keywords: cable-driven parallel system, 6 DOF, dynamics modeling, simulation

1. Introduction

Multi-robot cooperative hoisting forms a tightly coupled system, which driving an object by cables actually is the cable-driven parallel manipulators. They not only have the characteristics of parallel robot such as high rigidity, high precision, strong flexibility and strong carrying capacity, but also have the advantages of cable-driven system like simple structure, small inertia, large working space and movement speed. They can cooperatively accomplish the tasks such as hoisting the heavy object, etc.

In the mid-1980s, MIT scholar Landsberger [1] first designed a kind of cable-driven parallel mechanism with 3 degrees of freedom for undersea operations and carried on mechanical analysis and control. From 1989, the US National Institute of Standards and Technology (NIST) has developed cable-

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driven parallel robot RoboCrane [2] for hoisting, which tow payload to achieve positioning and movement by six cables together. Ming et al. [3] first proposed that the cable-driven parallel robot is divided into two categories: (1) when $m \leq n$, it is the incompletely restrained positioning mechanism (IRPM). Where m is the number of cables, n is the number of degrees of freedom for positioning object, the same below. (2) When $m \geq 1+n$, it is the completely restrained positioning mechanism (CRPM). Williams et al. [4] proposed a class of translational planar cable-direct-driven robot and analysed kinematics, dynamics and statics workspace. Fattah and Agrawal [5] presented a workspace analysis methodology that can be applied for optimal design of cable-suspended planar parallel robots. The mechanisms in literature [4-5] both belong to CRPM. Because of less constraints, when $m<n$, this kind of research are relatively few. And the inverse dynamics and trajectory control has been studied for the incompletely restrained positioning mechanism by Yamamoto et al. [6-7]. Yu and Zheng [8] have analysed inverse kinematics for three configurations of 6 DOF cable-driven gantry crane robots with 3 cables. Moreover, Zheng analysed the differential flatness of its inverse kinematics and dynamics for 6 DOF cable-driven gantry crane robots [9] and motion trajectory planning of 6 DOF cable-driven robots have been addressed based on inverse kinematics equations by Zheng [10]. Because the trolleys of the mechanism in literature [6-9] are restricted in one direction for motion, therefore the above mechanisms can be regarded as that the 3 cables tow the object of 6 degrees of freedom by adding some constraints to achieve. And the projection points of the endpoints of 3 mobile cranes are always maintained a equilateral triangle, which obtain special 6 degree of freedom of the object only through the change of cable length in literature [11]. Kumar's team [12-15] established the static equilibrium equation of multiple aerial robots which cooperative tow the load and analysed inverse kinematics, planning and control for 3 aerial robots. Through the above examples, it can be seen that 3 cables can be also achieved towing the object of 6 degrees of freedom by adding some constraints. In this paper, the three sets of robots cooperatively hoist an object of 6 DOF, which also belongs to this kind of problem, but the difference from the above mechanisms is that the end effector of each robot is considered with three degrees of freedom. So it makes the system more flexible, reconfigurable between multiple robots and larger or smaller for work space. In this paper, the mathematical model of the system will be re-established and it will be divided into two types of variable cable length and fixed cable length to discuss some ways that how to hoist the object of 6 degrees of freedom by 3 cables.

Since the single serial robot has been developed relatively maturely, this paper focuses on movement of connection point between the cable and the robot endpoint.

2. Dynamics modeling

Systemic model that 3 robots cooperatively hoist an object by cables is shown in Fig.1, which composed of three manipulators, three cables and an equilateral triangle heavy object(moving platform). The endpoints of manipulators are respectively represented by A_1, A_2, A_3. The connection points between the cables and the heavy object are respectively represented by B_1, B_2, B_3, which are also the vertexes of the equilateral triangle heavy object. A global coordinate system O-XYZ on the ground is established and a local coordinate system O'-X'Y'Z' in the center of mass of heavy object is established.

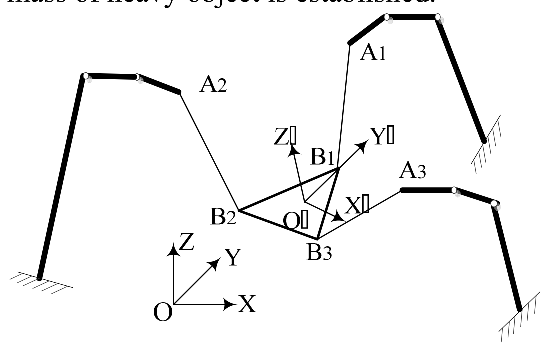

Fig. 1. 3 robots cooperatively hoist an object

The location of the origin of the local coordinate system relative to the global coordinate system is as Eq. (1).

$$\boldsymbol{r} = [x\, y\, z]^{\mathrm{T}} \tag{1}$$

The orientation of the moving platform can be described by a general rotation matrix R. The Euler representation is adopted to describe the absolute moving platform posture, which is composed of three successive relative rotations, namely: first rotation of angle α around the Z1-axis, followed by second rotation of angle β around the rotated Y2-axis and finally followed by third rotation of angle γ around the X'-axis of the moving frame O'-X'Y'Z' attached to the moving platform. With respect to the fixed global coordinate system O-XYZ, the general rotation matrix can be expressed by a multiplication of three basic rotation matrices as follows:

$$R = R1(\mathrm{Z1}, \alpha)R2(\mathrm{Y2}, \beta)R3(\mathrm{X'}, \gamma) \tag{2}$$

The skew-symmetric matrix operator $\Omega = (\mathrm{d}R/\mathrm{d}t)R^{\mathrm{T}}$ is associated to the vector of angular velocity of the moving platform, expressed in the global fixed base O-XYZ [16-18].

$$\omega = [\omega_{\mathrm{x}}\ \ \omega_{\mathrm{y}}\ \ \omega_{\mathrm{z}}]^{\mathrm{T}} = (\mathrm{d}\gamma/\mathrm{d}t)R1R2\mathrm{u}_1 + (\mathrm{d}\beta/\mathrm{d}t)R1\mathrm{u}_2 + (\mathrm{d}\alpha/\mathrm{d}t)\mathrm{u}_3 \tag{3}$$

with following components:

$$\omega_{\mathrm{x}} = \mathrm{c}\alpha\mathrm{c}\beta(\mathrm{d}\gamma/\mathrm{d}t) - \mathrm{s}\alpha(\mathrm{d}\beta/\mathrm{d}t)$$

$$\omega_{\mathrm{y}} = \mathrm{s}\alpha\mathrm{c}\beta(\mathrm{d}\gamma/\mathrm{d}t) + \mathrm{c}\alpha(\mathrm{d}\beta/\mathrm{d}t) \tag{4}$$

ω_z = - sβ(dγ/dt) + dα/dt

where $u_1 = [1\ 0\ 0]^T$, $u_2 = [0\ 1\ 0]^T$, $u_3 = [0\ 0\ 1]^T$ are three unit vectors.

Also, the vector of angular acceleration of the moving platform yields:

$$\varepsilon = d\omega/dt = [\varepsilon_x\ \varepsilon_y\ \varepsilon_z]^T = (d2\gamma/dt^2)R1R2u_1 + (d2\beta/dt^2)R1u_2 + (d2\alpha/dt^2)u_3 +$$
$$+(d\gamma/dt)(d\alpha/dt)R1U_3R2u_1 - (d\beta/dt)(d\gamma/dt)R1R2u_3 - (d\alpha/dt)(d\beta/dt)R1u_1 \tag{5}$$

with the skew-symmetric matrix U_3 associated to third unit vector u_3.

The endpoints of manipulators are $A_i(x_{Ai}\ \ y_{Ai}\ \ z_{Ai})$. Assuming that b_i is the coordinate of connection point $B_i\,(x_{Bi}\ \ y_{Bi}\ \ z_{Bi})$ in a local coordinate system, then the following equation is obtained:

$$B_i = Rb_i + \boldsymbol{r} \tag{6}$$

The cable length formula can be obtained by A_i , B_i .

$$l_i = \sqrt{(x_{Ai} - x_{Bi})^2 + (y_{Ai} - y_{Bi})^2 + (z_{Ai} - z_{Bi})^2} \tag{7}$$

The velocity of the heavy object in the global coordinate system is v, then the following equation is obtained:

$$\boldsymbol{v} = \dot{\boldsymbol{r}} = [\dot{x}\ \dot{y}\ \dot{z}]^T \tag{8}$$

The inertia matrix of the heavy object in the local coordinate system is I, then the inertia matrix I' in the global coordinate system is as follows:

$$I' = RIR^T \tag{9}$$

Assuming that the intensities of three tensions of the cables are T_1, T_2, T_3 , then the dynamic equations of the heavy object based on the Newton-Euler modelling, applied with respect to mass centre O' of the moving platform, can be expressed in the fixed frame O-XYZ in following matrix form:

$$m\mathrm{d}v/\mathrm{d}t = -mgu_3 + \sum (T_i/l_i)\overline{B_iA_i}$$
$$I'\mathrm{d}\omega/\mathrm{d}t + \Omega I'\omega = \sum (T_i/l_i)R\hat{B}_iR^T\overline{B_iA_i} \tag{10}$$

where $\hat{B}_i$ is the skew-symmetric matrix associated to the vector b_i expressed in local coordinate system , i={1, 2, 3} and some important variable symbols are defined in Table 1.

Table 1

Various symbols being used

A_i	Endpoint of the i^{th} manipulator
B_i	The connection point between the i^{th} cable and the heavy object
$\boldsymbol{r}$	Position vector of mass centre O' of the moving platform
R	Rotation matrix of the moving platform
Ω	Skew-symmetric matrix operator of the moving platform
ω	Vector of angular velocity of the moving platform
ε	Vector of angular acceleration of the moving platform

b_i	Coordinate of connection point B_i in local coordinate system
l_i	Length of the i^{th} cable
$\boldsymbol{v}$	Velocity of the moving platform in the global coordinate system
I	Inertia matrix of the moving platform in the local coordinate system
I'	Inertia matrix of the moving platform in the global coordinate system
T_i	Tension of the i^{th} cable
m	Mass of the moving platform
g	Acceleration due to gravity

3. Inverse dynamics analysis of system under constraints

Based on the analysis of dynamics modelling in the above section, it can be seen that if the position $\boldsymbol{r}$ and orientation γ, β, α of the heavy object and b_i are known, B_i can be obtained from Eq. (6). Then the mass m and the inertial matrix I are given, there are 9 equations according to Eq. (7) and (10) and there are 15 unknown parameters which are A_i with 9, l_i with 3 and T_i with 3 in total. Because 9 equations can't be resolved to obtain 15 unknown parameters and equations contain quadratic term, so it is needed to add constraints to reduce the number of unknown parameters, then the analytical or numerical solution can be obtained. This paper is divided into two types of variable cable length and fixed cable length to discuss, and the number of unknown parameters becomes less by adding constraints for A_i, which aims at achieving the purpose that one solution or a few amount of solutions can be obtained, specifically it can be described as follows:

3.1 Variable cable length

When the cable length changes, in order to reduce the number of unknown parameters, we consider that the endpoints of manipulators A_i remain still. For simplifying calculation, we assume that the endpoints of manipulators A_i maintain an equilateral triangle in the horizontal plane and the projection points of A_i on the ground is an equilateral triangle which takes the origin O of the global coordinate system as the center. Triangle side length is *D,* the Z axis value of A_i is *Z* and the projection point of A_1 is on the Y axis, hence A_i can be expressed as：

$$A_1(0,\frac{D}{\sqrt{3}},Z)\ ,\ A_2(-\frac{D}{2},-\frac{\sqrt{3}D}{6},Z)\ ,\ A_3(\frac{D}{2},-\frac{\sqrt{3}D}{6},Z)$$

thus Eq.(7) can be transformed into as follows:

$$\begin{cases} l_1 = \sqrt{(-x_{B1})^2 + (\frac{D}{\sqrt{3}} - y_{B1})^2 + (Z - z_{B1})^2} \\ l_2 = \sqrt{(-\frac{D}{2} - x_{B2})^2 + (-\frac{\sqrt{3}D}{6} - y_{B2})^2 + (Z - z_{B2})^2} \\ l_3 = \sqrt{(\frac{D}{2} - x_{B3})^2 + (-\frac{\sqrt{3}D}{6} - y_{B3})^2 + (Z - z_{B3})^2} \end{cases} \tag{11}$$

So that there are only 6 unknown parameters that are l_i, T_i (i=1,2,3). According to the 9 equations for Eq.(10) and (11), we can finally obtain analytical formulas of l_i , T_i , which are associated with the position ***r*** and orientation γ, β, α of the heavy object, and the analytical formulas are unique. Because the expressions are cumbersome, they are not detailed here.

3.2 Fixed cable length

When the cable length is unchangeable, we assume that l_i =*L*. In order to reduce the number of unknown parameter, we consider limiting A_i only at the height *Z* for motion, thus Eq.(7) can be transformed into as follows:

$$\begin{cases} \sqrt{(x_{A1} - x_{B1})^2 + (y_{A1} - y_{B1})^2 + (Z - z_{B1})^2} = L \\ \sqrt{(x_{A2} - x_{B2})^2 + (y_{A2} - y_{B2})^2 + (Z - z_{B2})^2} = L \\ \sqrt{(x_{A3} - x_{B3})^2 + (y_{A3} - y_{B3})^2 + (Z - z_{B3})^2} = L \end{cases} \tag{12}$$

So there are 6 unknown parameters for A_i and 3 unknown parameters for T_i .According to the 9 equations for Eq. (10) and (12), we can't obtain analytical formulas of l_i , T_i , which are associated with the position ***r*** and orientation γ, β, α of the heavy object, but when specific values of the position and orientation of the heavy object are given, we can get multiple groups of numerical solution.

3.2.1 The method of choosing feasible inverse solution

Due to the existence of multiple groups of numerical solutions, we can consider choosing feasible inverse solution from the following two aspects:

(1) Try to keep the tension of each cable changing smoothly.

(2) Try to keep the trajectory of endpoint of each manipulator smooth.

The (2) method is selected in this paper. Because from the view of the practical control, the (2) method is convenient to control the endpoint of each manipulator. Although the tension change is smooth in the (1) method, but the mutation of the manipulator endpoints' trajectory may be relatively large in some time.

So the selection step is as follows:

(1) For the multiple groups of numerical solutions above, the solutions that are imaginary, $T_i \leq 0$ and exceeding the cable tension limit should be eliminated. Since each numerical solution have a corresponding relationship among A_1, A_2, A_3, so the corresponding solutions should be also eliminated.

(2) For the multiple groups of numerical solutions above, the same coordinate values of A_1, A_2, A_3 at same time should be eliminated, otherwise the manipulators will touch together.

(3) After the above processing，the number of numerical solution of A_i is different in different time. Then the moment that the number of numerical solution of A_1, A_2, A_3 is the least are regarded as starting points respectively. From the starting points, we start to choose the solution of adjacent time which must be quite close with the solution of this time. Meanwhile we must guarantee the entire curve smooth as far as possible, so as to realize the manipulators moving smoothly.

(4) Since each numerical solution have a corresponding relationship among A_1, A_2, A_3, we can select several trajectories respectively. We compare these trajectories to choose the optimal one and the curves of T_i corresponding to A_i can also be selected.

4. Simulation results

Based on the analysis of the model above, we set parameters of the mechanism in Fig. 1 are as follows:

The mass of the heavy object m = 1 kg, distance between the mass center O ' of the heavy object and vertexes b_i is d=0.1m. The desired trajectory of the mass center O ' of the heavy object in the global coordinate system can be expressed as:

$$\begin{cases} x = 0.05 \times t \times \cos t \\ y = 0.05 \times t \times \sin t \\ z = 0.05 \times t \\ \alpha = \pi \times t / 100 \\ \beta = \pi \times t / 100 \\ \gamma = \pi \times t / 100 \end{cases} \qquad 0 \leq t \leq 20(s) \tag{13}$$

So the desired trajectory of the heavy object is shown in Fig. 2 and B_1, B_2, B_3 are the vertexes of the triangle heavy object.

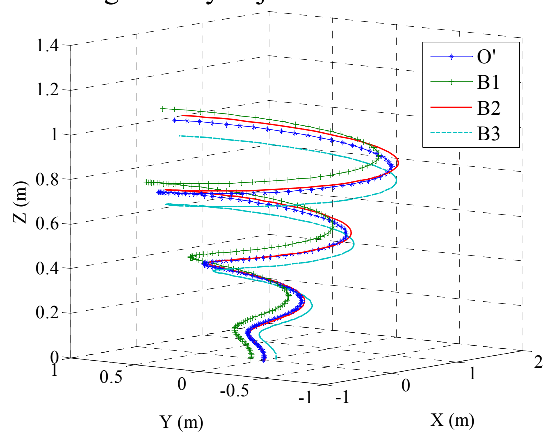

Fig. 2. The desired trajectory of the heavy object

4.1 Variable cable length

We set the side length D=5m and the height Z=1.5m. For the realization of the desired trajectory in Fig. 2, according to the analytic formula obtained by 9 equations of Eq. (10) and (11), we can obtain the cable length with time, and cable tension with time, which are shown in Fig. 3 and Fig. 4.

It can be seen from Fig. 2 that the trajectory of the heavy object is a helical line whose radius becomes large with time. Therefore, the curves in Fig. 3 and Fig. 4 are similar to the sinusoidal or cosine curve and the amplitude and period of the curves increase gradually. While comparing Fig. 3 and Fig. 4, we can see that the length and tension of the cable is roughly inverse proportional relationship.

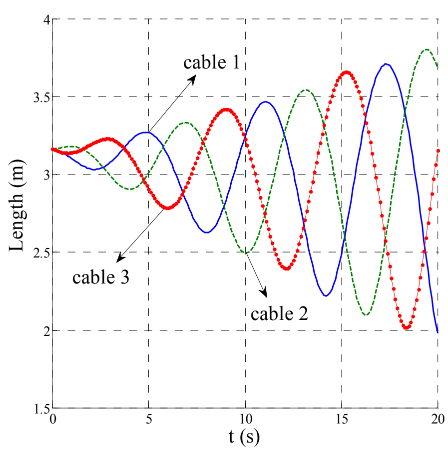

Fig. 3. The changes of cable length

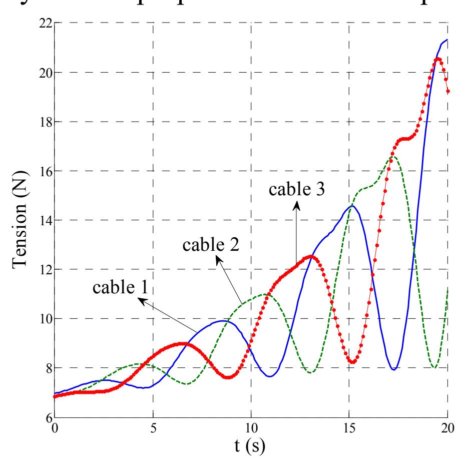

Fig. 4. The changes of cable tension

4.2 Fixed cable length

We set that l_i =2m and limit the A_i only at the height $Z=1.5m$ for motion. For the realization of the desired trajectory in Fig. 2, according to the 9 equations of Eq. (10) and (12), we can obtain multiple groups of numerical solution. According to the selection method in section 3.2.1, we get only 2 groups of solution in some moment. In order to realize the manipulators moving smoothly and avoid the collision, we finally obtain an optimal trajectory 1 and suboptimal trajectory 2 of A_i respectively. The contrast curves of two trajectories of points A_i respectively in the X, Y axis direction with time as shown in Fig. 5- Fig. 10.

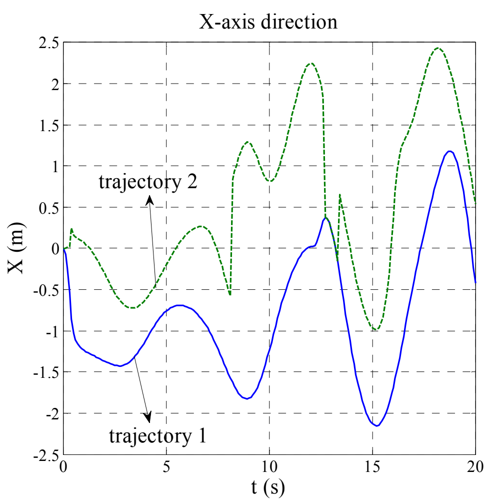

Fig. 5. Comparison chart of A_1

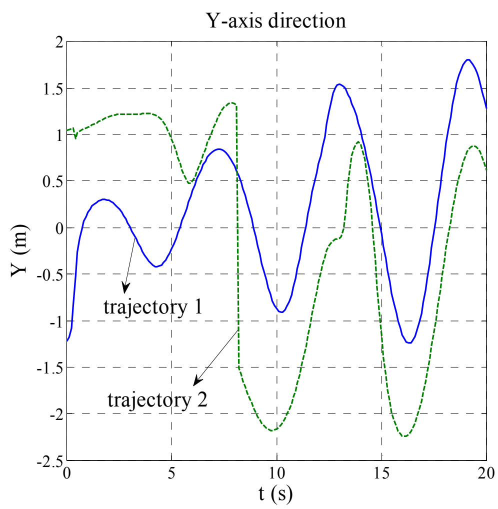

Fig. 6. Comparison chart of A_1

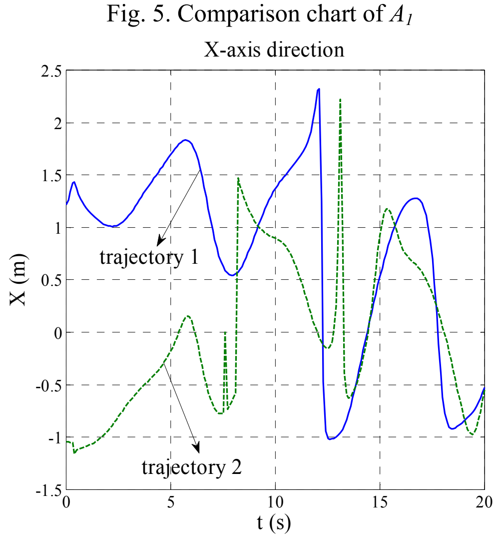

Fig. 7. Comparison chart of A_2

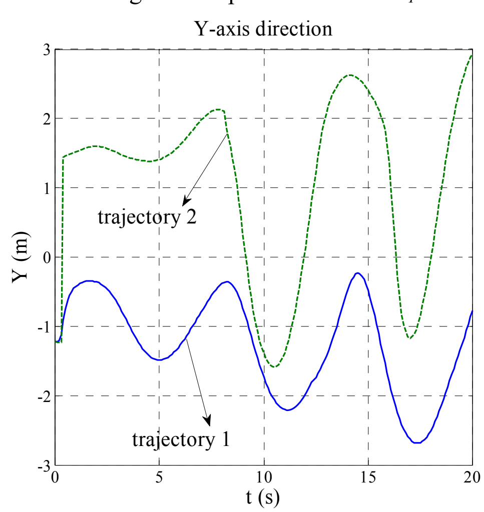

Fig. 8. Comparison chart of A_2

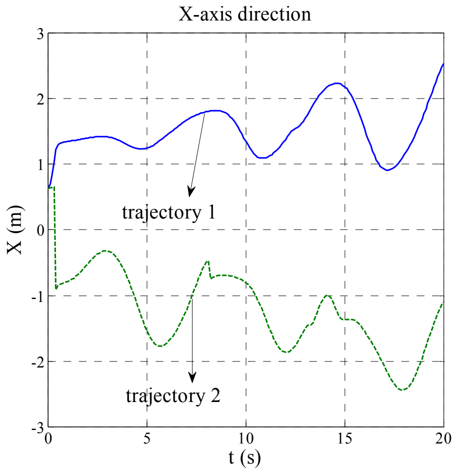

Fig. 9. Comparison chart of A_3

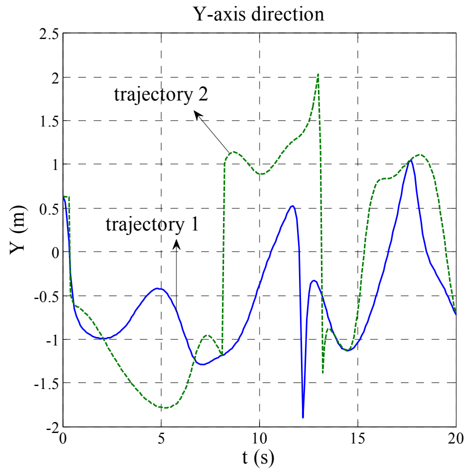

Fig. 10. Comparison chart of A_3

The comparison charts of A_i's trajectories in the Z=1.5m plane as shown in Fig. 11. The comparison charts of cable tension corresponding to two A_i 's trajectories are shown in Fig. 12.

It can be seen from Fig. 5 and Fig. 6 that the optimal trajectory 1 of A_1 in the X, Y axis direction on the entire time period is relatively smooth. So the corresponding trajectory 1 in Fig. 11 is also relatively smooth.

Fig. 7 and Fig. 8 show that the optimal trajectory 1 of A_2 in the X axis direction at the 12s-13s moments has relatively large changes and in the Y axis direction overall is relatively smooth. So the corresponding trajectory 1 in Fig. 11 shows relatively large changes only in individual local.

Fig. 9 and Fig. 10 show that the optimal trajectory 1 of A_3 in the X axis direction overall is relatively smooth and in the Y axis direction at the 12s-13s moments has relatively large changes. So the corresponding trajectory 1 in Fig. 11 overlaps together, which is a bit poor relative to the trajectories of A_1 and A_2.

Fig. 12 shows that the tension changes of cable 1 and cable 2 corresponding to the optimal trajectory 1 are relatively smooth. Since A_3 has relatively large changes at the 12s-13s moments, so the corresponding tension of cable 3 shows relatively large changes.

The above comparison charts show that the trajectory 1 is more smooth than the trajectory 2 and the corresponding tension is also more smooth. Because the desired trajectory of the heavy object is more complex, the optimal trajectory 1 of point A_i has relatively large changes inevitably in some individual time.

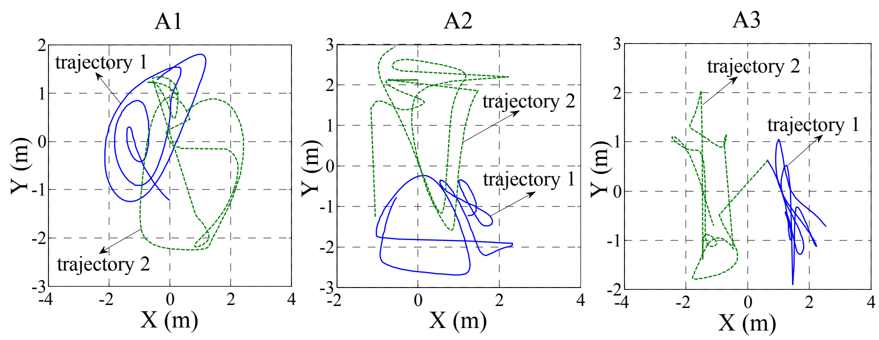

Fig. 11. Comparison chart of A_i 's trajectories in the plane

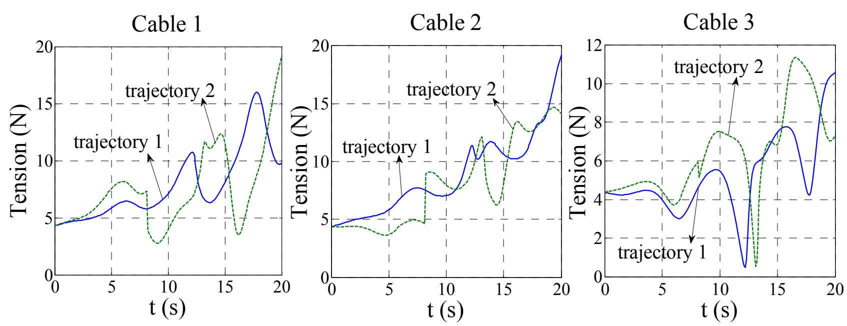

Fig. 12. Comparison chart of 3 cable tension over time

5. Conclusions

In this paper, a dynamic model of 3 robots cooperative hoisting system is established. For 3 cables towing the object of six degrees of freedom, we consider adding constraints for the system to reduce the number of unknown parameters, and the number of equations can't be less than the number of unknown parameters. Thus we will be able to obtain analytical or numerical solution. This paper specifically discusses two cases of adding constraints. In variable cable length, we can obtain the unique analytical solution. In fixed cable length, we can get multiple groups of numerical solution and then choose the optimal solution. Since the trajectory of the heavy object has six degrees of freedom，it means that 3 cables can hoist the object of 6 degrees of freedom. The results will be used to further study that the cooperative control of hoisting system, stiffness and stability analysis of the system. Since in this paper, we only analyse the case of fixed base robot, we can also analyse the case of the mobile robot as the further research.

Acknowledgements

This project is supported by National Natural Science Foundation of China (Grant No. 51265021). This paper is also co-supported by Specialized Research Fund for the Doctoral Program of Higher Education of China (20126204120004) and Key Project of Ministry of Education of China (212184).

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