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MARKOV CHAINS AND DECOMPOSITION METHOD USED FOR SYNCHRONIZING THE MANUFACTURING PRODUCTION RATE WITH REAL MARKET DEMAND

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This paper presents an approach to find a strategy for stabilizing production of a real manufacturing line. In the first phase the flow line is analyzed and then Markov chains and decomposition method are applied, in order to optimize the production rate by reallocating the buffers. Finally, a model is proposed to focus on stabilization of the production and, in the same time on synchronizing the production rate with the market demand. The mathematical model presented is developed and coded in C++.

Keywords: Markov chain, production, synchronize, demand, stabilization.

1. Introduction

Nowadays, many companies are in difficulty to find a strategy for *stabilizing* production. In order to solve this problem, an analytical model using Markov chains is proposed.

In this paper an approach of Markov chain is applied for analyzing a real flow line and scheduling algorithms are as well developed. The main objective of the paper is to optimize the buffers by maximizing production rate at critical resources in order to make enough products for customer's satisfaction and maintaining of the delivery on-time.

The proposed model integrating the analytical model with Markov chains and decomposition method can be used by researchers and practitioners to estimate the production rate in order to *synchronize* the production and the market demand.

Markov Chains were introduced in 1907 by the Russian mathematician A.A. Markov. Markov chains were rapidly recognized for their important power of representation and their possibility to model a wide range of real life problems as well as for the quality of performance indices they give with a relatively small computing effort. Markov chains can be used for modeling and performance

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evaluation of manufacturing systems when they reveal some random behavior (breakdowns, random time to process a part).

Decomposition methods were proposed by Zimmern (1956) and Sevastyanov (1962) for the analysis of large systems with unreliable machines and by Hillier and Boling (1967) for the analysis of large systems with reliable machines. An accurate decomposition method was proposed by Gershwin (1987) for the analysis of the synchronous model [1].

2. Preliminary concepts

This section is devoted to provide the mathematical definitions which will be used in the next section [1-5].

Definition 2.1. Let $X(t)$ a parameter. A *stochastic process* is a random variable indexed by $X(t)$, where t is considered as a continuous variable $t \in (-\infty, +\infty)$.

Definition 2.2. A *Markov chain* is a stochastic process for which the domain of the variable is a countable set and the following relation is satisfied:

$$P[X(t_k) = x_k \,/\, X(t_{k-1}) = x_{k-1}, \dots, X(t_1) = x_1] = P[X(t_k) = x_k \,/\, X(t_{k-1}) = x_{k-1}], \quad (1)$$

$$\forall k,\ \forall t_1 \le \dots \le t_k \text{ and } \forall x_1, \dots, x_k \text{ in the domain of the variable,}$$

where $P[.\,/\,.]$ is the usual notation for a conditional probability.

To a finite Markov chain, one can associate a graph with n vertices. In this graph each vertices corresponds to a state. Then there is an edge (x_i , x_j). If $\lambda_{ij} \neq 0$, λ_{ij} is called the *transition rate* from state x_i to x_j.

Example 2.1. A machine can have two states: *up* and *down*. When the machine is *up*, it can work and may have failures, with the *failures rate* λ. When the machine is *down*, we repair it with a *repair rate* μ. In this situation the graph of the Markov chain is as follows:

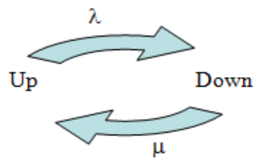

Fig. 1. Graph of the Markov chain of the two-state machine

Definition 2.3. For the state x_i, the *outgoing rate* is defined as the sum of all transition rates from x_i to all x_j's where $i \neq j$.

Consider a particular state x of a Markov chain with outgoing rate λ. We are interested in the time τ we will spend in state x before leaving it. Let us first define the function:

$$G(t) = P[\tau \ge t] \quad (2)$$

From the definition of the outgoing rate we can write the first order equation:

$$G(t+dt) = G(t)(1 - \lambda dt) \quad (3)$$

which can be understood as: '*we remain in x until time t+dt if we are in x at time t and if we do not leave x between t and t+dt'*.

Assuming that G(t) has a derivative, we get:

$$G(t) + (dG/dt)dt = G(t)(1- \lambda dt) \quad (4)$$

which finally gives:

$$(dG/dt) = -\lambda G(t) \quad (5)$$

This differential equation with the obvious initial condition $G(0) = 0$, gives the solution:

$$G(t) = e^{-\lambda t} \quad (6)$$

Consider now the *distribution function* of the variable τ defined by:

$$F_\tau(t) = P[\tau \leq t] \quad (7)$$

It appears that:

$$F_\tau(t) = (1 - e^{-\lambda t}) \quad (8)$$

which is called an *exponential distribution*. The corresponding *probability density function* (*pdf*) is therefore:

$$f_\tau(t) = \lambda e^{-\lambda t} \quad (9)$$

We have proved that the *sojourn time* in a markovian state with outgoing rate λ is exponentially distributed with rate λ.

Using the *pdf* (probability density function) of the random variable τ, one can compute its various moments as the average value, variance… For the average value $m(\tau)$ we get:

$$m(\tau) = \int_0^\infty t\lambda e^{-\lambda t} dt \quad (10)$$

and finally:

$$m(\tau)=1/\lambda \quad (11)$$

the variance $var(\tau)$ and the standard deviation $\sigma(\tau)$ can be computed in the same way; in particular, we get also:

$$\sigma(\tau) = \frac{1}{\lambda} \quad (12)$$

An important parameter used to characterize random variables is the *coefficient of variation,* which is defined as:

$$C_v{}^2 = \frac{\sigma(\tau)}{m^2(\tau)} \quad (13)$$

This parameter C_v is a normalized measure of the variability of the random variable. It is a remarkable feature of the exponential variable that we have:

$$C_v(\tau) = 1 \quad (14)$$

Further, we need to introduce the following notation:

$$\pi_i(t) = P[X(t) = x_i], \quad 15)$$

and the row vector *:*

$$\pi(t) = (\pi_1(t), \ldots, \pi_n(t)). \quad 16)$$

The vector $\pi(t)$ is a probability distribution, which means that

$$0 \leq \pi_i(t) \leq 1, \text{ for } i = 1, \ldots, n \text{ and } \sum_{i=1}^{n} \pi_i(t) = 1 \quad (17)$$

Starting with an initial probability distribution $\pi(0)$, we would like to evaluate $\pi(t)$ for any $t \geq 0$. We can write the first order equation:

$$\pi_i(t+dt) = \pi_i(t)(1 - \sum_{\substack{j=1 \\ j \neq i}} \lambda_{ij} dt) + (\sum_{\substack{k=1 \\ k \neq i}}^{n} \pi_{ki}(t) \lambda_{ki} dt) \quad 18)$$

This equation explains that being at time $t+dt$ in state x_i two situations may result:

- we were in a state at time t, then we travel from this state to x_i between t and $t+dt$,
- we are in state x_i at time t and we do not move between t and $t+dt$.

Notice also that at the first order, multiple state movements cannot occur.

If we denote:

$$\lambda_{ii} = -\sum_{\substack{j=1 \\ j \neq i}} \lambda_{ij} \quad (19)$$

The equation can simply be written as:

$$\pi_i(t+dt) - \pi_i(t) = (\sum_{k=1}^{n} \pi_{ki}(t) \lambda_{ki} dt) \quad (20)$$

Introducing the *(n.n)* matrix Λ, which has for (i-j)th entry λ_{ij} , we get the matrix differential equation:

$$\dot{\pi}(t) = \pi(t)\Lambda \text{, with initial condition } \pi(0) = \pi_0. \quad (21)$$

This first order matrix differential equation gives us the probability of being in a given state at time t, given the initial probability distribution. The matrix Λ is called the *infinitesimal generator,* or simply the generator of the Markov chain. The sum of the entries of any row of Λ is 0. Notice that λ_{ii} is *not* a transition rate, in fact λ_{ii} is the opposite of the outgoing rate of state x_i.

The solution has exactly the same form as in the scalar case, that is:

$$\pi(t) = \pi(0)e^{\Lambda t} \quad (22)$$

where $e^{\Lambda t}$ is the exponential of a matrix. There are several ways for computing the exponential of a matrix.

Given a Markov chain with generator Λ and initial probability distribution π_0 we would like to evaluate:

$$\pi = \lim_{t \to \infty} \pi(t) \tag{23}$$

This limit indeed always exits, and from the evolution equation it must satisfy the relation:

$$\pi \Lambda = 0 \tag{24}$$

This equation is called the *stationary equation* since it implies that starting with initial distribution $\pi_0 = \pi$, the probability distribution will not change with t. In general, the limit probability distribution will depend on the initial situation, but it is important to identify the cases where this is not necessary. The answer is in the following theorem.

Theorem 2.1. The limit distribution is independent of the initial probability distribution if and only if the Markov chain has only one *ergodic* class.

The necessity of the condition is obvious, if there are several ergodic classes, starting in a state of one ergodic class will lead to a limit behavior in the same class.

Indeed, when the condition is satisfied, the limit distribution is obtained by solving the equation with the additional normalization constraint:

$$\sum_{i=1}^{n} \pi_i = 1 \tag{25}$$

The equation (24) amounts to n algebraic equations but only *(n-1)* of them are independent.

The i^{th} equation is:

$$0 = \sum_{j=1}^{n} \pi_j \lambda_{ji} \tag{26}$$

which can be rewritten as :

$$-\pi_i \lambda_{ii} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \pi_j \lambda_{ji} \tag{27}$$

This equation can be seen as a '*balance equation*' at state x_i; recall that $-\lambda_{ii}$ is the outgoing rate from state x_i . The equation expresses on the left hand side the '*flow in*' state x_i and the right hand side the '*flow out'*. As in the usual flow problems we can also write '*balance equations'* when the state set is partitioned in two parts. Consider $I \subset \{1, \ldots, n\}$ and $J = \{1, \ldots, n\}/I$, we assume that I and J are non-empty, then :

$$\sum_{\substack{i \in I \\ j \in J}} \pi_i \lambda_{ij} = \sum_{\substack{k \in I \\ l \in J}} \pi_l \lambda_{lk} \tag{28}$$

This method (which is called the '*method of cuts*') is very effective for solving the stationary equation even for large Markov chains and has a good structure [1-4].

3. Problem description and Mathematical model
3.1. Model assumptions

Analytical performance analysis methods are based on the *stochastic* modeling of flow production systems. We use the analytical results of *queuing theory* for the modeling of a flow production system. The problem is stated under the following assumptions (for an ideal case) [1, 6]:

- single product manufacture,
- limited capacity of stocks,
- the product has a successive flow in the line (it goes from one machine to another within the same line),
- the raw parts are always stocked in front of the first machine (it is never starved) and the last machine can always deposit a finished product (it is never blocked).
- No breakdowns

3.2 Mathematical model

The authors are interested in the performance evaluation of particular production systems called *lines*. A line is made up of a succession of machines, to which parts go from one to another, successively. A single type of product is treated.

It is proposed to experiment by analytical modeling, a real case study, a flow line called *Headrest support*, consisting of M=8 stations.

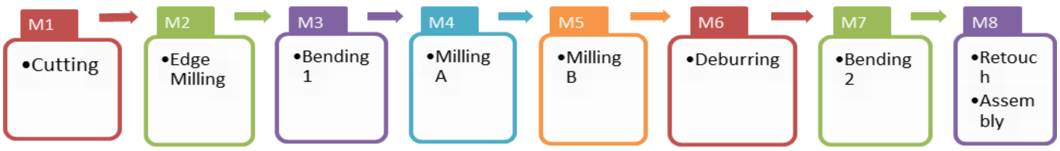

Fig. 2. Headrest support line with 8 stations

The case study is related to a manufacturing line of a car headrest support work piece, which is manufactured in an industrial company from Arges County.

Fig. 3. Headrest support work piece

The main activity of the company is production and marketing of parts and assemblies for automotive industry, especially for DACIA - RENAULT.

The main problem encountered in the factory was to find solution in order to estimate the production rate of the system and also, to synchronize the production and the market demand. In order to solve this inconvenient the authors will set two algorithms to calculate the production rate (number of parts produced in a time unit).

First of all, it is calculated the production rate for 2 machines; the Markov chains are applying. The necessary algorithm (with stock 0) is presented below:

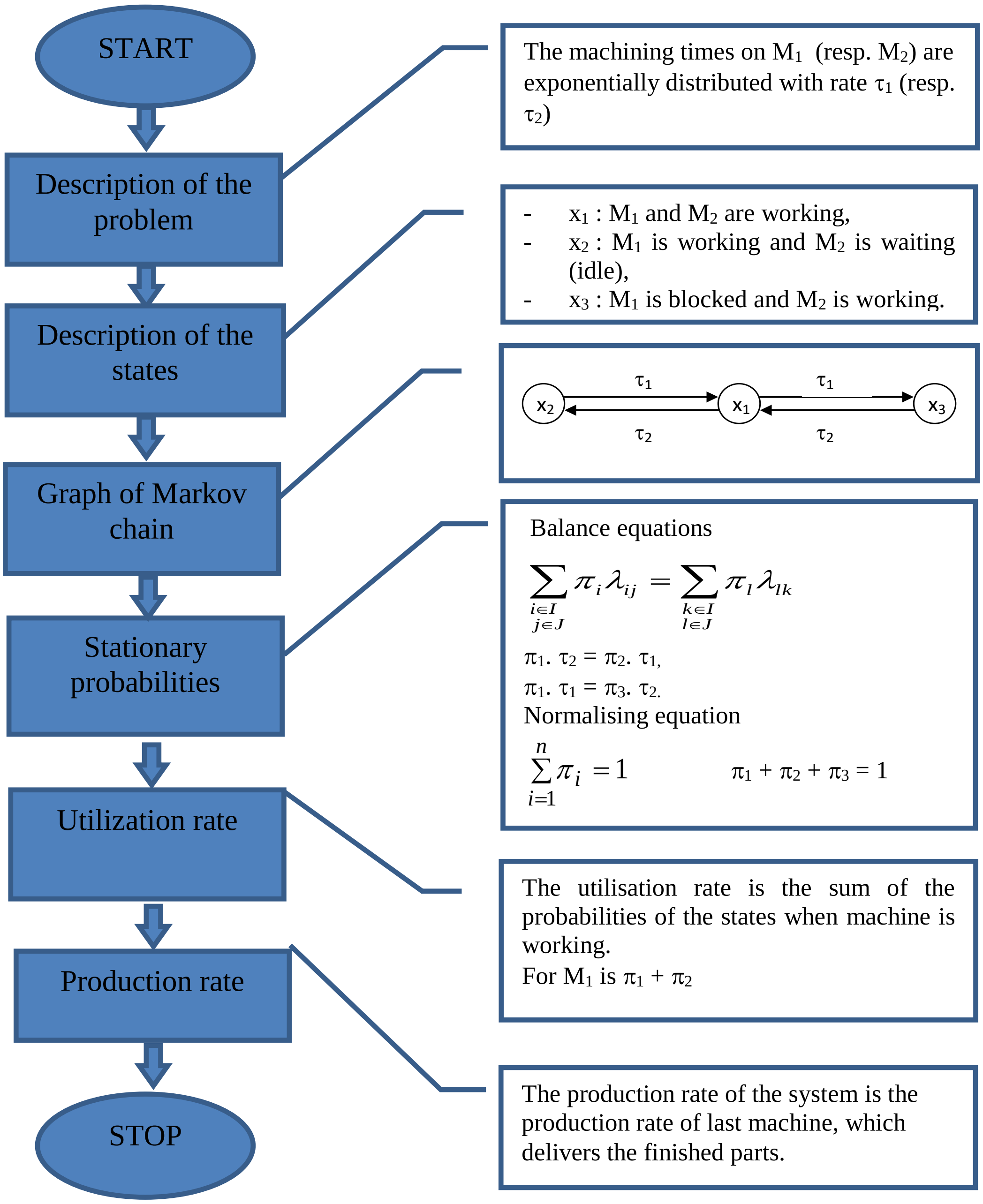

Fig. 4. Production rate of a workstation (Markov chain method)

As the Markov chains need more time to provide solutions in queues theory, they will be used only for small production lines. For larger production lines (in our case 8 machines), there is another method - “decomposition method” – which will be used [7-11].

Given the solution of the two machines case, approximate methods for the general lines were developed. The system is decomposed into (M-1) = 7 subsystems consisting of two stations each. Each of the two-station subsystems is analyzed with the help of an exact or approximate evaluation method. The parameters of the two stations of the subsystem are then adjusted, such that they account for the effects of all stations located outside the subsystem. All results are then adjusted in an iterative procedure [2, 12-15].

The general steps of the decomposition method are described in the following algorithm:

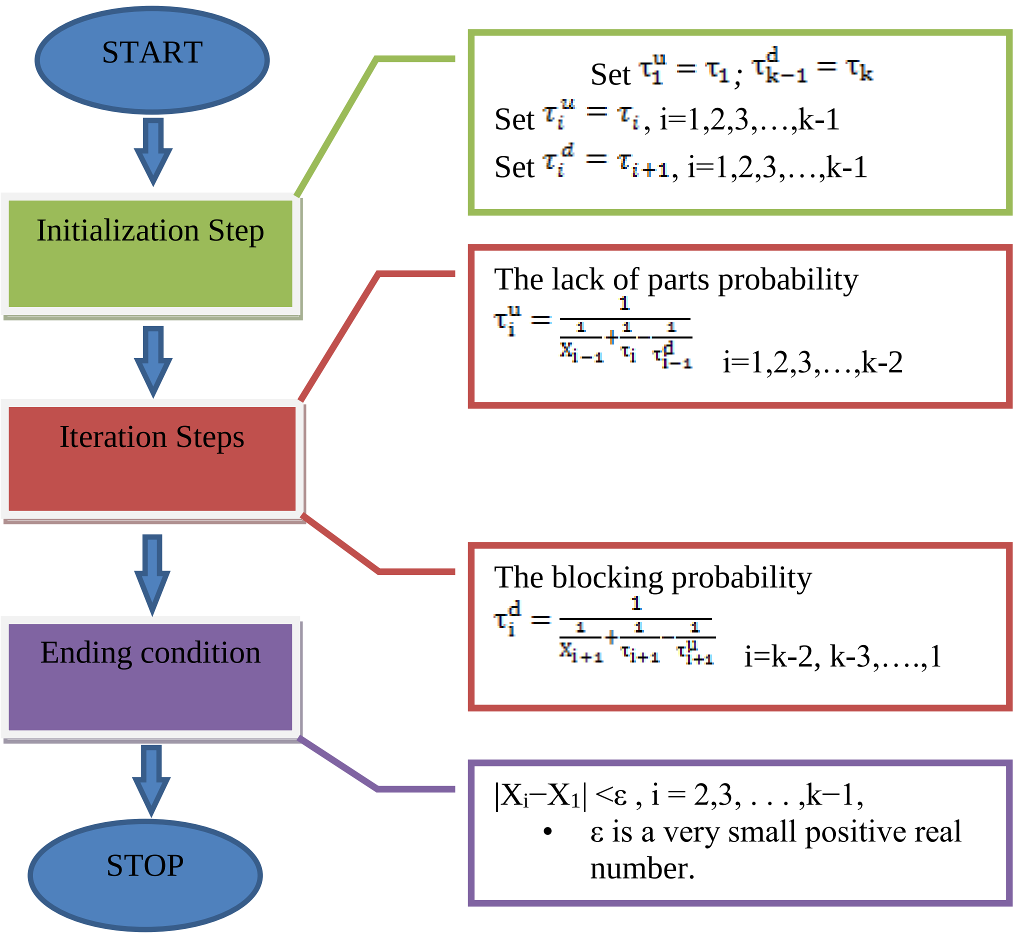

Fig. 5. Production rate of a large flow line (decomposition method)

Applying the algorithm coded in C/C++, the authors can found very easily the production rate for all the system.

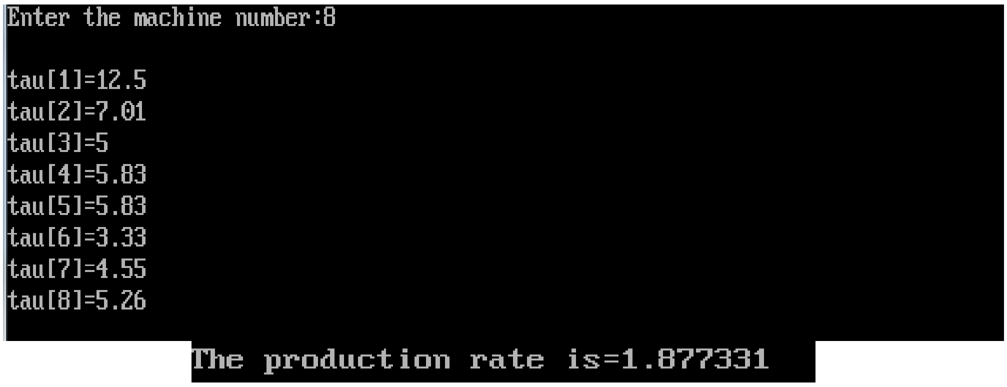

Fig. 6. Results of the model implemented in C++

4. Solution Approach - Strategy for stabilizing production

It is quite difficult to *synchronize* the production with the demand, taking into account that *synchronizing* to unequal demand patterns may not be feasible either for the internal operation or for suppliers. To respond properly to the demand, a company must *stabilize* the manufacturing operation [16,17,18].

The authors propose to explore a specific technique for stabilizing production. It is known that the machines in a flow line are decoupled with the help of buffers. The technique proposed in the article is to increase the production through the reallocation of buffers.

To demonstrate this purpose, gradually, one buffer after another is added to the system.

When we add buffers in the line, within the assumptions some changes will appear.

The states are:

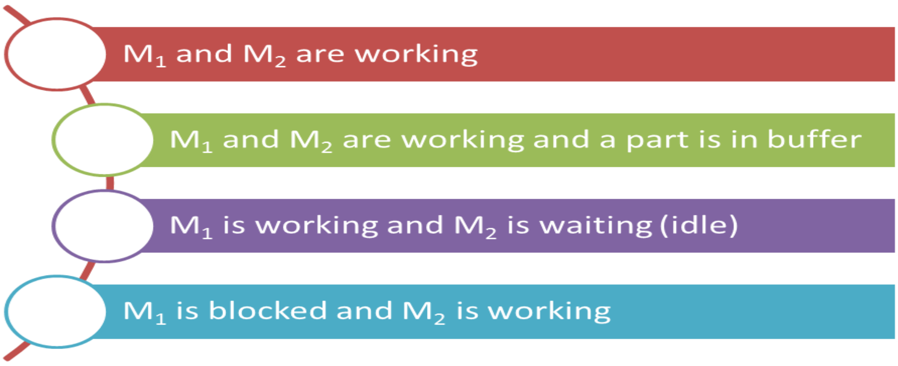

Fig. 7. The states of the machine

Our goal is to synchronize the production with the predictable demand. So, the proposed method is to find how many parts are required in the buffers in order to increase the production rate. The production rate of the system is calculated using stochastic processes and C++ programming (see Fig. 6). The figure above shows a comparison between the production rates of the our flow line and the predictable demand.

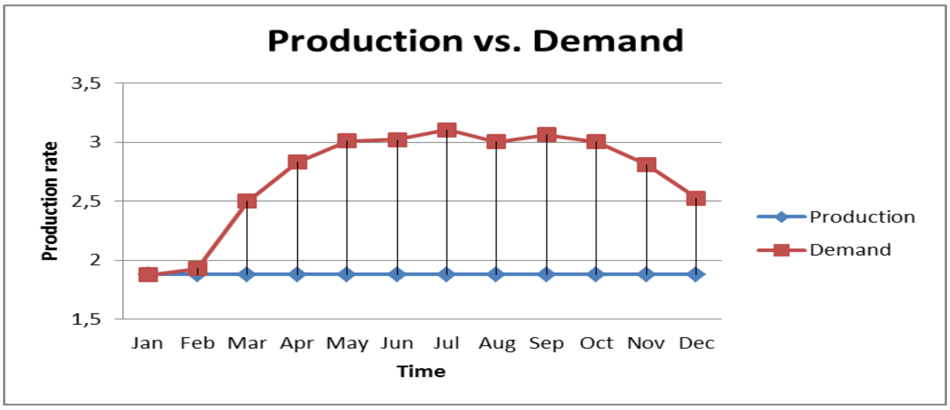

Fig. 8. Comparison between Production and Demand

The production of the system is liniar, but the demand of the products has many fluctuations. The following diagram shows us the number of the parts available in the buffers in order to set a balance between production and demand.

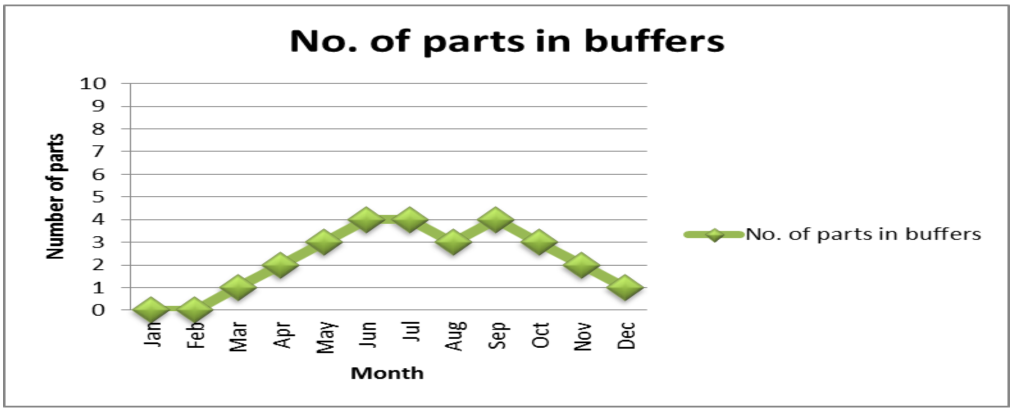

Fig. 9. Number of stocks in the buffers

Using Markov chains and decomposition method, the authors have synchronized the production with the demand of the products by buffer rellocation

(Fig. 10). By simple comparison of the two graphs (Fig. 8 - initial state and Fig. 10 – products synchronization after buffer rellocation), we can see that the balance was established.

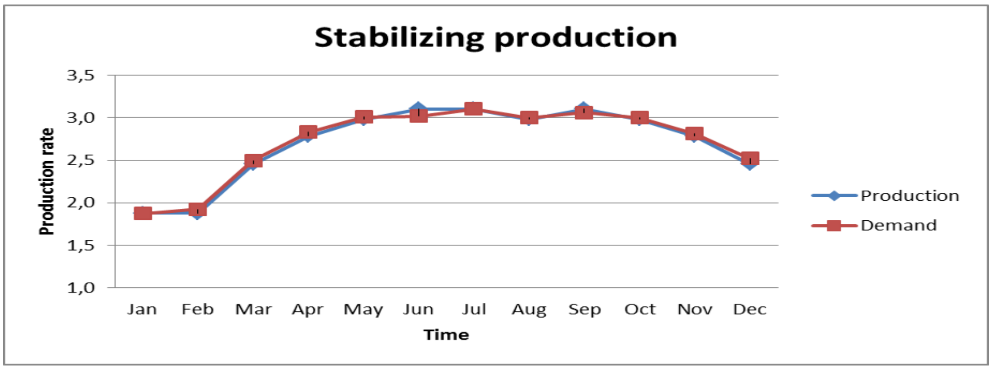

Fig. 10. Stabilizing production

5. Conclusions

The results presented in this paper demonstrate a strategy to stabilize the production of a real case study.

The first goal of this work was to evaluate the production rate of a real-life manufacturing line with an analytical approach in form of the software C++.

The second goal of this work was the production rate optimization of the flow line by restructuring its buffers.

Using the methods, Markov chains and decomposition method, described so far, the authors can offer a solution for both products manufacturing problem, by avoiding intermediary stocks at the same time, and a predictable market demand of these products, getting a balance between the demand and production.

New directions for scientific work in this field must focus on incorporating into the decomposition method the state that a machine can be under failures. The machines may fail while working; a machine fails with a failure rate. When it is down, a machine, if the repairman is working on it, is repaired with at repair rate. An alternate effort is to give the Markov chain the behavior specific to the system, to evaluate the average availability of the system, or to compute the *MTTF* (Mean Time To Failure) and *MTTR* (Mean Time To Repair).

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