2×3 SPACE-TIME BLOCK CODE AS EXTENSION OF ALAMOUTI'S SCHEME FOR IMPROVED PERFORMANCE OF DIGITAL TRANSMISSION OVER MOBILE FADING CHANNELS

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We consider a wireless mobile communication system equipped with three transmit antennas and a single receive antenna. The 2×3 space-time block code used by this system was built by combining two versions of the Alamouti's scheme, that is the best space-time block code known to date. We compared our new code with Alamouti's scheme by way of extensive computer simulations using a one ring scattering radio channel simulator and found an obvious superiority of the new code in terms of BER and FER performance. The price to be paid in order to enjoy such performance is using three transmit antennas instead of only two as is required for Alamouti's space-time block code. We studied the two space-time block codes, ours and Alamouti's, for a distance between transmit antennas of λ/2 and λ, where λ is the wavelength of the carrier frequency. We deem our results as useful for building improved wireless communication systems.

Keywords: Alamouti's scheme, mobile communications, space-time block codes

1. Introduction

The physical layer of telecommunication networks is still an active field of research in order to discover novel coding and modulation techniques [1]. Multiple-input multiple-output (MIMO), also known as antenna diversity, is known to much improve the performance of wireless links. Not only a plethora of papers, but also a lot of books have been already published on this topic. We mention here just a few books [1]-[6].

Antenna diversity means using more than a single antenna at one or both ends of a wireless radio channel. That is, the source signal is simultaneously transmitted out of two or more antennas with some form of modulation like phase-shift keying (PSK) or quadrature-amplitude modulation (QAM).

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It is also advantageous to apply some form of coding. There are two large classes of space-time coded modulation: space-time block coding and space-time trellis coding.

Space-time block coding means that a fixed number of different digital symbols are grouped together into a matrix that is transmitted and decoded independently of any other matrix. Space-time trellis coding is an extension of trellis-coded modulation (TCM) to more than a single transmit antenna. Thus, the transmitter includes a convolutional encoder.

The earliest space-time block code was invented by Alamouti [7]. It shall be described in the next section. It suffices to mention here that it is a half-rate code since it transmits two complex-valued 2D data symbols in two consecutive channel uses by using two transmit antennas. However, full-rate space-time block codes exist, like the celebrated Golden code [8]. It was independently rediscovered in [9]. Remarkably, despite the endeavor of many scientists around the world, the Alamouti space-time block code is still the best found until today, as it was shown in [10]. In this paper, we do not challenge this fact, on the contrary, we extend it by proposing to use three transmit antennas instead of only two. In so doing, we show that a large gain can be obtained in the BER and FER performance. In [11], the authors recursively extend the Alamouti space-time code to $2^n \times 2^n$ antenna elements as well as to unsymmetrical arrays of size $m \times n$, where m and n are powers of 2. Note that in our contribution $n = 3$. In [12], the authors derive the theoretical performance of the Alamouti space-time code in time-varying Rayleigh fading channels with noisy channel estimates. It is interesting to note that, according to this conference paper, the Alamouti space-time code with linear combining scheme is outperformed by the no transmit diversity system at high Doppler frequency or low pilot SNR. In [13], the authors consider the downlink MIMO communication scenario where sparse code multiple access (SCMA) is used for multiuser access. To improve the performance, they make use of the Alamouti space-time block code and of a quasi-orthogonal 4×4 space-time block code.

In [14], the authors propose a full-rate full-diversity space-time block code for 2×2 reconfigurable multiple-input multiple-output systems that require a low complexity maximum likelihood detector. In [15], the authors cleverly exploit the fact that the Alamouti transmit matrix is available in more than a single version in order to introduce space-time super-modulation. The same property was previously used to build super-orthogonal space-time codes [16].

The paper is organized as follows. Section 2 describes the Alamouti's scheme. In Section 3, we describe the 2×3 space-time block code. In Section 4 we present our simulation results, that consists in plots showing the BER and FER performance of the new code as compared to Alamouti's. In Section 5, some conclusions and recommendations are offered.

2. Alamouti's scheme

The signal model we apply is as follows. A string of input bits generated by a digital information source is mapped into complex-valued symbols taken from a 2D signal constellation like PSK and QAM. As usually, a form of modulation is implied but not shown here explicitly. We thus consider a stream of complex-valued baseband symbols $s_0, s_1, s_2, \cdots$ and divide it into blocks of two consecutive symbols $\{s_{2n}, s_{2n+1}\}$, where $n = 0,1,2,\cdots$. Such string of 4D symbols can be transmitted by a single antenna. To improve the performance, however, the same symbols are also transmitted in a different order and form by a second, redundant, antenna. The Alamouti's scheme is best explained using the transmit matrix

$$\boldsymbol{M}_n = \begin{pmatrix} s_{2n} & -a \cdot s_{2n+1}^* \\ s_{2n+1} & a \cdot s_{2n}^* \end{pmatrix}. \tag{1}$$

Of course, eq. (1) is well-known, one cannot speak of Alamouti matrix without mentioning it. Both [10] and the present paper start from Alamouti space-time block code, but they are completely different developments originating from the same root, so to say. In (1), the asterisk denotes complex conjugation and a is a scalar. Usually, it assumes the values +1 or -1. Note that the columns (as well as the rows) of M_n are orthogonal. A first antenna transmits s_{2n} in a first channel use $2n$ and s_{2n+1} in a second channel use $2n$ + 1. A second antenna transmits $-a \cdot s_{2n+1}^*$ in a first channel use and $a \cdot s_{2n}^*$ in a second channel use. We say that this space-time block code is half-rate since it transmits two 2D symbols s_{2n} and s_{2n+1} in two consecutive channel uses with two transmit antennas. At the receiver end, a single antenna suffices, but using two or more receive antennas will improve the performance.

We consider a flat fading mobile channel and denote by $h_i, i = 1,2$, the fading factor from the transmit antenna i to the single receive antenna as sketched in Fig. 1. It is a classical setting and we do not claim any originality with it. It is used only to help entering our main topic.

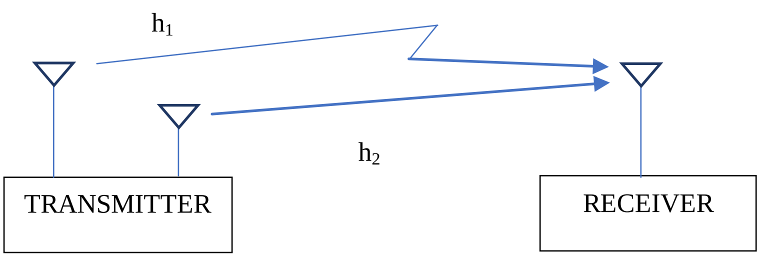

Fig. 1. Sketch of the communication system.

We then have:

$$r_{2n} = h_1 \cdot s_{2n} - a \cdot h_2 \cdot s_{2n+1}^* + z_{2n} \quad (2)$$

$$r_{2n+1} = h_1 \cdot s_{2n+1} + a \cdot h_2 \cdot s_{2n}^* + z_{2n+1} \quad (3)$$

where r_{2n} and r_{2n+1} are the complex-valued received symbols after the demodulator in two consecutive channel uses $2n$ and $2n + 1$, and z_{2n} and z_{2n+1} are noise factors, assumed here to be independent samples of zero-mean complex Gaussian random variables each with variance $Z_0/2$ per dimension. The transmission proceeds by frames. A frame is a block of successive matrices over which the fading is considered as constant. However, the fading can vary from frame to frame at random. It is assumed that the receiver can recover the fading factors h_1 and h_2; we say that it acquires channel state information (CSI). For maximum-likelihood detection, the receiver forms the following estimations:

$$\hat{s}_{2n} = (h_1^* \cdot r_{2n} + a \cdot h_2 \cdot r_{2n+1}^*)/\delta \quad (4)$$

$$\hat{s}_{2n+1} = (h_1^* \cdot r_{2n+1} - a \cdot h_2 \cdot r_{2n}^*)/\delta \quad (5)$$

and

$$\delta = |h_1|^2 + |h_2|^2. \quad (6)$$

The task of the receiver is then to minimize the metrics $m_{2n} = \|s_{2n} - \hat{s}_{2n}\|$ and $m_{2n+1} = \|s_{2n+1} - \hat{s}_{2n+1}\|$. Note that the decision is made independently for the symbols s_{2n} and s_{2n+1}, which is a significant advantage of the Alamouti's scheme.

3. 2×3 Space-time block code

Consider the following transmit matrix:

$$\boldsymbol{M}_n = \begin{pmatrix} s_{2n} & s_{2n+1}^* & -s_{2n+1}^* \\ s_{2n+1} & -s_{2n}^* & s_{2n}^* \end{pmatrix}. \quad (7)$$

Now, a third transmit antenna will send out the elements of the third column of the matrix. The fading factor from it to the reception side is denoted by h_3. This is illustrated in Fig. 2.

We thus have

$$r_{2n} = h_1 \cdot s_{2n} + h_2 \cdot s_{2n+1}^* - h_3 \cdot s_{2n+1}^* + z_{2n} \quad (8)$$

$$r_{2n+1} = h_1 \cdot s_{2n+1} - h_2 \cdot s_{2n}^* + h_3 \cdot s_{2n}^* + z_{2n+1}. \quad (9)$$

This system is equivalent to:

$$r_{2n} = h_1 \cdot s_{2n} + (h_2 - h_3) \cdot s_{2n+1}^* + z_{2n} \tag{10}$$

$$r_{2n+1} = h_1 \cdot s_{2n+1} - (h_2 - h_3) \cdot s_{2n}^* + z_{2n+1}. \tag{11}$$

By comparing to equations (4)-(6), it is easy to see that the estimates are given by

$$\hat{s}_{2n} = [h_1^* \cdot r_{2n} - (h_2 - h_3) \cdot r_{2n+1}^*]/\delta \tag{12}$$

$$\hat{s}_{2n+1} = [h_1^* \cdot r_{2n+1} + (h_2 - h_3) \cdot r_{2n}^*]/\delta \tag{13}$$

where:

$$\delta = |h_1|^2 + |h_2 - h_3|^2. \tag{14}$$

Standards and regulations usually limit the power of the signal radiated by a transmit antenna in the air. Let us denote by P_1, P_2, and P_3 the power of a 2D data symbol when using a single antenna, and two and three antennas, respectively. Then, in order to keep constant the transmit power, we should have $P_1 = 2P_2 = 3P_3$.

That is, when using three transmit antennas instead of only two, each one of the three antennas must radiate a slightly smaller electromagnetic power. This smaller power is overcompensated by a larger antenna diversity.

Assuming the receiver acquires perfect CSI, at first glance, the single source of errors is the noise. However, this is not so. The fading factors h_1, h_2 and (in case of the new code) h_3 are random variables and thus it happens that one of them or even two or three are small or even vanishing. The probability of such events is small, but not zero. In such a poor condition of the channel, correct decoding is harder or even impossible.

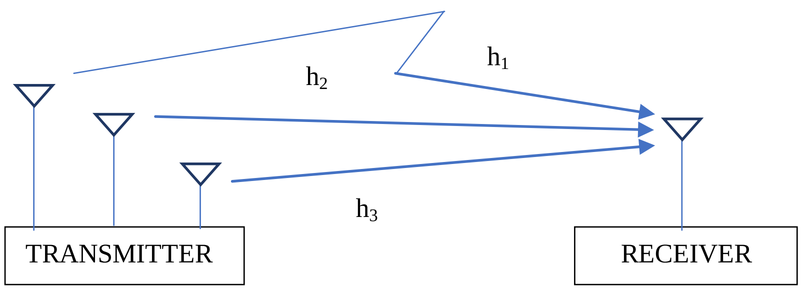

Fig. 2. Sketch of the transmission system using the new space-time block code

Note that the probability of both h_2 and h_3 to be in such a poor condition is much smaller than the probability that only one of them, but not both, to have a small value. This is the reason why we expected that the performance of (7) to be better than that of (1). We show that this is indeed the case in the next section.

4. Simulation results

We used a space-time channel simulator developed for MIMO channels based on the geometrical one-ring scattering model [16]. The transmission is assumed to proceed by frames. A *frame* is a block of consecutive digital symbols over which the fading is assumed to be constant. The fading varies randomly from frame to frame. It is also assumed that the receiver can acquire perfect *channel state information* (CSI), that is, it can measure in real time the values of the fading factors $h_1, h_2,$ and h_3. In this setting, the main source of errors is the additive noise, modelled as additive white Gaussian noise (AWGN) with zero mean and variance $Z_0/2$ per dimension. Note that the fading contributes to errors as well, since for vanishing fading coefficients, it is hard if not impossible for the receiver to correctly demodulate the received signal. For simulations, we have used MATLAB programs written by the authors. In our illustrative example, 16QAM modulation format was applied. The signal constellation is shown in Fig. 3. Note that this is a classical figure (see for instance Fig. 1 of [16] and Figs. 2.4 and 3.3 of [6]).

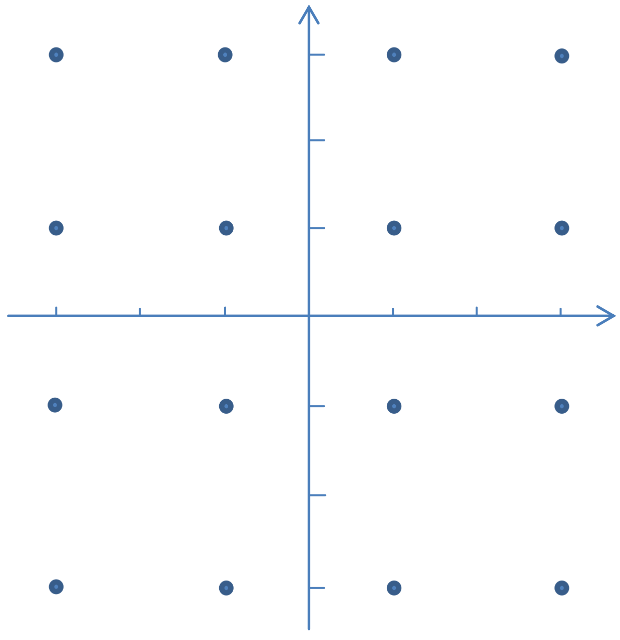

Fig. 3. Sixteen-point 2D signal constellation with points labeled according to the Gray code

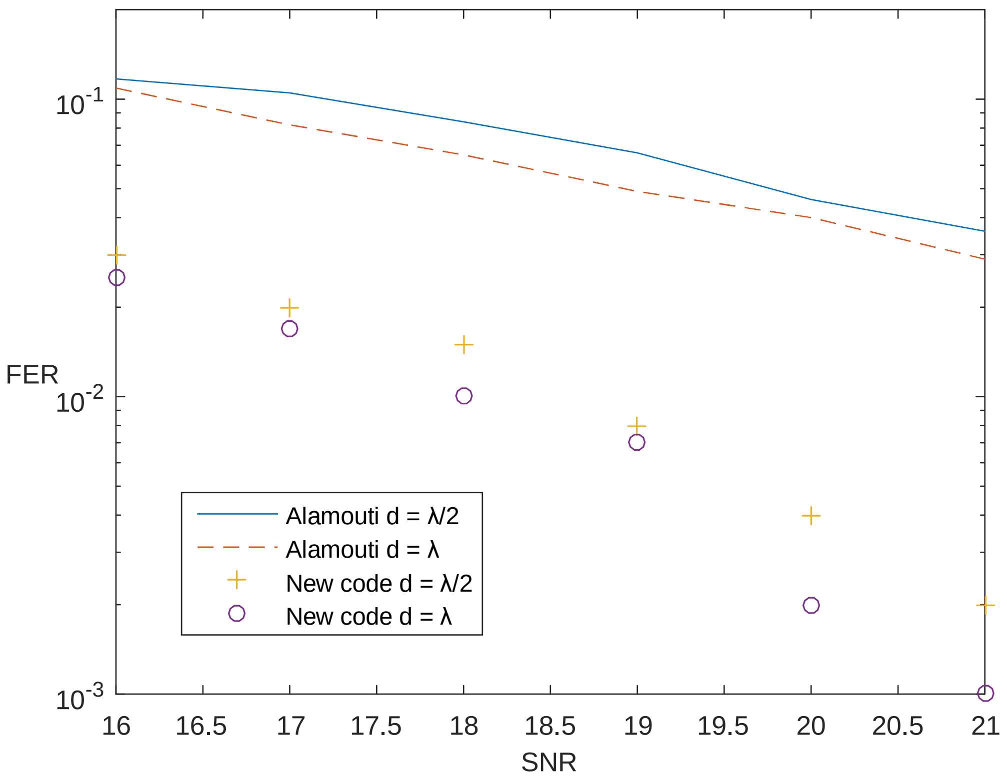

Fig. 4. FER performance of the new code as compared to Alamouti's for d = λ/2 and d = λ.

It is no originality whatsoever with it. We included it for the convenience of the reader. For each value of the signal-to-noise ratio (SNR), 1000 frames were transmitted, where a frame contains 65 transmit matrices. The value of 65 was selected in order to comply to the literature. The results of the simulations are shown in Fig. 4 for FER and Fig. 5 for BER performances for two values of the distance between the transmit antennas: d = λ/2 and d = λ, where λ is the wavelength of the carrier frequency. Since the comparison was made to the classical Alamouti scheme, the results for this one are borrowed from [1]. They are bound to be identical.

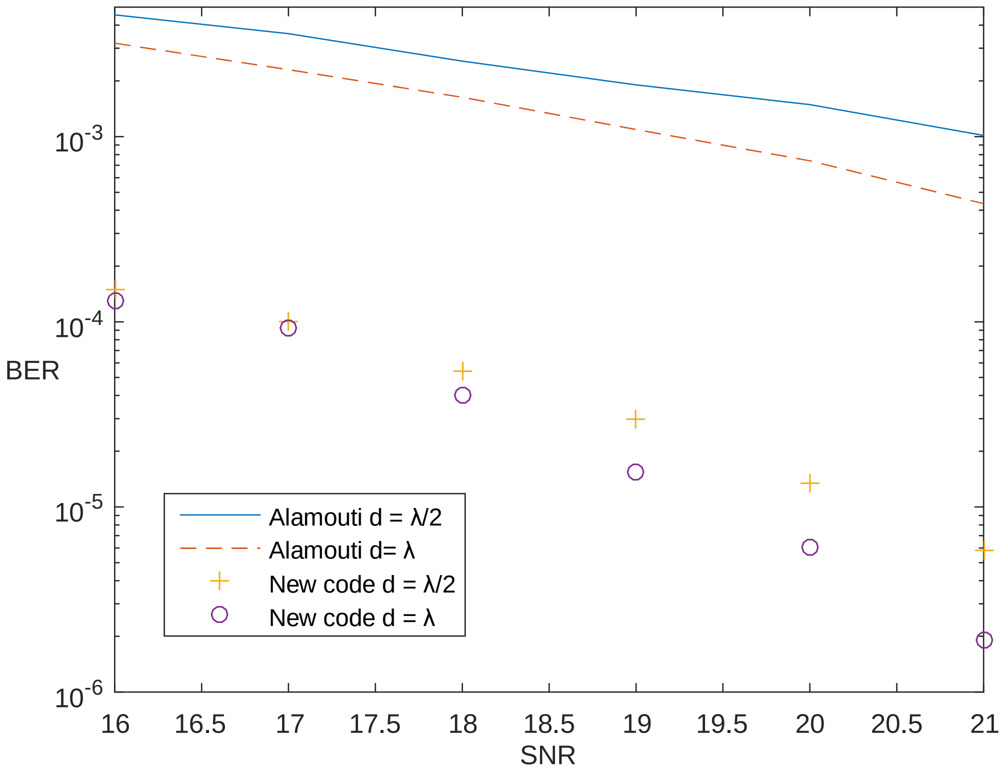

Fig. 5. BER performance of the new code as compared to Alamouti's for d = λ/2 and d = λ.

The superiority of the new space-time block code is obvious from these plots. However, we stress that, in fact, the comparison is not quite fair, since the new code provides a diversity order of 3, while the Alamouti was designed for a diversity order of only 2. Nevertheless, no other term of reference would be more fit than this one, with Alamouti's scheme whose development our new space-time block code is.

5. Conclusion

Despite its apparent simplicity, the Alamouti matrix enjoys many remarkable properties, maybe not all of them being fully exploited to date. The basis of this richness is the orthogonality of the two columns (as well as that of the two rows). By keeping fixed one of the columns, there is an infinity of columns that provide for the orthogonality of the two columns of the matrix. In most of the applications, only one of them will do. In some other applications, like in [15] and [16], using two of them can effectively enhance the transmission rate. In our paper, we have combined two 2×2 versions of the Alamouti's scheme into a 2×3 space-time block code and shown by way of computer simulations that its FER and BER performance is clearly superior. We think that our work will be useful in cases where it is worth adding a supplementary transmit antenna in order to much improve the performance of wireless communications systems. The price to be paid for this gain is using three transmit antennas instead of only two. Of course, more transmit antennas translates in a larger diversity gain. However, until now, only an even number of transmit antennas has been considered in the literature. Moreover, a supplementary number of channel uses is necessary for this purpose. This is not the case with our proposal, since only two consecutive channel uses are required, as in the case of the classical Alamouti matrix.

In conclusion, the main point of originality we claim is using a single supplementary transmit antenna that greatly improves the performance of the Alamouti's scheme.

R E F E R E N C E S

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