ACCELERATION OF THE MOVEMENT OF THE INSTANTANEOUS CENTER OF ROTATION ON THE FIXED CENTRODE

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In the work [1] the expression of the speed of movement of the instantaneous center of rotation (I.C.R.) was established on the fixed centrode, which has been customized for the case of the antiparallelogram mechanism.

We aim to extend the problem addressed by determining the I.C.R. acceleration on the fixed centrode. We have customized the layouts presented also for the antiparallelogram mechanism.

Keywords: parallel plane movement of the rigid, instantaneous center of rotation (I.C.R.), antiparallelogram mechanism

1. Introduction

In the technique are sometimes used in the structure of machines and noncircular toothed wheels. In the work [2] for the dividing contour of noncircular toothed wheels, the term centrode is used by extension.

The acceleration of I.C.R. movement on the fixed centrode may also suggest the variation of the force of interaction between the teeth of noncircular tooth wheels.

In the kinematics of the plane-parallel motion of the rigid it is convenient to establish the components and the magnitude of the acceleration of a point of the rigid by reference to the Frenet reference system.

For this it is necessary to know the expressions of the analytical quantity of the velocity of the point and of the radius of curvature of its trajectory.

2. Components of the speed acceleration I.C.R. on the fixed centrode

Size of I.C.R. travel speed on fixed centrode ,v_{1I} is given by the relationship:

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$$v_{1I} = \sqrt{v_{Ix_1}^2 + v_{Iy_1}^2} = \sqrt{\dot{\xi}_1^2 + \dot{\eta}_1^2} \tag{1}$$

In which: ξ_1 and η_1 represents the I.C.R. coordinates in relation to the reference fixed system $x_1O_1y_1$.

The tangential component of the movement acceleration of the I.C.R. on the fixed centrode is obtained by deriving in relation to time the relation (1) [3,4]:

$$a_{1I}^{\tau} = \dot{v}_{1I} = \frac{\dot{\xi}_1\ddot{\xi}_1 + \dot{\eta}_1\ddot{\eta}_1}{\sqrt{\dot{\xi}_1^2 + \dot{\eta}_1^2}} \tag{2}$$

The normal component of this acceleration is given by the relationship [3-6]:

$$a_{1I}^{\nu} = \frac{v_{1I}^2}{\rho_{c1I}} \tag{3}$$

In which:

ρ_{c1I} – represents the radius of curvature of the fixed centrode corresponding to position I.C.R. at the moment considered.

The analytical expression of the radius of curvature of a point of the trajectory, in the general case is [3-6].

$$\rho_c = \frac{|\bar{v}|^3}{|\bar{v} \times \bar{a}|} \tag{4}$$

The magnitude of the I.C.R. movement acceleration on the fixed centrode results from:

$$a_{1I} = \sqrt{(a_{1I}^{\tau})^2 + (a_{1I}^{\nu})^2} \tag{5}$$

3. Example of calculation

We will establish the component expressions of the movement acceleration of the fixed centrode I.C.R. , also for the case of the antiparallelogram mechanism, presented in the paper [1] and figure 1.

Centrodes for this mechanism may represent the dividing contour of two elliptical noncircular gears.

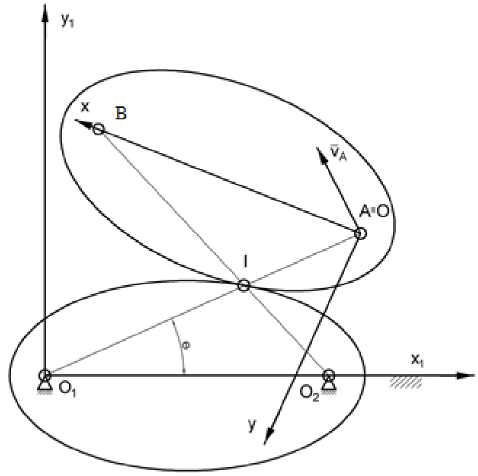

Fig. 1. The antiparallelogram mechanism -centrodes.

For the antiparallelogram mechanism in Fig. 1, is considered the motor element, the crank $O_1A = 2l$. The variation of the angle $\theta = \theta(t)$,defines the law of motion of the motor element. Noted $O_1O_2 = AB = 2b$ and $O_1A = O_2B = 2l$ $(2l > 2b)$. In this situation for the bar AB which is in plane-parallel motion, I.C.R. is at the intersection of the bars O_1A with O_2B which generates the fixed and mobile centrode, two congruent ellipses having the focus in the points O_1 O_2 respectively in A and B, with the semiaxes l and $\sqrt{l^2 - b^2}$.

In order to determine the normal component of the I.C.R. movement acceleration on the fixed centrode, the curvature expression of the fixed centrode will have to be established.

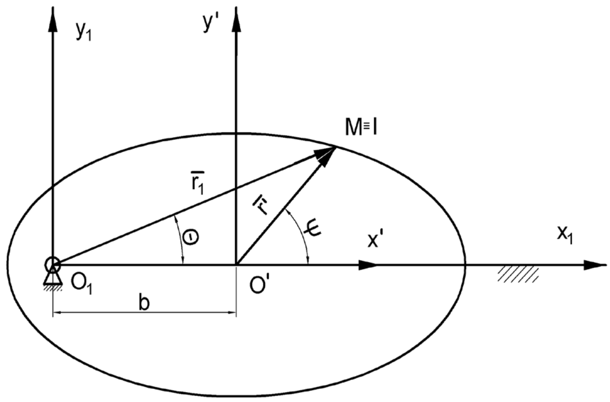

Fig.2. Fixed centrode.

To determine the radius of curvature of the fixed centrode we chose two reference systems shown in Fig. 2. For the system $O_1\, x_1 y_1$, has its origin O_1 in the focus of the ellipse. The second reference system originates in the center O' of the ellipse and the axis $O'y'$ superimposed on the small axis of symmetry of the ellipse. Noted: $\bar{r}_1 = \overline{O_1 M}$, $\bar{r}' = \overline{O'M}$, and angles $\theta = \sphericalangle\, MO_1x_1$ respectively $\psi = \sphericalangle MO'\, x'$.

You can write geometric relationships (see also [7]):

$$x_{1M} = b + l\cos\psi = r_1\ \cos\theta \tag{6a}$$

$$y_{1M} = \sqrt{l^2 - b^2}\ \sin\psi = r_1 \sin\theta \tag{6b}$$

After summarizing the squares of the relationships (6a) and (6b), the expression is obtained from elementary calculations:

$$r_1 = (l + b\cos\psi) \tag{7}$$

Returning to relations (6a) and (6b), with the help of relation (7) we can determine:

$$\cos\psi = \frac{l\cos\theta - b}{l - b\cos\theta} \tag{8}$$

Respectively:

$$\sin\psi = \frac{\sqrt{l^2 - b^2}\,\sin\theta}{l - b\cos\theta} \tag{9}$$

As it is known the expression of the radius of curvature of the trajectory can be obtained by means of the relation [3,4,6]:

$$\rho_{1c} = \frac{|\bar{v}|^3}{|\bar{v} \times \bar{a}|} = \frac{(\dot{x}_{1M}^2 + \dot{y}_{1M}^2)^{\frac{3}{2}}}{|\dot{x}_{1M}\ddot{y}_{1M} - \dot{y}_{1M}\ddot{x}_{1M}|} \tag{10}$$

By successively deriving in relation to time the coordinates I.C.R. x_{1M} și y_{1M} the relations are obtained:

$$\dot{x}_{1M} = -l\,\dot{\psi}\,\sin\psi \tag{11a}$$

$$\dot{y}_{1M} = \sqrt{l^2 - b^2}\,\dot{\psi}\ \cos\psi \tag{11b}$$

$$\ddot{x}_{1M} = -l\ddot{\psi}\sin\psi - l\dot{\psi}^2\cos\psi \tag{12a}$$

$$\ddot{y}_{1M} = \sqrt{l^2 - b^2}\,\ddot{\psi}\cos\psi - \sqrt{l^2 - b^2}\,\dot{\psi}^2\sin\psi \tag{12b}$$

Which introduced in relation (10) leads to:

$$\rho_{1c} = \frac{(l^2 - b^2 + b^2\sin^2\psi)^{\frac{3}{2}}}{l\sqrt{l^2 - b^2}} \tag{13}$$

Taking into account the relation (9) the relation of the radius of curvature of the trajectory as a function of the angle θ becomes:

$$\rho_{1c} = \frac{(l^2-b^2)(l^2+b^2-2lb\,cos\theta)^{\frac{3}{2}}}{l(l-b\,cos\theta)^3} \quad (14)$$

The tangential component of the I.C.R. movement acceleration on the fixed centrode becomes:

$$a_{1I}^{\tau} = \dot{v}_{1I} =$$

$$= (l^2 - b^2)\frac{\ddot{\theta}(l^2+b^2-2lbcos\theta)(l-bcos\theta) + b\dot{\theta}^2 sin\theta(-l^2-2b^2+3lbcos\theta}{(l-bcos\theta)^3\sqrt{l^2+b^2-2lbcos\theta}} \quad (15)$$

The normal composition of the I.C.R. movement acceleration on the fixed centrode is given by the relation (3) which particularized for our calculation example leads to the expression:

$$a_{1I}^{v} = \frac{(l^2-b^2)l\dot{\theta}^2}{(l-b\,cos\theta)\sqrt{l^2+b^2-2lbcos\theta}} \quad (16)$$

And the magnitude of the movement acceleration of I.C.R. on the fixed centrode results:

$$a_{1I} = \sqrt{(a_{1I}^{\tau})^2+(a_{1I}^{v})^2} \quad (17)$$

4. Numerical application

For a didactic antiparallelogram mechanism, a program was drawn up in the C++ language with the help of which the following sizes were calculated for a complete a_{1I}^{τ}cycle; a_{1I}^{v}; a_{1I}(formulae (15); (16); and (17) for a uniform mode movement of the engine element (OA bar in Fig.1).

The input data were l = 64 mm; b = 49 mm; $\theta = \frac{\pi}{2}\,t$ (laugh).

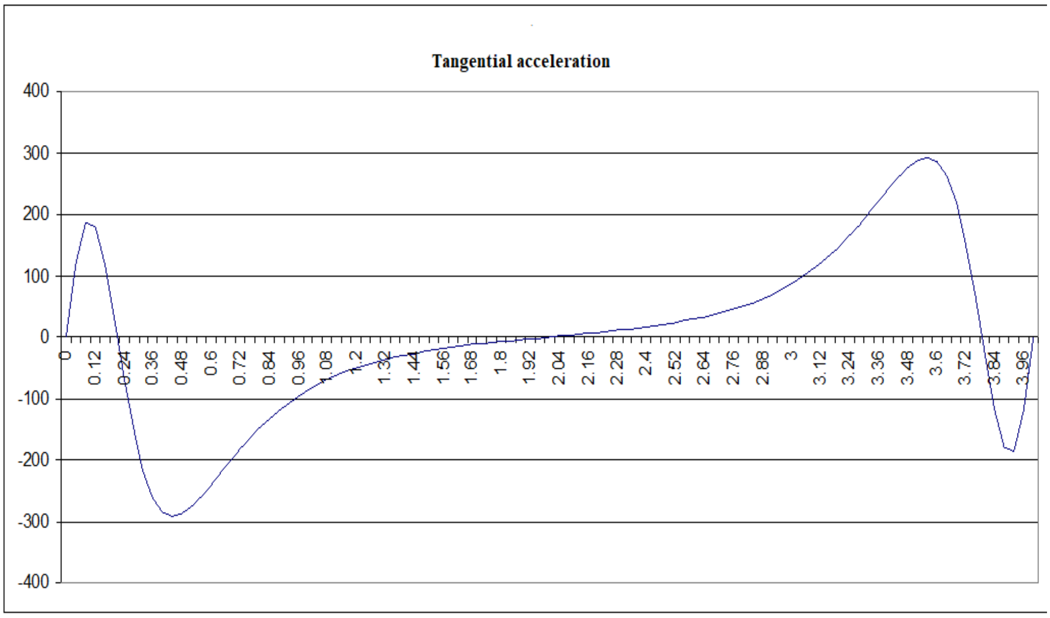

Fig.3 Variation in the size of the tangential component of the C.I.R. movement acceleration on the fixed centroid for a full cycle.

Figures 3, 4 and 5 are shown to graphs of the tangent acceleration variation a_{1I}^{τ}; of normal acceleration a_{1I}^{ν} respectively, of the total acceleration of the I.C.R. movement on the fixed centrode for a full cycle.

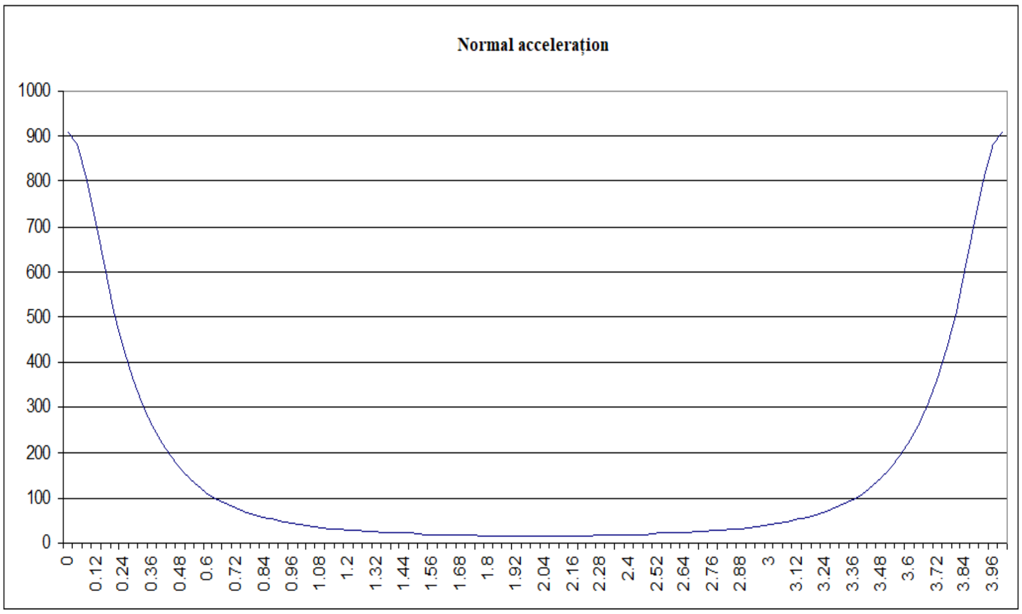

Fig.4 Variation in the size of the normal component of the movement acceleration I.C.R on the fixed centrode for a full cycle.

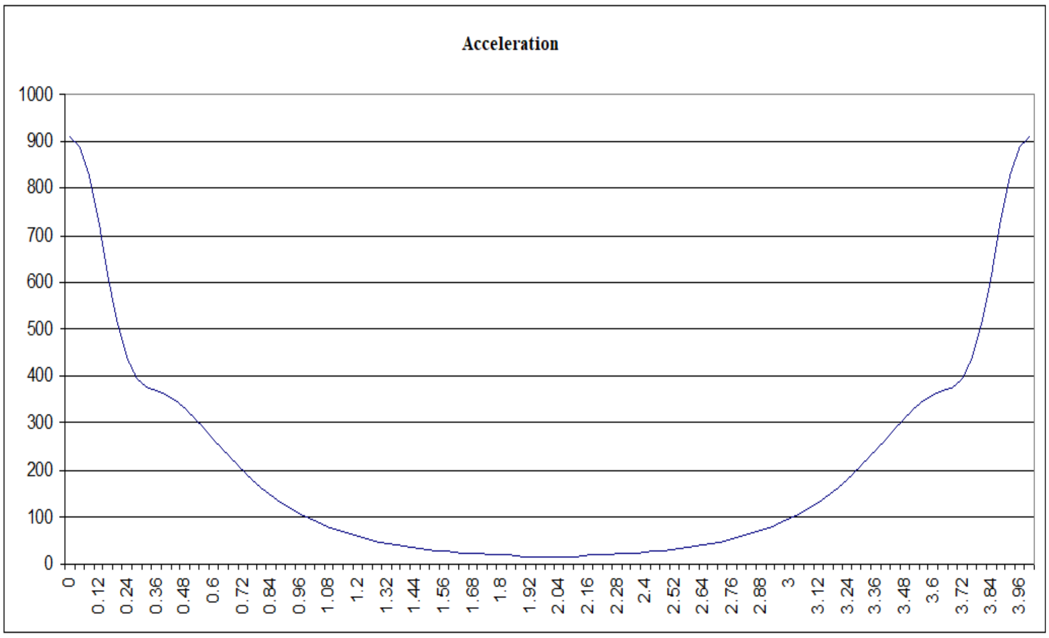

Fig.5 Variation in the total acceleration size of I.C.R. motion on the fixed centrode for a full cycle.

5. Conclusions

The component expressions and the magnitude of the I.C.R. movement acceleration on the fixed centrode were established.

It is also interesting a numerical analysis for the particular case of studying the variation of the quantities presented for a complete cycle of movement of the antiparallelogram mechanism.

This analysis may be the subject of another article, in which the situations of the regime movement $(\theta = \omega_0 \cdot t)$ and the transition movement from the rest movement to the regime movement are treated $\left(\theta = \frac{\varepsilon_0 \cdot t^2}{2}\right)$. A numerical application was carried out for the uniform softening of the motor element for a full cycle.

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