IDENTIFICATION OF RESPONSE FACTORS IN TRANSIENT CONDUCTION PROBLEM, FOR A HOMOGENEOUS ELEMENT EXPOSED TO ENVIRONMENT CONDITIONS, IN HEATING LOADS COMPUTATIONS

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Current paper aims to identify the internal flow using conductive heat transfer function (CTF) for homogeneous element, without internal heat sources. The CTF has a wide applicability, being extended for the design of heating, cooling and ventilation (HVAC) systems. For the simulation's purposes, we used weather conditions as input data, collected by the monitoring system of the passive house. The energy performance algorithm of the simulated element allowed us to validate the model and to extend our research in order to determine the energy needs of the entire building.

Keywords: Laplace transform, conduction transfer function, heat flux, differential equation

1. Introduction

Energy efficiency, thermal comfort and automated HVAC systems create valences for smart buildings and represents a priority for future house owners. According to Romanian national strategy for 2007-2020 it was identified that the residential sector has the potential to reduce energy consumption between 35-50%.

Energy requirements and fuel consumption of HVAC systems directly affect a building's operating cost and indirectly affect the environment [1]. Predicting energy performance of the building in dynamic conditions, development of techniques for heating, ventilation and air conditioning simulation represents a current concern, which started in the early '60s [2].

Heat conduction through buildings' construction elements has the most important influence on designing HVAC system. Therefore, a solution can be formulated through CTF models.

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The 2009 Ashrae Handbook Fundamentals divided buildings modeling methods in two categories: **forward modeling** and **data-driven modeling**. [3]
The *forward* approach is based on the fact that all physical description of the building is known in details. This means that buildings geometry, construction materials, whether condition, geographic location, type of HVAC system are subject to input variables.

In order to ensure a better accuracy, models tend to become more complex, especially now when computing power is cheap. The peak and average energy use of such a building can then be predicted or simulated by the forward simulation model. Thus, this approach is ideal in the preliminary design and analysis stage and it is most often used then.

The *data-driven* approach lies on the fact that all input and output variables are known and measured. Based on known data a mathematical description of the system can be developed.

Choosing the right modeling method is driven by the project requirements. The method should give sufficient information with high accuracy. In any case there are some general lines, which can be applied to all models: accuracy, ease of use, versatility, cost effective and these should provide results at a great speed.

Nomenclature	
A,B,C,D	transmission matrix element
a	thermal diffusivity (m^2s^{-1})
c_p	specific heat ($J\ kg^{-1}\ K^{-1}$)
λ	thermal conductivity ($W\ m^{-1}\ K^{-1}$)
ρ	density ($kg\ m^{-3}$)
L	thickness (m)
s	Laplace variable
T	temperature (^{0}C or K)
t	time (s or h)
δ	time (s)

Another classification was presented by Gupta et al 1971 for space loads who divided predicting methods in three classes [4]: (1) *numerical methods*, (2) *harmonic methods*, and (3*) response factor methods*.

Numerical methods use lumped parameter approximations to the heat conduction equation and were originally implemented using resistor-capacitor circuits on analog computers.

Harmonic methods can be used to solve the heat conduction equation if the boundary conditions are represented as periodic functions. These methods require that the building heat transfer parameters, including convection coefficients, be constant with time and that radiant heat transfer be linearized.

Response factor methods represent yet a third approach to solving the heat conduction equation. The major advantages of these methods are that they are not numerical in the sense of finite differences techniques, and they do not require that the heat conduction boundary conditions be periodic and linear.

The current paper aims to validate the energy performance of a building's element using a response factor method for solving heat conduction equation for one-dimensional heat transfer.

For the simulation's purposes, we have used weather conditions as input data, collected by the monitoring system of the passive house. Finally, the results have shown that the heat flux varies depending on external climatic conditions.

2. Model foundation

The determination of heat conduction through walls is described by the general equation of heat conduction for a transient regime without any internal heat sources and Fourier's law related to heat fluxes and temperature gradient:

$$c_p(T)\rho(T)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z \frac{\partial T}{\partial z}\right) \quad (1)$$

$$q = -\lambda\left(\frac{\partial T}{\partial x};\frac{\partial T}{\partial y};\frac{\partial T}{\partial z}\right) \quad (2)$$

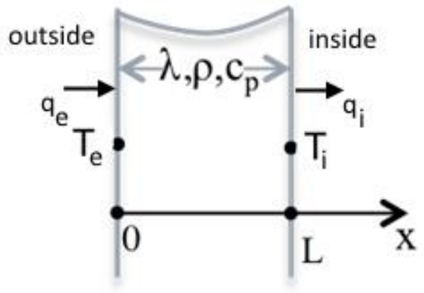

Fig. 1. Detail of one-dimensional heat conduction problem in a homogeneous wall

Considering one dimensional conduction heat transfer (Fig 1.) the above equation can be written as follows:

$$\frac{\partial T}{\partial \tau} c_p \rho = -\frac{\partial q}{\partial x} \quad (3)$$

$$\frac{q}{\lambda_x} = -\frac{\partial T}{\partial x} \quad (4)$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{c_p \rho}{\lambda_x}\frac{\partial T}{\partial \tau} \quad (5)$$

Solving the equation (5) will provide the conductive transfer function (CTF) model of heat conduction through building's wall. Stephenson and Mitalas developed the CTF using frequency-domain approach [5]. The method is based on solving the heat conduction equation and boundary conditions using the Laplace transformation and applying Z-transform for time sampling at regular intervals.

For a homogeneous wall with constant thermal properties (λ, c_p, ρ) using Laplace transform function, the equation (5) becomes:

$$-\frac{\partial T(x,s)}{\partial x} = \frac{q(x,s)}{\lambda_x} \tag{6}$$

$$-\frac{\partial q(x,s)}{\partial x} = sT(x,s)c_p\rho \tag{7}$$

$$\frac{\partial^2 T(x,s)}{\partial x^2} = sT(x,s)\frac{c_p\rho}{\lambda_x} \tag{8}$$

Noting $a = \frac{\lambda_x}{c_p\rho}$ and assuming that T (x, 0) = 0, the equation (8) becomes:

$$\frac{\partial^2 T(x,s)}{\partial x^2} - sT(x,s)\frac{1}{a} = 0 \tag{9}$$

The solution for the differential equation (9) is:

$$T(x,s) = A\cosh\left(x\sqrt{\frac{s}{a}}\right) + B\sin\left(x\sqrt{\frac{s}{a}}\right) \tag{10}$$

The transformed equation (6) can be written using equation (10) as:

$$q(x,s) = -\lambda_x\sqrt{\frac{s}{a}}A\sinh\left(x\sqrt{\frac{s}{a}}\right) - \lambda_x\sqrt{\frac{s}{a}}B\cosh\left(x\sqrt{\frac{s}{a}}\right) \tag{11}$$

Considering the heat flux ($q_e(s)$, $q_i(s)$) and temperatures ($T_e(s)$, $T_i(s)$) at the surfaces of the wall (x=0 and x=L) and substituting A and B, the equation (10) and (11) can be written as:

$$T_i(s) = \left(\cosh\left(L\sqrt{\frac{s}{a}}\right)\right)T_e(s) + \left(\frac{1}{\lambda_x\sqrt{\frac{s}{a}}}\sinh\left(L\sqrt{\frac{s}{a}}\right)\right)q_e(s) \tag{12}$$

$$q_i(s) = \left(\lambda_x\sqrt{\frac{s}{a}}\sinh\left(L\sqrt{\frac{s}{a}}\right)\right)T_e(s) + \left(\cosh\left(L\sqrt{\frac{s}{a}}\right)\right)q_e(s) \tag{13}$$

In scientific literature [4-7] based on equation (12) and (13) a related matrix expression is defined in the form:

$$\begin{bmatrix} T_i(s) \\ q_i(s) \end{bmatrix} = \begin{bmatrix} A_1(s) & B_1(s) \\ C_1(s) & D_1(s) \end{bmatrix}\begin{bmatrix} T_e(s) \\ q_e(s) \end{bmatrix} \tag{14}$$

By noting thermal resistance and thermal capacitance per unit area with $R_1C_1 = \frac{L^2}{a}$, the transmission matrix can be written as follows:

$$\begin{bmatrix} A(s)_1 & B(s)_1 \\ C(s)_1 & D(s)_1 \end{bmatrix} = \begin{bmatrix} \cosh(\sqrt{sR_1C_1}) & \frac{R_1}{\sqrt{sR_1C_1}}\sinh(\sqrt{sR_1C_1}) \\ \frac{\sqrt{sR_1C_1}}{R_1}\sinh(\sqrt{sR_1C_1}) & \cosh(\sqrt{sR_1C_1}) \end{bmatrix} \tag{15}$$

The determinant of the equation (15) is $\cosh^2(\sqrt{sR_1C_1}) - \sinh^2(\sqrt{sR_1C_1}) = 1$. The equation (14) is valid for a single layered wall. For a multilayered wall the square matrix is multiplied by the equivalent square matrix for each layer, as shown in equation (16).

$$\begin{bmatrix} T_i(s) \\ q_i(s) \end{bmatrix} = \begin{bmatrix} A_1(s) & B_1(s) \\ C_1(s) & D_1(s) \end{bmatrix} \cdots \begin{bmatrix} A_n(s) & B_n(s) \\ C_n(s) & D_n(s) \end{bmatrix} \begin{bmatrix} T_e(s) \\ q_e(s) \end{bmatrix} \tag{16}$$

Knowing the inside and outside temperatures, the expression (14) can be written as follows:

$$\begin{bmatrix} q_1(s) \\ \\ q_e(s) \end{bmatrix} = \begin{bmatrix} \frac{B_1(s)}{D_1(s)} & \frac{-1}{B_1(s)} \\ \frac{1}{B_1(s)} & \frac{-A_1(s)}{B_1(s)} \end{bmatrix} \begin{bmatrix} T_i(s) \\ \\ T_e(s) \end{bmatrix} \tag{17}$$

Hittle introduces a procedure to solve the equation (17) by generating a response factors by applying a unit triangular temperature pulse to the inside and outside surface of the multilayered slab. The response factors are defined as the discretized heat fluxes on each surface due to both the outside and inside temperature pulse. The response factors are an infinite series. Hittle also described an algebraic operation to group response factors into CTFs, and truncate infinite series of response factor by the introduction of flux histories coefficients [8].

To calculate conductive heat flux on inside surface, the boundary condition was set to ramp unit with the slope of $\frac{1}{s^2}$. Hence, if this transformed boundary condition is applied to outside surface of a single-layer wall while the temperature on inside surface 'i' is held at zero, equation (17) for $q_i(s)$ becomes:

$$q_{X\_1}(s) = \frac{1}{s^2}\frac{D_1(s)}{B_1(s)} \tag{18}$$

To calculate conductive flux on outside surface, the boundary condition was set to ramp unit with the slope of $\frac{1}{s^2}$. Hence, if this transformed boundary condition is applied to inside surface of a single-layer wall while the temperature on outside surface is held at zero, equation (17) for $q_1(s)$ becomes:

$$q_{Y\_1}(s) = -\frac{1}{s^2}\frac{1}{B_1(s)} \tag{19}$$

The classical approach to this inversion consists of using the well-know Riemann inversion formula. The inverse Laplace transform of the equations (18) and (19) yields heat flux variation on inside surface and outside surface, as a function of time $q(t)$:

$$q(t) = \frac{1}{2\pi j}\int_{\Gamma} q(s)e^{st}\,ds \tag{20}$$

where $\Gamma=(c-i\infty,c+i\infty), c\in\mathbb{R}$ is a path (named Bromwich contour) parallel to the imaginary axis located to the right of every singularity of $q(s)$.

The above integral can be solved by summing the residues at the poles of $q(s)e^{st}$. Summing up the residues we can write response factors with the pulse start time $t = \delta$:

$$X_1 = \frac{1}{\delta}\left[\frac{C_1}{3} + \frac{\delta}{R_1} - 2\sum_{n=1}^{\infty}\frac{C_1 e^{-\delta\beta_n}}{n^2\pi^2}\right] \tag{21}$$

$$X_2 = -\frac{1}{\delta}\left[\frac{C_1}{3} + 2\sum_{n=1}^{\infty}\frac{C_1(e^{-2\delta\beta_n} - 2e^{-\delta\beta_n})}{n^2\pi^2}\right] \tag{22}$$

$$X_m = -\frac{1}{\delta}\left[2\sum_{n=1}^{\infty}\frac{C_1((e^{-m\delta\beta_n}(1 - e^{\delta\beta_n})^2)}{n^2\pi^2}\right] \tag{23}$$

$$Y_1 = -\frac{1}{\delta}\left[\frac{C_1}{6} - \frac{\delta}{R_1} + 2\sum_{n=1}^{\infty}\frac{(-1)^n C_1 e^{-\delta\beta_n}}{n^2\pi^2}\right] \tag{24}$$

$$Y_2 = \frac{1}{\delta}\left[\frac{C_1}{6} + 2\sum_{n=1}^{\infty}\frac{(-1)^n C_1(2e^{-\delta\beta_n} - e^{-2\delta\beta_n})}{n^2\pi^2}\right] \tag{25}$$

$$Y_m = -\frac{1}{\delta}\left[2\sum_{n=1}^{\infty}\frac{(-1)^n C_1(e^{-m\delta\beta_n}(1 - e^{\delta\beta_n})^2)}{n^2\pi^2}\right] \tag{26}$$

Where $\beta_n = \frac{n^2\pi^2}{R_1 C_1}, m \geq 3; R_1 = \frac{L}{\lambda}; C_1 = L\rho c_p$

According to Hittle for one dimensional and linear heat conduction the surface heat fluxes and temperatures can be related by the three response factors

X, Y and Z.

For the inside heat flux, we can identify as:

$$q_i(t) = \sum_{m=1}^{\infty} T_{i,t-m+1}X_m - \sum_{m=1}^{\infty} T_{e,t-m+1}Y_m \qquad (27)$$

3. Simulation scenario

The simulations included two scenarios: 14 days of warm seasons and 14 days of cold seasons. The weather conditions were collected by the monitoring system in the University Politehnica of Bucharest's campus [10-12].

The inside temperature was set up constant in time (22 ^{0}C), while the outdoor / outside temperature has been mean hourly observed. The following thermal properties of the wall were used: L=30 cm, λ=0,04 W/m/K, ρ=10 kg/m^3, c_p=800 J/kg/K.

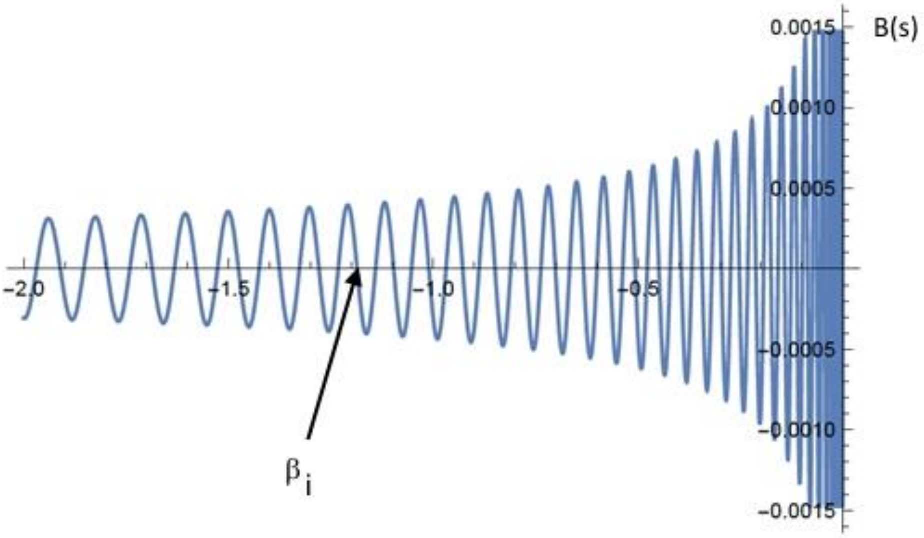

Fig. 2. Evolution of $\boldsymbol{\beta}_i$ poles related to response factors coefficients

For the simulation purposes the response factors (X_m, Y_m) were calculated using 200 poles (Fig. 2) for the integral presented in equation (20). A higher number of poles lead to a higher accuracy. According to Giaconia [9], percentage mean error (PME) for response factors coefficients related to the ramp input

coefficients does not change n ≥ 6, $\boldsymbol{\beta}$≥100 are less than 1,16% in comparison with

ASHRAE coefficients. In figure 3, 4,5 are presented heat fluxes determined using equation (27) for 7 days during during winter and summer period.

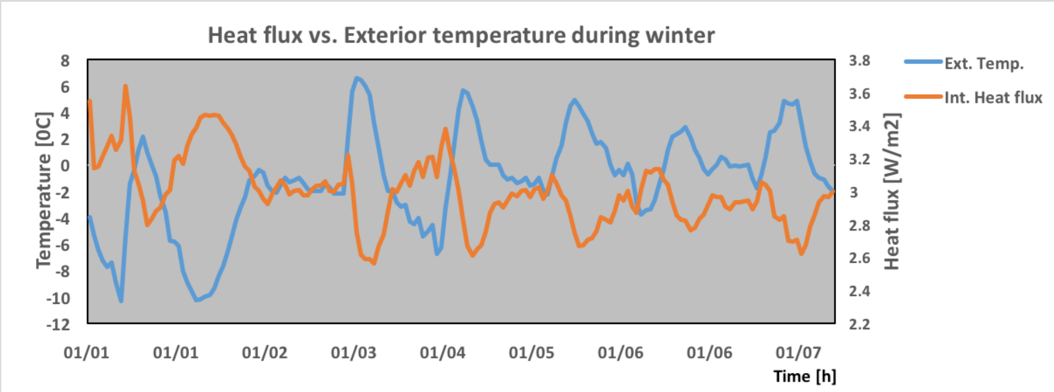

Fig. 3. Influence of exterior temperature on the evolution of the interior heat flux during 01-07 January.

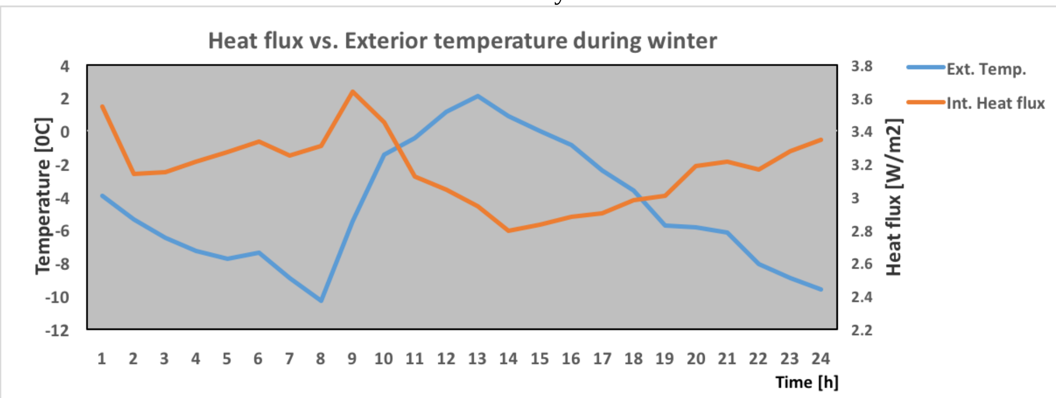

Fig. 4. Influence of exterior temperature on the evolution of the interior heat flux during 24 hours.

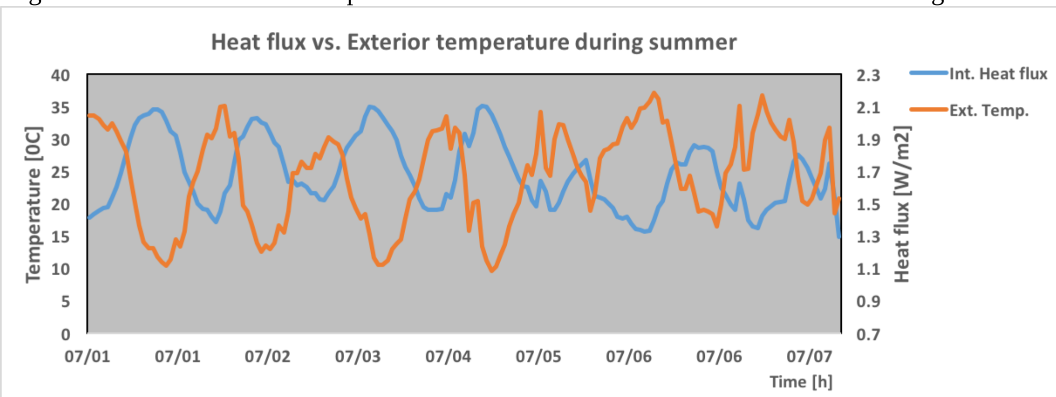

Fig. 5. Influence of exterior temperature on the evolution of the interior heat flux during 01-07 July

It can be observed in both figures that the heat flux varies as a function of exterior temperature. Energy losses are 0.5 – 3.5 times higher during winter.

The slight delay of between exterior temperature and heat flux evolution, which is especially observed in figure 4 is due to thermal inertia of the wall which represents the responsiveness of a material to variations in temperature.

4. Conclusions

The article has presented a dynamic simulation of heat transfer through homogenous wall exposed to real environmental conditions. The simulations had showed promising results. As it can be observed in Fig. 3 and Fig. 4 the interior heat flux varies as a function of external temperature.

The research is important for both future research and development of an analysis tool for a complex prediction of energy demand using a constant or intermittent regime for human comfort. Moreover, the study is important for understanding building reaction and energy requirement to increase the temperature by one degree through intelligent control system allowing management of energy demand based on house occupancy, destination and user schedule.

The impact of computational time is important for generating the CTF function for an entire house but the response factors are generated once per simulation for each wall.

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