AN EMPIRICAL METHOD FOR SIGNAL STATIONARITY ESTIMATION

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The paper presents a novel method for estimation of the wide-sense stationarity of signals / time series / data sequences without a priori knowledge of the underlying process and without efforts to make an inference about it. The method is entirely empirical and threatens the signal "as it is".

At the beginning of the paper, a clarification of the stationarity concept is provided concerning both the processes and the corresponding signals, with particular attention to the problems related to the application of some known signal stationary estimation techniques. After that, a logical explanation and a mathematical description of the newly proposed method are given, incl. an algorithm. Several numerical experiments are performed to verify the performance of the proposed method. Finally, some conclusions are made regarding the presented work.

Keywords: signal, stationarity, estimation, assessment, empirical, test, method.

1. Introduction

The estimation of whether a given process is stationary or not is a classic problem in signal processing (*e.g.*, speech processing) [1]; econometrics (*e.g.*, modeling of market behavior) [2]; environmental science (*e.g.*, climate modeling) [3]; medicine (*e.g.*, patients monitoring [4]), *etc*.

In many cases, one deals only with a single realization (*e.g.*, signal / time series) instead of the corresponding underlying process. Hence, the fundamental consideration is whether a resulting signal is stationary or not. Moreover, the signal shall be considered "as it is", that is, without *a priori* knowledge about the underlying process. Also, surprisingly (or not), there are situations when the process is stationary, but the resulted signal is not (*e.g.*, data sequence containing ten outcomes from a Bernoulli process – coin flipping).

A signal is said to be wide-sense stationary (WSS) if its local statistical properties up to the second-order are time-invariant [5] or which is the same – the local spectral content of the signal does not vary by time [6]. In other words, using purely engineering terms, the stationarity of a signal concerns its DC value, RMS value and spectral content.

This problem is not trivial since it directly affects the proper choice of signal analysis tools. For instance, the power spectral density could be calculated using the autocorrelation function (ACrF) or Bartlett's or Welch's methods only if the

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signal is WSS. Otherwise, a proper time-frequency analysis technique (*e.g.*, short-time Fourier transform) should be used [6].

Three basic techniques exist for assessing a data sequence's stationarity, described in the next section. It would be shown that each has serious drawbacks (*e.g.*, subjectivity, no possibility for automation of the assessment process, too many erroneous results, inappropriateness for some signal types, *etc*.). So, a *niche* exists for a novel, more accurate, and robust stationary estimation method.

To overcome the above problems, a novel approach for signal stationarity estimation is proposed and described below. This work extends and further develops the study presented in [7]. The developed method is implemented in the Matlab® environment and is ready-to-use for real-world scientific and engineering applications.

The paper is organized as follows: (i) at the beginning, a clarification of the stationarity concept is provided concerning both the processes and the corresponding signals; (ii) special attention is paid to the problems related to the application of the known signal stationary estimation techniques; (iii) an explanation and a description of the newly proposed method for signal WSS estimation is given; (iv) results from experiments for sake of verification and demonstration of the method are shown and conclusions are made regarding the presented work.

2. Background

In this section, we clarify the concept of stationarity. First, let's consider a discrete *a priori* unknown process (phenomenon) $\left\{X\left(s,t\right): s \in S \in \mathbb{R}, t \in T \in \mathbb{N}\right\}$ defined in the index set (*e.g.*, time domain) *T*, which is an object of observation (*e.g.*, measurement) by an experiment. For the observation time, a set of outcome results (events) are collected, forming the event space $E \in S$, part of the sample space *S* (population) – the aggregate observations of the process $X\left(s,t\right)$ [8].

The members of the event space *E* may be arranged in an event matrix $X\left(e,t\right) \in \mathbb{R}^{M \times N} : M, N \in \mathbb{N}$ as shown in Fig. 1. For instance, one may think of it as a dataset from meteorological temperature measurements from *M* different spatial locations containing *N* data values each.

For a fixed event e_m, $X_e(t)$ is a time series (a realization) $x(t)$, and for a fixed time t_n, $X_t(e)$ is an ensemble of realizations (a random variable) $x(e)$. In terms of our example, each time series is a set of data from a single measurement location as a function of time, and the cross-section of the time series at time *t* is the state of the thermal process at time *t*. In this light, the process may be considered as a collection of events ensembles defined at specific time instances [9].

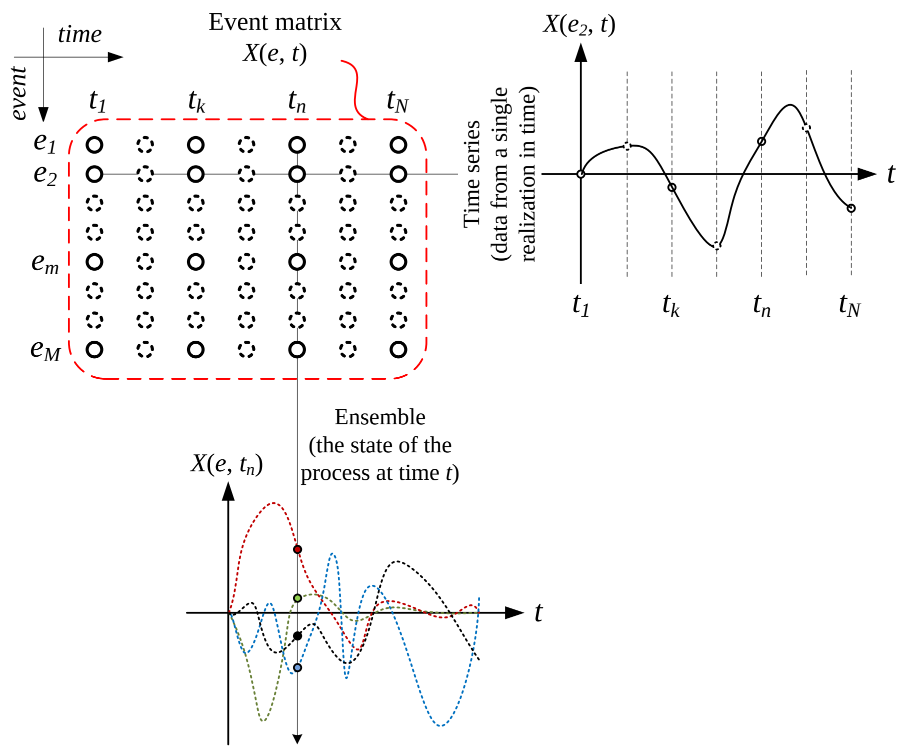

Fig. 1. A graphical representation of an event matrix $X(e,t)$ and the related concepts for an ensemble of realizations and a single realization (time series).

The dynamic behavior of the processes can be analyzed by answering how the statistical properties of its ensembles vary in time. The process is considered stationary if they remain unchanged in time (*i.e.*, time-invariant). This property is of great significance since unless the stationarity is established for the stochastic process under consideration, the analysis of the last becomes intractable [9]. Many stochastic processes have that property, so their average statistical properties are stable and time-independent. However, many other processes and their corresponding signals are nonstationary, so special methods for handling them should be implemented. Hence, an analysis tool should be used to assess whether the signal is stationary or not.

From a practical point of view, the so-called wide-sense / weak-sense / covariance stationarity is of particular interest. It refers to the stationarity of the statistical properties of the successive data ensembles up to the second-order (first- and second-order moments) – sample (empirical) mean m_x and sample (empirical) cross-covariance s_{xy} [10].

In the typical case, $m_x(t)$ and $s_{xy}(t)$ are functions of time [5, 11]:

$$m_x(t_n) \approx \frac{1}{M}\sum_m x(e_m, t_n), \tag{1}$$

$$s_{xy}(t_k, t_n) \approx \frac{1}{M}\sum_m x(e_m, t_k)\, x(e_m, t_n). \tag{2}$$

For a WSS process [5, 11]:

$$m_x(t_n) = const., \forall n, \tag{3}$$

$$s_{xy}(t_k, t_n) = s_{xy}(\tau) : \tau = t_n - t_k, \forall k, n, \tag{4}$$

so that m_x is not a function of time t (*i.e.*, it is time-invariant) and $s_{xy}(\tau)$ is a function only of the time-shift τ between the considered ensembles of realizations.

One should be aware that for most of the real-world processes, the sample space S is not fully available and so the process is not fully cognizable (*e.g.*, the monthly financial benefit on each household in New York /USA/ for a specific year or the exact position and velocity of any car in Varna /Bulgaria/ during a specific day).

Moreover, in many cases, only one realization of the process is available (due to physical restrictions or due to the nature of the process), for instance, the output of flipping a coin several times (Bernoulli process) or the daily USD/EUR exchange rate for the last month (Lévy process). In such a case, a new question arises concerning the stationarity of the available single realization itself (*e.g.*, signal / time series) instead of the process as a whole. At this stage, one should be aware that one does not make assumptions about the process but only about a single realization that is not statistically representative of the underlying process.

The already presented paradigm of WSS of a process can be applied to a single time series but with modifications. A time series is said to be WSS if Eqs. (3) and (4) holds true [11]. Now, one does not dispose with the process' ensembles of realizations but only one time series; hence, the averaging should take place locally in the time domain (*cf*. Fig. 2), that is, one should work with the time-localized statistical properties of the signal instead of the ensemble's one.

In light of the above discussion, in this paper, the object of consideration is assessing whether a given signal is stationary without considering the underlying process. We defined the signal as WSS if: (i) its (time-localized) mean value m_x (*i.e.*, the first-order raw moment) is constant over time (in the boundary of the signal existence) and if (ii) its (time-localized) autocovariance function (*i.e.*, the second-order central cross-moment) $C_{xx}(t_k, t_n)$ depends only on the difference $\tau = |t_k - t_n|$, but not on their particular values [11].

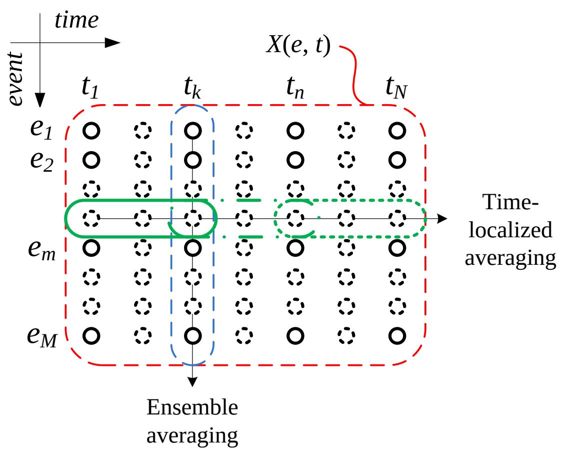

Fig. 2. A graphical representation of the ensemble averaging in one particular time instant *vs*. the time-localized averaging in one particular signal realization. The presented WSS estimation technique adopts the second approach, as only one signal realization is available in most cases.

It is known that the *j*th-order stationarity guarantees the *i*th-order stationarity for all $i \leq j$, that is, if a process is second-order (*i.e.*, covariance) stationary, the process is also first-order (*i.e.*, mean) stationary [9]. As a matter of principle, this property allows one to estimate the WSS of a given signal by checking directly for second-order stationarity. However, performing a check for both the mean and covariance is practically advisable. Moreover, for completeness, we also suggest a test for stationarity about variance along with autocovariance. This measure ensures more robustness of the proposed estimation method and is more informative regarding the reasons for possible nonstationarity of the signal under test.

3. The Problem of the Stationarity Estimation

Three basic techniques exist for assessing the stationarity of a data sequence – visual inspection of the run-plot of the data, visual inspection of the correlogram, and statistical tests.

The researcher's expertise and experience limit the visual inspection of the data graph, so this method is too subjective. Also, it cannot be automatized, so it is impractical for large datasets.

The second approach to estimate the stationarity of the signals by the correlogram (the plot of the ACrF) is shown in [12]. This technique turns out to be entirely irrelevant since there are pieces of evidence that the ACrF has no relation to the stationarity of the signal *i.e.*, the shape of the ACrF is not indicative of the stationarity. Hyndman and Athanasopoulos stated: “As well as the time plot of the data, the ACrF plot is also useful for identifying nonstationary time series. For a stationary time series, the ACrF will drop to zero relatively quickly, while the ACrF of nonstationary data decreases slowly. Also, for nonstationary data, the value of $r1$ (*i.e., ACrF for a lag of one sample)* is often large and positive."

Fig. 3 shows the ACrF of a linear frequency-modulated (LFM) signal, which contradicts the above statement insofar as the LMF signal is a notorious example of nonstationary behavior. In addition, Fig. 4 depicts a correlogram of a sine-wave signal, showing that the ACrF decreases slowly with a significant positive $r1$ spike regardless of the signal's heavy stationarity.

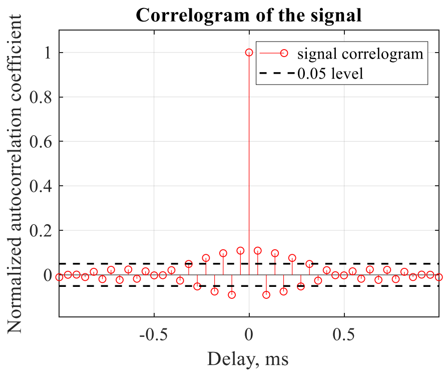

Fig. 3. Correlogram of an LFM signal (duration 0.1 s, start frequency 0 Hz, end frequency 10000 Hz, sampling frequency 22050 Hz). The ACrF decreases rapidly, although the signal is nonstationary.

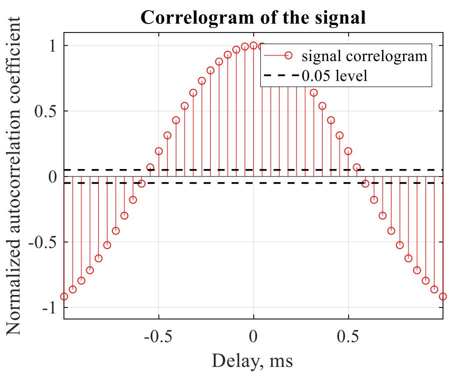

Fig. 4. Correlogram of a sine-wave signal (duration 0.1 s, frequency 400 Hz, sampling frequency 22050 Hz). The ACrF decreases slowly with a significant positive $r1$ value, although the signal is stationary.

The third method is to use some of the unit root tests. The most famous of them are the augmented Dickey–Fuller (ADF) test and the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test [13].

Both tests (and many others) fail to detect stationarity for many real-world signals. They work well predominantly with data from autoregressive processes. Another weakness is that the tests are limited to trend-stationarity or unit-root nonstationarity hypotheses. It is often the case that using different statistics gives contradictory results.

A simulation[2] was performed in the Matlab® environment, confirming the weakness of the ADF, KPSS, and also the Phillips-Perron and Leybourne-McCabe tests. All of them erroneously classified the example LFM signal as stationary.

These inconsistent and contradictive results must ensure the reader how tricky the topic is. The drawbacks mentioned above lead to the idea that a new method must be developed and applied to obtain more reliable results.

4. Method Description

First, let's introduce the statistics of interest in our study – the mean, the standard deviation, the variance and the autocovariance.

The sample mean and the sample standard deviation (STD) of a signal are given as [14, 15]

$$m_x \triangleq \frac{1}{N}\sum_n x[n], \tag{5}$$

$$s_x \triangleq \sqrt{\frac{1}{N-1}\sum_n \left|x[n]-m_x\right|^2} \,. \tag{6}$$

The sample variance s_x^2 is defined as the square of the STD.

The autocovariance function (ACvF) is a classic tool for assessment of the similarity between a signal and its sliding delayed copy as a function of the delay (sample lag) h [14, 15]

$$C_{xx}(h) \triangleq \sum_{n=1}^{N-|h|}\left(x[n+|h|]-m_x\right)\cdot\left(x[n]-m_x\right). \tag{7}$$

Often, the autocovariance sequence is normalized by its value at zero lag [14, 15]

$$\hat{C}_{xx}(h) \triangleq \frac{C_{xx}(h)}{C_{xx}(0)}, \tag{8}$$

so that $-1 \le \hat{C}_{xy}(h) \le 1$.

Another tool that measures the linear relationship between two sequences is the Pearson correlation coefficient (PCC) [14, 15]

[2] It can be found in the supplementary material given in [16]. There are also two examples that show the failure of the correlogram as a WSS estimation tool and a demonstration of the efficiency of the developed method to detect even weakly nonstationarities in the signals under test.

$$R_{xy} = \frac{\sum_{n=1}^{N}\left(x[n]-m_x\right)\cdot\left(y[n]-m_y\right)}{\sqrt{\sum_{n=1}^{N}\left(x[n]-m_x\right)^2}\cdot\sqrt{\sum_{n=1}^{N}\left(y[n]-m_y\right)^2}}. \tag{9}$$

If the signal is WSS and if it is fragmented in a few (let's say K) frames (with or without overlapping) with constant length, the mean and the autocovariance will be identical for all of them – they will not vary from frame to frame. If the signal is nonstationary, the above statement will not be valid. This fact plays a central role in developing the signal stationary estimation technique described below.

The stationarity estimation procedure itself starts with splitting the signal of interest $x[n]$ into three equally length parts termed "partial signals" – $x_1[m]$, $x_2[m]$ and $x_3[m]$ (so, $K = 3$), where $n = \{1,...,N\} \in \mathbf{N}$, $m = \{1,...,M\} \in \mathbf{N}$, and N and M are the corresponding signals' lengths. After that, the parameters of interest – the means, the variances, and the autocovariance sequences of the three partial signals are compared empirically, as specified below.

First, one should check the statistical significance of the mean values compared to the STD for each partial signal. We proposed the mean to be considered statistically insignificant if the logical statement

$$\left[\!\left[\left|m_{X_k}\right| / s_{X_k} < \delta \right]\!\right] \tag{10}$$

holds true for each partial signal ($k = \{1,...,K\} \in \mathbf{N}$, and in this particular case $k = \{1,2,3\}$), where $[\![\cdot]\!]$ denotes the Iverson bracket, and δ is a predefined tolerance. If this is the case, then one may assume that the overall signal $x[n]$ is mean-stationary since the mean values and their difference are negligible (with respect to the STD of the signal). If not, one should test additionally using the logical statement

$$\left[\!\left[\left| \frac{m_{X_1} - m_{X_2}}{\min\left(m_{X_1}, m_{X_2}\right)} \right| < \delta \right]\!\right]. \tag{11}$$

If the statement holds true, the overall signal is estimated as stationary about the mean since the difference between the partial means is negligible.

Similarly, for the variances, the following logical statement should hold true to determine a signal as stationary about its variance:

$$\left[\!\left[\frac{\left|s_{X_1}^2 - s_{X_2}^2\right|}{\min\left(s_{X_1}^2, s_{X_2}^2\right)} < \delta \right]\!\right]. \tag{12}$$

Note that one does not check for the significance of the individual partial signals' variances since this parameter is always non-zero.

The identity of the ACvF of the partial signals is estimated in two steps. First, we check for the equality of the variances of the ACvF sequences using

$$\left[\frac{\left| s^2_{C_{X_1X_1}} - s^2_{C_{X_2X_2}} \right|}{\min\left(s^2_{C_{X_1X_1}}, s^2_{C_{X_2X_2}} \right)} < \delta \right]. \tag{13}$$

The second check is performed using the PCC $R_{C_1C_2}$ and by estimating the p-value for testing the hypothesis of getting a PCC as large as the observed one by random chance when the true PCC is 0 [17]. The *p*-value ranges from 0 to 1, where values close to 0 correspond to a significant correlation between the ACvFs under test and a low probability of observing the null hypothesis that there is no relationship between them. Presented in the form of a logical condition, the second check is satisfied when

$$p < \delta \ . \tag{14}$$

This double-check is mandatory since the PCC checks only for “pattern” similarity but does not consider the possible scale differences.

In Eqs. (10) to (14) δ is the tolerance level of the test. It determines the tolerable dissonance between the time-localized values of the particular statistical parameter (mean, variance, autocovariance) acceptable by the user in the specific context. If all parameters of interest are virtually time-invariant, that is, if their drift is within the tolerance, it may be neglected, and the signal is considered as WSS. In contrast, if some of the statements (10) ÷ (14) are not met, the signal is nonstationary regarding the corresponding property. The user chooses the tolerance depending on the task and requirements; typical values are 0.01, 0.05, 0.1 (*i.e.*, 1%, 5%, 10%).

The algorithm of the proposed stationarity estimation method is shown in Fig. 5 (left), and the core of the method is the statistical identity procedure shown in Fig. 5 (right). It is entirely empirical, that is, it does not rely on statistical tests (in comparison with the previous version presented in [7]) but on a comparison of the time-localized summary statistics of the signal under test *i.e.*, no *a priori* assumptions are made about the underlying process or population.

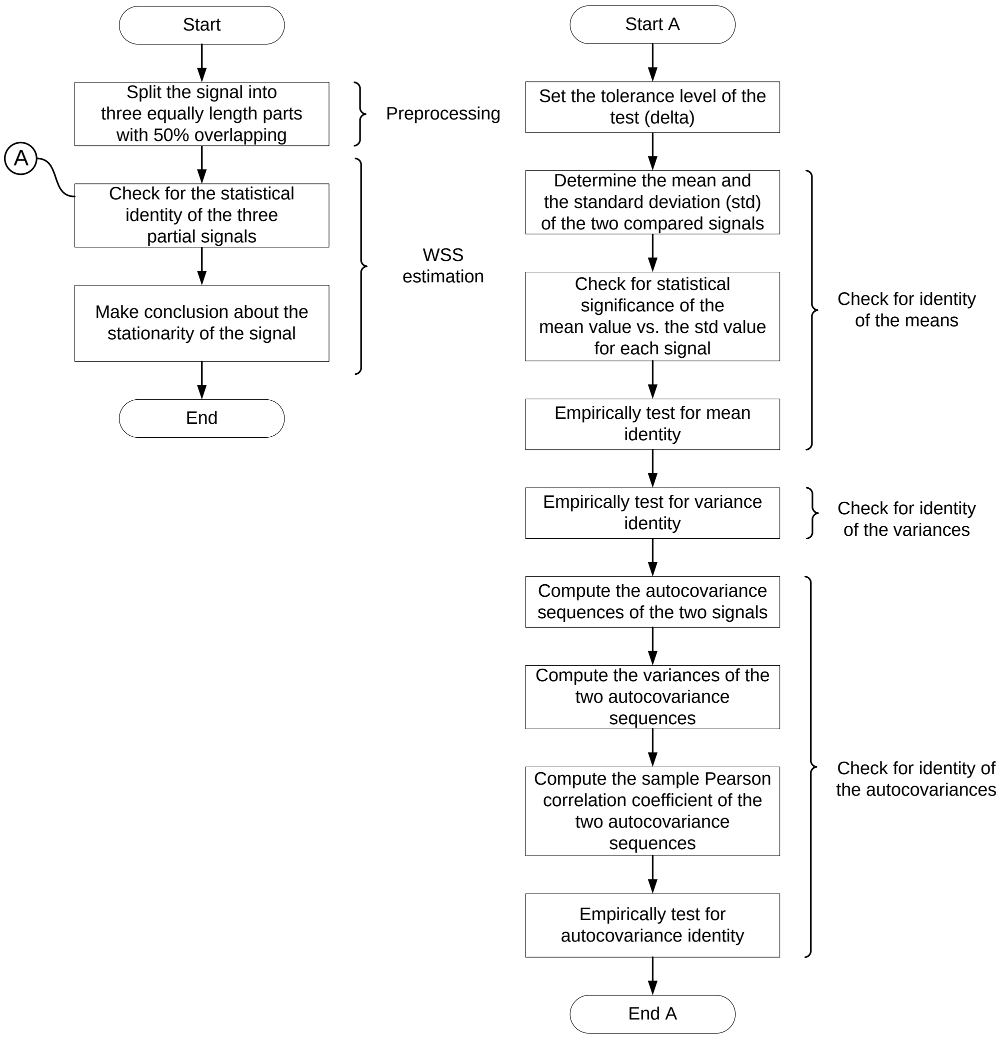

Fig. 5. The general algorithm of the novel signal stationarity estimation procedure (left). The signal is split into three equally length parts with 50% overlapping. Then, a check is made for their statistical identity using the core algorithm (right). Finally, a conclusion is made about the signal stationarity based on the results of the statistical identity checks.

5. Experimental results

A software implementation of the algorithm is done by the author in the Matlab® environment as a user-defined function named “isstationary”. This function, along with some supportive examples and test data are freely accessible at the Matlab Central File Exchange repository [16], which allows full reproducibility of the experimental results.

Twelve representative signals are selected for test purposes. The formal descriptions of them are given in Tab. 1. They are separated into four groups: (i) TS*x*A which represents four deterministic signals of the type "sine wave"; (ii) TS*x*B are four signals of type "color noise"; (iii) TS*x*C are two signals representing autoregressive–moving-average (ARMA) processes and (iv) TS*x*D are two real-world sound signals – human speech and music. Some signals are strongly stationary (*e.g.*, TS1A, TS1B), and others are heavily nonstationary (*e.g.*, TS4B, TS1D). The tolerance level δ is set to 0.1 (*i.e.*, 10% tolerance) for all cases, and the sampling frequency is 44100 Hz, excluding the TS*x*D signals.

Table 1

Mathematical descriptions of the representative test signals

Test signal	Signal type	Signal parameters
TS1A	Sine-wave (stationary signal) $x(t) = U_{m0}\sin\left(2\pi f_0 t\right)$	$x(t) = 1.0\sin\left(2\pi 440t\right)$, $t = \{0,...,0.5\}$.
TS2A	Sequence of sine-waves (nonstationary signal) $x(t) = \begin{cases} U_{m1}\sin\left(2\pi f_1 t\right), & 0 \le t < t_1 \\ U_{m2}\sin\left(2\pi f_2 t\right), & t_1 \le t < t_2 \end{cases}$	$x(t) = \begin{cases} 1.0\sin\left(2\pi 440t\right), & 0 \le t < 0.1, \\ 2.0\sin\left(2\pi 440t\right), & 0.1 \le t \le 0.5. \end{cases}$
TS3A	Sequence of sine-waves (nonstationary signal) $x(t) = \begin{cases} U_{m1}\sin\left(2\pi f_1 t\right), & 0 \le t < t_1, \\ U_{m2}\sin\left(2\pi f_2 t\right), & t_1 \le t < t_2 \end{cases}$	$x(t) = \begin{cases} 1.0\sin\left(2\pi 440t\right), & 0 \le t < 0.1, \\ 1.0\sin\left(2\pi 1000t\right), & 0.1 \le t \le 0.5. \end{cases}$
TS4A	Linear chirp (nonstationary signal) $x(t) = U_m \sin\left(2\pi f_1 t + \pi \frac{f_2}{T} t^2\right)$	$x(t) = 1.0\sin\left(2\pi 20t + \pi \frac{20000}{0.5} t^2\right)$, $t = \{0,...,0.5\}$.
TS1B	Violet noise (stationary signal) $x(t) = \mathcal{VN}(\mu, \sigma^2)$	$x(t) = \mathcal{VN}\left(0,1\right)$, $t = \{0,...,0.5\}$.
TS2B	Blue noise (stationary signal) $x(t) = \mathcal{BN}(\mu, \sigma^2)$	$x(t) = \mathcal{BN}\left(0,1\right)$, $t = \{0,...,0.5\}$.
TS3B	Pink noise (nonstationary signal) $x(t) = \mathcal{PN}(\mu, \sigma^2)$	$x(t) = \mathcal{PN}\left(0,1\right)$, $t = \{0,...,0.5\}$.
TS4B	Red noise (nonstationary signal) $x(t) = \mathcal{RN}(\mu, \sigma^2)$	$x(t) = \mathcal{RN}\left(0,1\right)$, $t = \{0,...,0.5\}$.

Mathematical descriptions of the representative types of test signals (cont.)

TS1C	AR(p) process (stationary signal) $x(t)=\mu+\varepsilon_t+\theta_1 x_{t\text{-}1}+\ldots+\theta_p x_{t\text{-}p}$	AR(2) $x(t)=0.5+\varepsilon_t+0.6x_{t\text{-}1}+0.1x_{t\text{-}2}$ $\varepsilon_t=\mathcal{WN}(0,0.1)$, $t=\{0,\ldots,0.5\}$.
TS2C	MA(q) process (stationary signal) $x(t)=\mu+\varepsilon_t+\theta_1\varepsilon_{t-1}+\ldots+\theta_q\varepsilon_{t-q}$	MA(12) $x(t)=0.5+\varepsilon_t+0.6\varepsilon_{t-1}+0.1\varepsilon_{t-12}$ $\varepsilon_t=\mathcal{WN}(0,0.1)$, $t=\{0,\ldots,0.5\}$.
TS1D	Human speech (nonstationary signal)	Record “DR2_FRAM1_SI522” from the TIMIT database [18].
TS2D	Music (nonstationary signal)	Built-in Matlab® audio sample of the “Hallelujah Chorus” from Handel

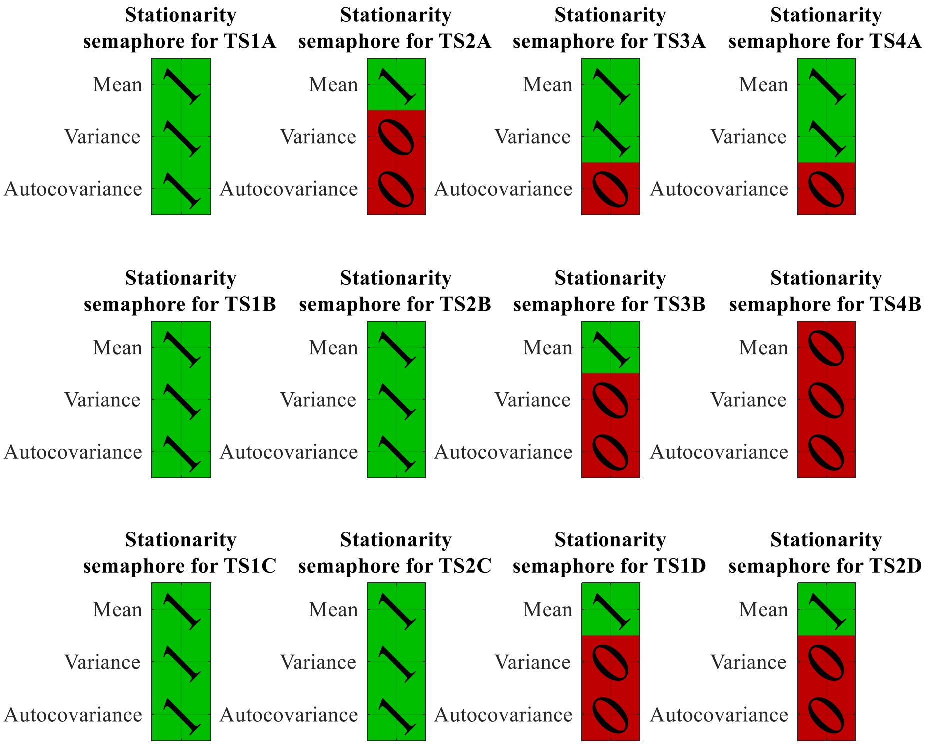

Fig. 6. Results from applying the developed empirical stationary estimation method on: TS1A – sine-wave signal; TS2A – sine-wave with variable amplitude; TS3A – sine-wave with variable frequency; TS4A – linear chirp; TS1B – violet noise; TS2B – blue noise; TS3B – pink noise; TS4B – red noise; TS1C – data sample from AR(2) process; TS2C – data sample from MA(12) process; TS1D – sample of human speech; TS2D – music sample.

The routine is applied to any test signal, and in all cases, fair results are obtained, as shown in Fig. 6, using the “stationary semaphore” [7].

The test confirmed that TS1A is a pure sine-wave signal – a *par excellence* WSS signal. TS2A is a sine-wave signal with an abrupt change in the amplitude, so the test confirms that the mean is stable, but the variance and, hence, the autocovariance of the signal are not stationary. TS3A is a sine-wave with an abrupt change in the frequency, which implies a stable mean and variance but nonstationary autocovariance, as detected by the test. The same result is obtained for TS4A, which has time-varying frequency but constant mean and variance. TS1B and TS2B are WSS stochastic noise signals, as the test confirms.

On the other hand, TS3B (pink noise) and TS4B (red noise) are progressively nonstationary, with a still stable mean of TS3B. TS1C (AR(2) process) and TS2C (MA(12) process) are detected as WSS in accordance with the theory. On the other hand, TS1D (speech) and TS2D (music) have assertive nonstationary behavior where only the mean value is stable, which is correctly detected by the test.

All results align well with the theory. The simulations provide clear evidence of the reliability of the suggested WSS estimation procedure.

6. Conclusions

The paper addresses the problem of signal stationarity estimation, treating the single realization itself instead of the underlying process. First, a brief description of the theoretical background is given, and then the challenges are highlighted regarding the assessment of the WSS, including the limitations of existing techniques such as visual inspection, correlogram analysis, and statistical tests.

Further, to overcome these limitations, a novel approach is proposed for signal WSS estimation. The method focuses on the signal's time-localized first and second-order statistical properties – mean, variance, and autocovariance. By comparing these properties across different signal segments, one can determine whether the signal can be considered stationary.

The effectiveness of the proposed method is demonstrated and validated through numerical experiments on various types of signals (representative for a wide range of processes). The obtained results aligned well with the theoretical expectations and the actual signal characteristics.

The method's advantages lie in its empirical nature, as it does not rely on statistical tests or make *a priori* assumptions about the underlying process or population. It provides a more accurate and reliable estimation of signal stationarity compared to existing techniques, which suffer from subjectivity, lack of automation, and inconsistent results. Also, it allows the determination of the source of the possible nonstationarity.

The developed method, implemented in the Matlab® environment, offers a ready-to-use solution for real-world scientific and engineering applications.

Future research will focus on applying the proposed empirical WSS estimation method on nonstationary signals to determine their short-time stationarity duration, that is, the short-time intervals at which a particular signal may be considered WSS.

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