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THEORETICAL AND COMPUTATIONAL ASPECTS OF NON LOCAL DAMAGE COUPLING WITH ELASTIC BEHAVIOUR

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Softening due to damage is the source of the strain localization in the materials as concrete. In such a situation, using the finite element analysis provides results that are directly depending to the spatio-temporal discretization. In this article, the adjustment of an isotropic elastic model coupled to non-local damage is presented in order to regularize the associated initial and boundary value problem. This formulation consists in delocalizing the damage variable "D". Dispersion analysis and numerical simulations are used to compare the local and the non local models.

Keywords: damage, softening behaviour, localization, non-local damage model, concrete.

1. Introduction

Deformation localization is a phenomenon often observed in a large class of materials, namely quasi-brittle materials such as concrete, rocks and soil. Based on the nature and intensity of the action-effect, concrete deformation takes place in a complex manner, bringing into play one or more combinations of basic mechanisms: elasticity, damage, sliding, rubbing, cracking, etc.

Many studies have focussed on the problem of deformation localization in a continuum [1, 2]. Localization can be defined as a zone where the deformations remain continuous, but are concentrated in a large strip, very small in terms of the structure, which depends on the load conditions [3, 4, 5]. This localization phenomenon quickly results in a ruined structure. In practical terms, the localization can be interpreted in a variety of ways: for metals, the localization strip is formed through the sliding of crystalline planes and the formation of cavities and for granular areas, a rearrangement of the granules can be at the source of the localization. However, in the case we are interested in, for quasi-brittle materials (concrete), the localization strip is formed by a collection of

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microcracks. This localization strip, micro-structural in size, cannot be described correctly using classic continuum models [4, 6, 7].

At the same time, the localization of the damage makes the mathematical problem to be resolved improperly since the softening causes a loss in the ellipticity of the differential equations system describing the deformation process [8, 9]. Lastly, the initial conditions and the conditions at the limits, which were correctly defined in the elliptical case, become improperly adapted in the hyperbolic case. Numerically, these difficulties result in the results being greatly dependent on the finesse and orientation of the mesh in a finite elements calculation [10]. The size of the localization zone becomes a function of the size of the finite elements where the localization criterion has been achieved. The result is a non-objectivity of the results in terms of spatial discretization, leading to the dissipation of less and less energy when the mesh is refined.

In order to resolve the physically-unrealistic results of a dissipated energy rupture that is nil as well as the numerical problem due the non-objectivity of the mesh, a variable of non-local damage is introduced into the local model. A non-local environment is an environment in which at least one variable field is subject to a spatial average in a finite neighbourhood of a point [11]. The purpose of this regularization technique is to prevent the sensitivity of the solution to the mesh.

This non-local model should contribute in helping to understand the major physical processes that govern the mechanisms for concrete deformation.

Therefore, special attention must be given to the material's behaviour in order to correctly reproduce the various phenomena put in play during its deformation.

2. Local Damage Model

Concrete, which is widely used in construction, has a very complex, non-linear behaviour. Depending on the nature and intensity of the action-effect, the quasi-brittle behaviour of concrete promotes the development of various modes for rupturing and crack propagation. Damage mechanics makes it possible to model the effects of microcracking on concrete's behaviour at the macroscopic scale. The basis for damage models is the introduction of a local damage variable impacting the stiffness of the material. This type of model was initially introduced for metallic materials (see work by Lemaitre and Chaboche) [12], reiterated by Montheillet and Moussy [13]. Mazars wrote a behaviour model based on these works and adapted to the uniaxial behaviour of concrete, both under compression and tension [3]. The model uses an isotropic scalar damage variable.

The Mazars model was developed based on damage mechanics [3, 14, 15] and the theory of elasticity coupled to isotropic damage (it ignores all manifestations of plasticity as well as the closing of cracks). A scalar and isotropic

D damage variable has been defined. This concept directly describes the loss of rigidity as well as the softening behaviour of the material and takes into account tension compression dissymmetry. The particular point of this model is that it uses a deformation criterion, by introducing the notion of equivalent strain.

The stress-strain relation can be expressed as follows**:**

$$\sigma = (1-D)C:\varepsilon, \tag{1}$$

where C and ε are respectively the components of the fourth-order elasticity tensor and the elastic deformation tensor.

Concrete damage in the Mazars criterion is governed by a variable called "equivalent strain" and which translates the local extension state of the material during loading. It is defined as:

$$\varepsilon_{eq} = \sqrt{\sum_{i=1}^{i=3}\left(\langle\varepsilon_i\rangle_+\right)^2}. \tag{2}$$

In which:

ε_i is the principal strain in *i* (*i*=1,3) and $\langle\varepsilon_i\rangle = \varepsilon_i$ si $\varepsilon_i > 0$

= 0 otherwise

The changes in the scalar damage are defined based on a threshold function:

$$f\left(\varepsilon, k_d\right) = \varepsilon_{eq} - k_d. \tag{3}$$

k_d is a parameter containing the load history. Initially, k_d is equal to damage threshold ε_{D_0} and takes on the maximum value achieved by ε_{eq} during the entire load history "t":

$$k_d = \frac{\max}{t}\left(\varepsilon_{eq}, \varepsilon_{D_0}\right). \tag{4}$$

In order to take into account the dissymmetric behaviour of the concrete under tension and compression, the damage is calculated as a combination of tension damage D_t and compression damage D_c. The linear combination of these two damages provides the global isotropic damage:

$$D = \alpha_t D_t + \alpha_c D_c, \tag{5}$$

with:

$$D_{t,c} = 1 - \frac{\varepsilon_{D_0}(1-A_{t,c})}{\varepsilon_{eq}} + \frac{A_{t,c}}{\exp\left(B_{t,c}\left(\varepsilon_{eq} - \varepsilon_{D_0}\right)\right)}, \tag{6}$$

and:

$$\alpha_{t,c} = \left(\sum_{i=1}^{i=3} \frac{\langle\varepsilon_i^{t,c}\rangle\langle\varepsilon_i\rangle_+}{\varepsilon_{eq}}\right). \tag{7}$$

A_t, B_t, A_c and B_c are four parameters of the model to be determined based on compression and bending tests. Coefficients α_t and α_c are dimensionless coefficients that represent respectively the impact of the contribution of each of the parts under tension and in compression. Under direct tension, $\alpha_t = 1, \alpha_c = 0$ and under direct compression, $\alpha_t = 0, \alpha_c = 1$ [3, 16]. ε_i^t are the strains that result from the main positive stresses and ε_i^c are the strains that result from the main negative stresses. Fig.1 presents the answers of the Mazars damage model in the tension and compression tests [15].

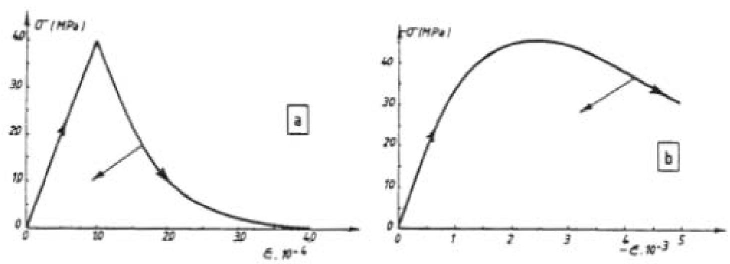


Fig. 1. Response of the Mazars damage model under direct tension (a) and under direct compression (b) [15]

The implementation of the model and its application in numerical simulations is more delicate: in fact, this type of model presents softening behaviours both under tension and compression (Fig.1), meaning that the numerical problem has not been set out properly. As soon as the material behaviour becomes soften, a strain localization phenomenon appears in a small zone [17]. For the finite element simulations, this results in a dependence of the results in relation to the mesh. In fact, based on the work of [5, 6, 16, 18, 19]. The localization zone becomes proportional to the size of the element. The more the mesh becomes refined, the smaller the localization zone becomes. An adjustment method therefore becomes necessary. Many regularizations techniques are proposed in the literature [18, 20, 21]. These various changes have led to rugged and usable models, both analytically and for finite element codes. The choice made here was to introduce a non-local change into the Mazars model.

3. Non-local damage model

Various works present varied formulations depending on the authors, physical for some, phenomenological for others as well as mathematical [22, 23]. They also make it possible to take into account the neighbouring effect for correcting the numerical dependence of the spatial discretization solution. Formulations based on the relaxation of potential energy [24], methods based on the introduction of new degrees of liberty [25], and others take into account a spatial neighbouring effect, either through a delocalization operator (non-local models) or by introducing state variable gradients (gradient theories), [7, 10, 26, 27, 28]. In this article, there is an attempt to limit the dependence on mesh by using a non-local variable in its integral form based on Pijaudier-Cabot [10, 21, 29, 30] and Saanouni [26, 31].

The idea of non-local models as described initially by Pijaudier-Cabot and Saanouni [10, 21, 26, 29, 31] for damage formulations and taken out again in the form of gradients by [5, 7, 32, 33, 34, 35, 36] involves taking into account a spatial neighbouring effect to describe the behaviour of a material point: there is remote interaction between the points of the structure Fig.2 [37]. These interactions take place in a neighbourhood of fixed size from the material point considered. More specifically, certain local sizes of the behaviour law are replaced by their non-local pendants. The choice of one or more non-local variables remains delicate to justify. However, it can be observed that the most effective models choose to go non-local where there are variables associated to the softening phenomena. This explains the choice of an isotropic strain-hardening scalar variable for a negative strain-hardening model where the choice of damage seems the most relevant. The work by Peerlings [32, 38] and by Engelen [33] for example proposes to adjust the cumulated plastic strain. However, Voyiadjis [39] proposes to adjust all of the internal variables. Germain [4] and Liebe [40] propose the choice of drive force variable for the damage. For damageable elastic behaviours, these formulations seem particularly effective; however, in this article, we have chosen to "delocalize" the local D damage variable, by replacing it with the non-local $\overline{D}$ variable. Therefore, the non-local $\overline{D}$ damage variable is calculated based on a volume that is representative of the damage.

The non-local $\overline{D}$ damage variable is expressed always as x in the structure:

$$\overline{D}(x) = \frac{1}{\psi(x)} \int_{\Omega} \psi(y,x) D(y) dV, \tag{8}$$

where V is the structure's volume and $\psi(x)$ is the volume represented in point x defined by:

$$\psi(x) = \int_{\Omega} \psi(y,x) dV. \tag{9}$$

$\psi(y,x)$ is a non-local weighting function that is meant to be homogenous and isotropic. It characterizes the geometry of the localization zone. It depends solely on the distance $R = \|x - y\|$ between source point x and reception point y. Generally, a Gaussian is chosen as a weight function. This choice is purely numerical. In fact, it has been demonstrated that a Gaussian optimizes the convergence rate of the finite elements solution [10, 29].

We will therefore use:

$$\begin{cases} \psi(y,x) = \exp\left(-\dfrac{\|2(x-y)\|^2}{R^2} \right) & \text{for}(x-y) < R \\ \psi(y,x) = 0 & \text{for}(x-y) > R\ . \end{cases} \quad (10)$$

R describes the area of the integration radius. In terms of plane calculation, this integration area is composed of a disc of radius R.

Before taking the first loading step, the function of a process is to define the volume that is representative of each point of the structure. Therefore, for each M integrating point on the mesh, we calculate the distance that separates it from other Gauss points and we consider as neighbours those that are included in the disc of the given R radius. For each of these N points, we calculate the contribution in the neighbourhood of the M point and we create a file in which we store, for all M points, the number of neighbours, the reference and contribution of each of these neighbours. It can be seen that with such a procedure, $R = 0$ makes it possible to use a classic local approach (the M point is considered to be a neighbour of itself with a unit contribution) (Fig. 2).

In cases where, in order to reduce the size of the finite elements problem, the symmetries of geometry and structure load are used, it is necessary to take into account fictional Gauss points that are not represented in the mesh but which exist in the real structure (Fig. 2).

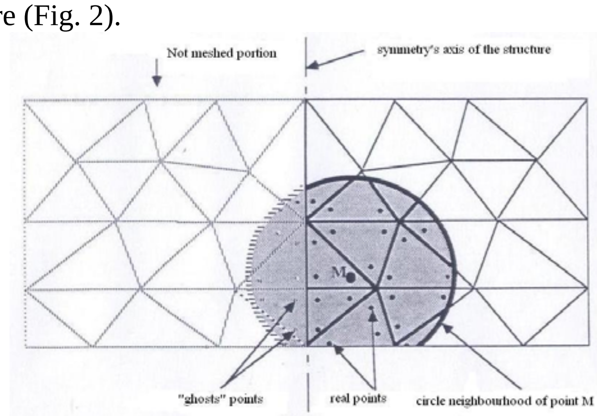


Fig. 2. Construction of fictional points in the neighbourhood of point M [37]

The modelling is implemented in a finite element calculation mode based on the integral formulation.

4. Numerical implementation

The model thereby described for the simulation of the main macroscopic phenomena, introduced in the case of a damageable elastic behaviour, is implemented into the finite element calculation program [41]**.** This program, initially developed for the numeric simulation of linear, isotropic, strain-hardening elastoplastic behaviour, has been modified for taking into account the coupled behaviour, elasticity-damage initially, then an additional block is added for calculating the non-local damaged, based on the organizational chart in Fig.3. In this same program, blocks were added so that the program would be interfaced with the GID Processor station. This program will be used to study the distribution of the damage, strain and stresses on a notched plate under tension, in a state of flat stress and strains.

Given the writing of the model, the natural way of resolving non-linear problems associated with its implementation using finite elements is to use an algorithm with a secant matrix because the damage affects the value of the secant module (Fig.3): with each iteration, the elastic characteristics of the elements are recalculated (the damage, at all instants in the process, is interpreted as a variation of its elastic characteristics).

The convergence test takes on a traditional form: in the current iteration i $(i \geq 1)$, we calculate the residues $\{R\}$ (or rebalancing forces) of the resolution which ensures the overall balance of the structure:

$$\{R\}_i = [K(D_{i+1})]\{u\}_i - \{F\}, \tag{11}$$

where $\{F\}$ is the effort imposed (to balance) to the current increment (in the event a displacement is imposed on the structure, this effort depends on iteration i).

$\{u\}_i$ the displacement calculated from $\{F\}$ and the damage of the previous iteration:

$$\{u\}_\mathrm{i} = [K(D_i)]^{-1}\ \{F\}, \tag{12}$$

and $[K(D_{i+1})]$ is the secant matrix introduced by the new damage calculated based on $\{u\}_i$.

It is felt we have reasonably converged on the solution when the following two tests are checked:

$$\frac{|R^\mathrm{i}_{\max}|}{|F_{\max}|} \leq n \qquad \text{and} \qquad \sqrt{\frac{\{F\}^T\{R\}}{\{F\}^T\{F\}}} \leq n\ , \tag{13}$$

where $|R^i_{\max}|$ and $|F_{\max}|$ are the components at maximum absolute value on the vectors $\{R\}_i$ and $\{F\}$ of iteration i, and n is a tolerance chosen by the user (10^{-3} for example).

$\{.\}^T$ indicates the Euclidean transposition .

The non-linear calculation algorithm is tested on a tensile bar in order to validate the implementation of the model. The non-local $\overline{D}$ damage value can be calculated using a numerical integration method, e.g. the Gauss method as part of the finite elements method.

Within this framework, the value of $\overline{D}$ on a geometric point x_i is calculated by:

$$\overline{D}(x_i) = \frac{\sum_{N=1}^{N=N_e} \sum_{N=1}^{N=Ng} \mathrm{w_g} \psi\left(y_\mathrm{g}, x_\mathrm{i}\right) \mathrm{D}\left(y_\mathrm{g}\right) \det(\mathrm{J})_\mathrm{g}}{\sum_{N=1}^{N=N_e} \sum_{N=1}^{N=Ng} \mathrm{w_g} \psi\left(y_\mathrm{g}, x_\mathrm{i}\right) \det(\mathrm{J})_\mathrm{g}}, \tag{14}$$

where N_e is the total number of elements, N_g is the number of Gauss points in an element, y_g is the position vector of the integration point, and w_g is the coefficient of the associated weight.

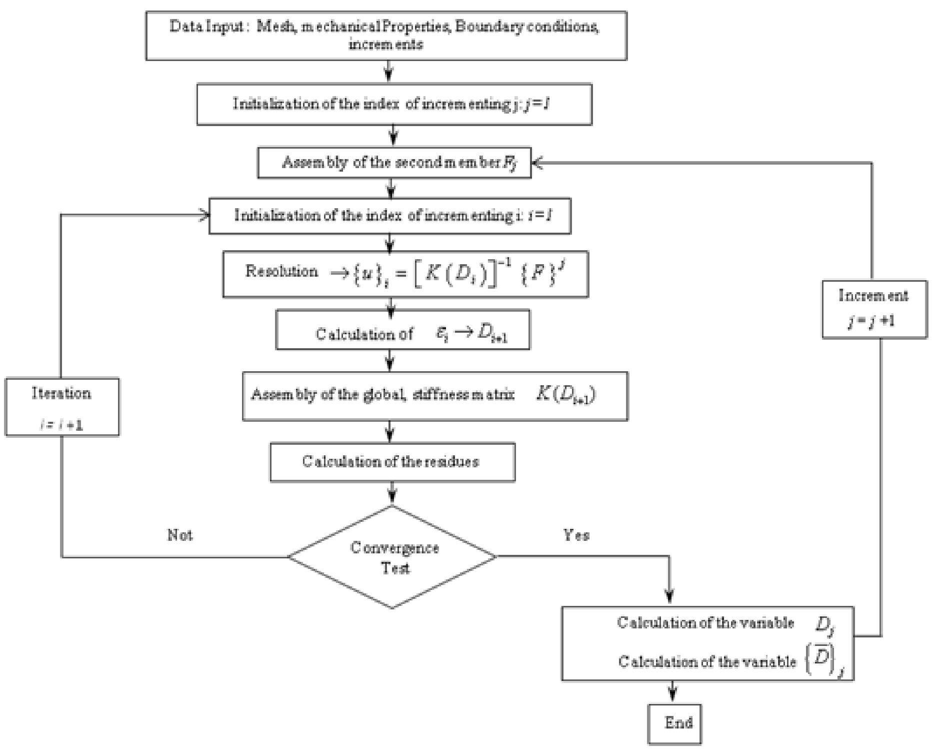


Fig. 3. Calculation Algorithm Chart

5. Application with a tensile bar

The mesh is completed using triangular, three-node elements with a Gauss integration point. An anchoring limit is applied on one side and displacement is imposed on the other. The dimensions, load and conditions at the limits of the tensile bar are represented in Fig.4.

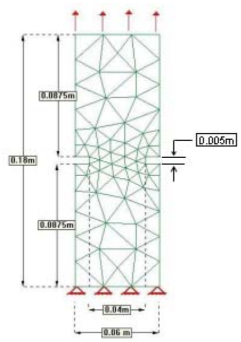


Fig. 4. Dimensions, load and conditions at the limits of the tensile bar

The parameters obtained are defined in the following table 1

Table 1

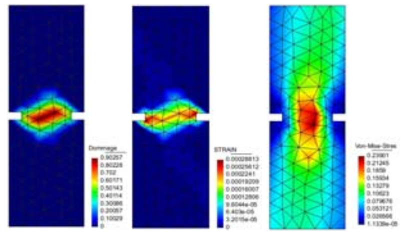
Model Parameters for a Tensile Bar

E (MPa)	υ	A_t	B_t	ε_{D_0}	A_c	B_t	R
21000	0.2	0.8	20000	0.0001	1.4	1800	0.03

Where E and υ are respectively the Young's modulus and the Poisson's ratio.

5.1. Influence of the R integration radius on the structure response

The non-local calculation is conducted for three values: R=0.03mm, R=0.04mm and R=0.05mm, whereas the local calculation is made at R=0. We show the distribution of damage D, the equivalent strain and the von Mises stress in the local case in Fig.5, and in the non-local case in Fig.6, Fig.7, and Fig.8, for the various values of R.



Damage Equivalent Strain von Mises Stress

Fig. 5. Distribution of the local damage, equivalent strain, and the von Mises stress (R = 0)

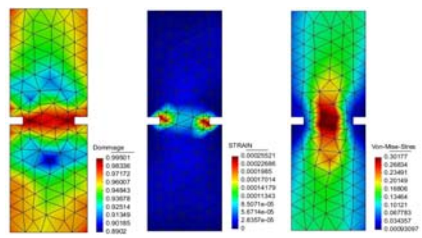


Fig. 6. Distribution of the non-local damage, the equivalent strain and the von Mises stress (R=0.03 mm)

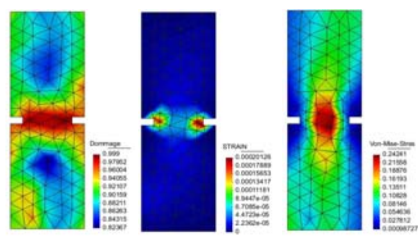


Fig. 7. Distribution of the non-local damage, the equivalent strain and the von Mises stress (R=0.04 mm)

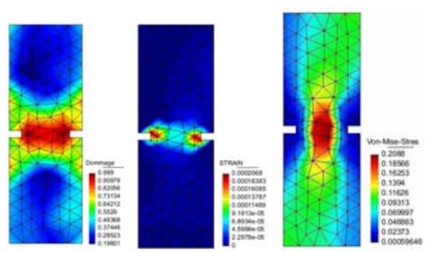


Fig. 8. Distribution of the non-local damage, the equivalent strain and the von Mises stress (R=0.05 mm)

The analysis of isovalues shows practically that the extent and form of the localization zone in the local case (R =0) takes place on a range of elements; however, in the non-local case (R = 0.03, R = 0.04 and R = 0.05), we see that the extent of the form of the localization zone forms a cloud of points around the central part of the plate. The localization zone volume therefore depends on the value of R. By increasing the value of R, we see that the extent of the localization zone increases.

Fig.9 presents the load-displacement curve based on the various values of R (local and non-local cases).

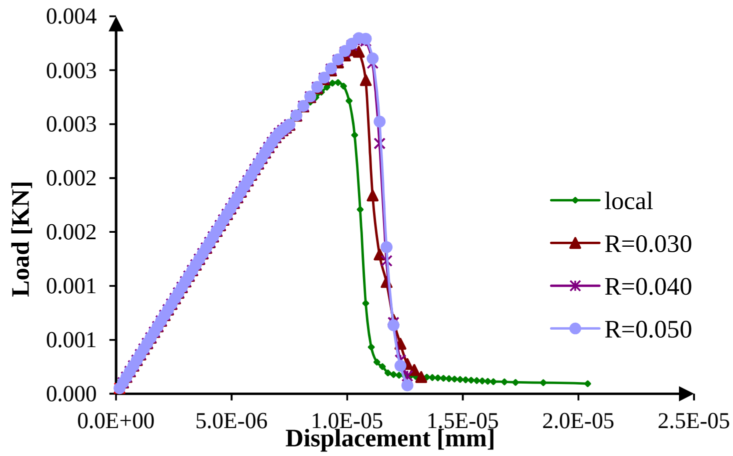
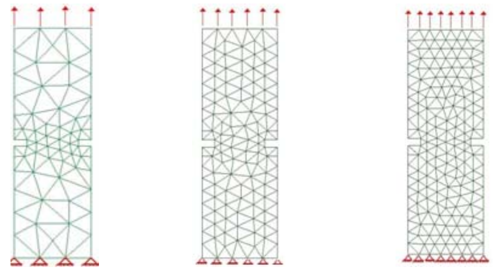


Fig. 9. Load-Displacement Curve for various values of “R”

We see that the rupture deformation increases with the value of R. We also see that during the softening phase, the load-displacement curves superimpose on one another as the R value increases. This shows that the non-nil values of R ensure the independence of the solution in terms of the mesh, and thanks to this non-local formulation, we were able to increase the load at rupture, which is one of the major inconveniences of the non-local formulations. We also see that the curves superimpose when the value of R is equal or greater to 0.04. The value R=0.04 seems to be representative of a zone where the damage is correctly captured by the approach considered in this work. Indeed, Hall and Hayhurst have observed that many materials have at microstructural level a characteristic volume where the damage distribution is almost uniform [42].The dimension of this characteristic volume is related to the material microstructure. Consequently, the correct choice the value of this parameter can only be accessed by combination of numerical and experimental research [43].

5.2. Sensitivity of the mesh

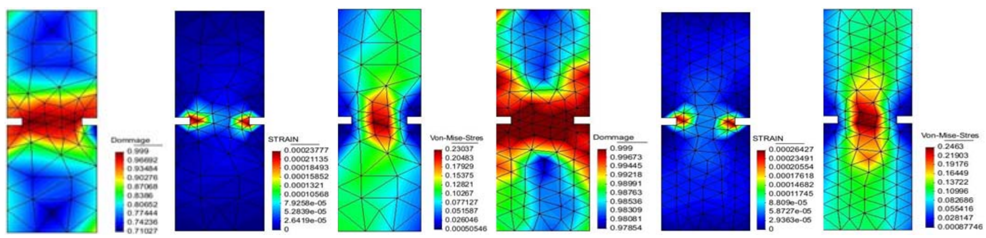
Three regular meshes are used with an element size in the neighbourhood of the crack that is h=0.019m for mesh M1, h=0.011m for mesh M2, and h=0.0088m for mesh M3. The non-local calculation is made for a single value of R =0.05 (Fig.10).



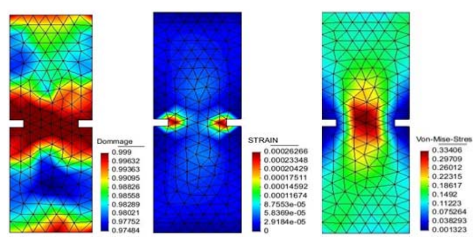
M1: 102 Elements M2: 210 Elements M3: 302 Elements

Fig. 10. Representation of the Various Meshes Used

In Fig. 11, the distribution of damage D, the equivalent strain and the von Mises stress are shown respectively for the various M1, M2 and M3 meshes.



M1: 102 Elements M2: 210 Elements



M3: 302 Elements

Fig. 11. Distribution of the non-local damage, the equivalent strain and the von Mises stress for the various meshes

The analysis of the isovalues shows that the extent and form of the localization zone in the local case (R =0) is done on a range of elements and this regardless of the mesh considered. The localization occurs as observed previously, always on a range of elements, the width of this strip is linked directly to the spatial discretization, which brings us back to saying that, if we take h steps (element size) that are very low, the strip width will be nearly nil [34] however, in the non-local case and for a given R value, we see that the localization zone is almost identical on the three meshes (Fig.11).

Fig.12 presents the load-displacement curve in the local case for the various M1, M2 and M3 meshes. The effort/displacement curves are different based on the mesh used. The consequences of the localization phenomenon at the numerical level are immediate. The mesh controls the size of the damage localization zone with the calculations becoming unstable and the entire deformation is localized in a single element. By refining the mesh, we modify the global response of the structure (which depends explicitly on the number of elements). Furthermore, the energy dissipated to break the plate tends towards zero when the mesh is refined: at the limit, the bar breaks without consuming energy, as stated previously. We might as well be saying that the results obtained numerically are not representative of the structure's real response. In fact, the calculation solution depends pathologically on the size of the elements, but also more generally on their shape, orientation, degree of interpolation, in a word, the approximation space.

Fig.13 presents the load-displacement curve in the non-local case and based on the various M1, M2 and M3 meshes. We can see the objectivity of these results. There is no mesh influence on the overall response of the structure. We can also note that the maximum strength of the plate is identical for the three meshes, which allows us to say that the expected objective of the non-local formulation has been achieved.

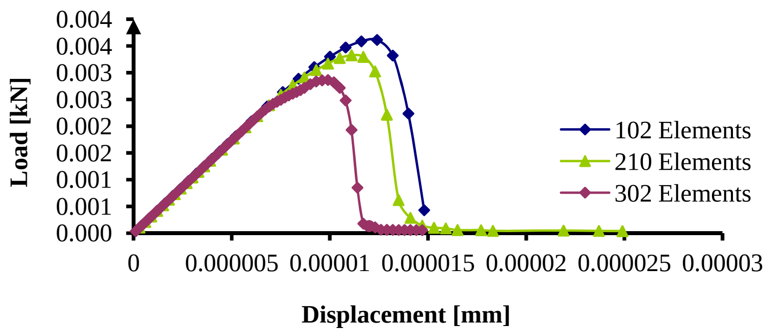


Fig. 12. Load-displacement curve in the local case and based on the various M1, M2 and M3 meshes

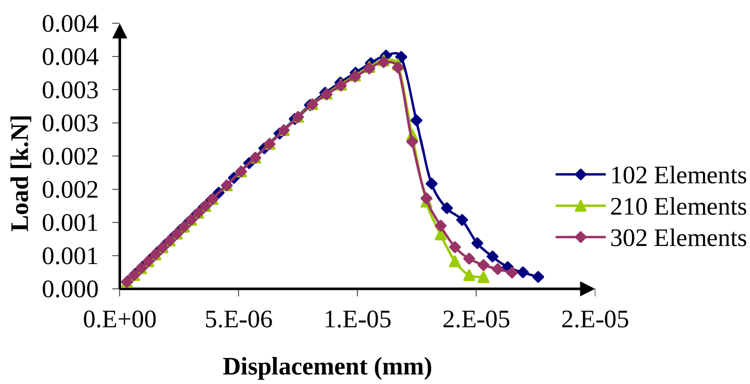


Fig. 13. Load-displacement curve in the non-local case and based on the various M1, M2 and M3 meshes

6. Conclusions

In this work an elastic isotropic model coupled with non-local damage is developed. It is well known that conventional formulations using the finite elements method that present a softening character are strongly dependent on the parameters of discretization during the post-critical phase and are the source of localization: the strong dependence of the structure on the meshing. The progression of the damage is affected by spatio-temporal discretization. The hypothesis pertaining to the local mechanics is thus being questioned. In order to compensate for the faults in the finite elements method and to resolve the problems of dependency on meshing, a model of non-local damage is being proposed. To make non-local calculations, it is necessary to choose the variable that will be delocalized. In this work, we have chosen to delocalize the variable " D" in the form of a Gaussian integral "$\overline{D}$". This model describes the continuous degradation of a medium that was initially presumed to have been free of any cracks or cavities. The problem is resolved using the finite elements method. The numerical and mathematical aspects of the non-local damage model were presented.

In terms of mapping, we chose to examine the effect of the formulation on a tension specimen. The mapping shows that the results obtained through the non-local model are practically independent from the mesh. With the help of the model, we were able to show that the deformation curves superimposed each other, and we were able to eliminate convergence and stability problems in the calculations – which was the set objective for this type of model.

However, there are a number of observations made on this work. We are citing only those that were considered at medium term. First of all, the use of micromorphic media or Cosserat media remains an open field that has only begun to make an appearance in the regularization of local models, the use of other forms of non-local damage can also be considered in convex damage, and, finally, the area of anisotropic models remains one to be further developed.

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