THE ONSET OF TRIPLE-DIFFUSIVE CONVECTION IN A WALTERS' (MODEL B') NANOFLUID LAYER SATURATING A POROUS MEDIUM

Gian C. RANA[1,*], Ramesh CHAND[2] and Veena SHARMA[3]

The onset of triple-diffusive convection in a viscoelastic nanofluid layer heated from below and salted from above and below in porous medium is studied. Walters' (model B') fluid model is used to express the rheological behavior of viscoelastic nanofluid. For porous medium, Darcy model is employed. In the governing equations, the effects of thermophoresis and Brownian diffusion parameters are also introduced through Buongiorno model. The dispersion relation describing for the effect of various parameters is derived by applying linear stability analysis and normal modes analysis method. The effects of solute-Rayleigh number, analogous solute-Rayleigh number, medium porosity, thermo-nanofluid Lewis number, modified diffusivity ratio and nanoparticle Rayleigh number on the stability of stationary convection are presented graphically. The necessary conditions for the existence of oscillatory modes are obtained analytically.

Keywords: Walters' (model B'), triple-diffusive, nanofluid, nanoparticles, Rayleigh number, viscosity, viscoelasticity.
MSC 2010: 76D05, 76E06, 76E20, 76E25

1. Introduction

Triple-diffusive convection is a mixing process of more than two fluid components which diffuse at different rates and has relevance in different areas such as geophysics, soil sciences, food processing, oil reservoir modeling, oceanography, limnology and engineering, among others. The examples of such multiple diffusive convection fluid systems include the solidification of molten alloys, geothermally heated lakes and sea water etc. The problems of triple-diffusive convection fluid in which the density depends on three independently diffusing agencies with different diffusivities have been studied by Griffiths [3], Lopez et al. [6], Pearlstein et al. [9], Rionero [17] and Kango et al. [5]. They found that small concentrations of a third component with a smaller diffusivity can have a significant effect upon the nature of diffusive instabilities.

[1] [1]Department of Mathematics, NSCBM Govt. College, Hamirpur-177005, Himachal Pradesh, INDIA Email: drgcrana15@gmail.com
[2] Department of Mathematics, Govt. College, Nurpur-176 202, District Kangra, Himachal Pradesh, INDIA Email: rameshnahan@yahoo.com
[3] [3]Department of Mathematics and Statistics, Himachal Pradesh University, Shimla-171005, Himachal Pradesh, INDIA Email: veena_math_hpu@yahoo.com

The problems of Walters' (model B') fluid under a considerable amount of different hydrodynamic and hydromagnetic assumptions has been studied by Sharma and Rana [18], Gupta and Aggarwal [4], Rana and Kumar [12], Rana and Kango [11], Rana and Sharma [10]. Shivkumara et al. [20] studied the effect of thermal modulation on the onset of thermal convection in Walters' B viscoelastic fluid in a porous medium while Rana et al. [13] studied the thermosolutal convection in compressible Walters' (model B') fluid permeated with suspended particles in a Brinkman porous medium

In recent years, study of nanofluids attracts many researchers as the nanofluids finds applications in several industries such as the automotive, pharmaceutical and energy supply industries. A considerable number of convection problems in a horizontal layer saturated by a nanofluid have been studied by Choi [2], Buongiorno [1], Tzou [21-22], Nield and Kuznetsov [7-8], Sheu [19], Yadav et al. [23], Rana et al. [14-15], Rana and Chand [16] etc.

In the present paper, triple-diffusive convection in a viscoelastic nanofluid layer saturating a porous medium heated from below and salted from above and below by salt S' and S'' respectively is studied by using the idea of Rionero [17] which include two additional parameters, namely, a viscoelasticity parameter F and solute Rayleigh number Rs''. The effects of solute-Rayleigh number, analogous solute-Rayleigh number, medium porosity, thermo-nanofluid Lewis number, modified diffusivity ratio and nanoparticle Rayleigh number on the stability of stationary convection are presented graphically. The necessary conditions for the existence of oscillatory modes are obtained analytically. The work presented in this paper has not been published as yet.

2. Mathematical model

Consider an infinite horizontal layer of Walters' (model B') viscoelastic nanofluid of thickness d, bounded by the planes $z = 0$ and $z = d$ heated from below and salted from above and below by salt S' and S'' respectively as shown in figure 1. The layer is acted upon by a gravity force $\boldsymbol{g} = (0, 0, -g)$ aligned in the z direction. The temperature T, concentrations C', C'' and the volumetric fraction of nanoparticles φ at the lower (upper) boundary is assumed to take constant values T_0, C_0', C_0'' and φ_0 (T_1, C_1', C_1'' and φ_1), respectively.

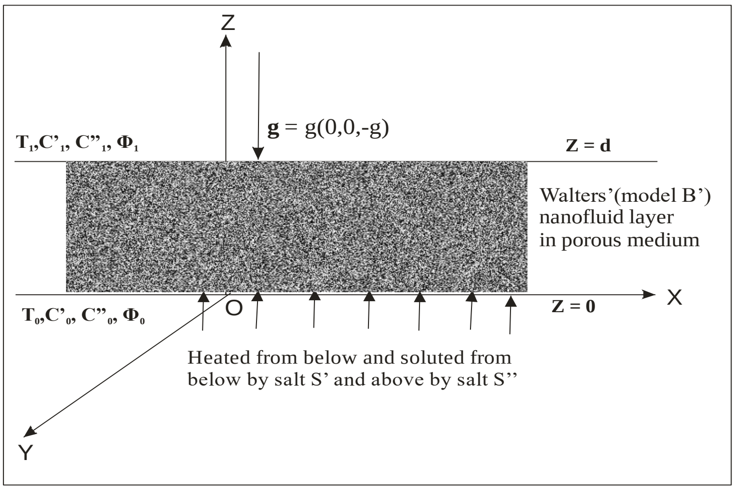

Fig. 1: Schematic sketch of the problem

The basic governing equations of Walters' (model B') viscoelastic nanofluid (Kango et al. [5], Boungiorno [1], Nield and Kuznetsov [8] and Rana and Chand [16]) in a triple-diffusive convection are

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\frac{d\mathbf{q}}{dt} = -\nabla p - \frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\mathbf{q} + \left(\varphi\rho_p + (1-\varphi)\left\{\rho_f\left(1-\beta_T(T-T_0) - \beta_{C'}\left(C' - C_0'\right) + \beta_{C''}\left(C'' - C_0''\right)\right)\right\}\right)\mathbf{g}, \tag{2}$$

$$\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}.\nabla\varphi = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T. \tag{3}$$

$$(\rho c)_m\frac{\partial T}{\partial t} + (\rho c)_f\,\mathbf{q}.\nabla T = k_m\nabla^2 T + \varepsilon(\rho c)_p\left(D_B\nabla\varphi.\nabla T + \frac{D_T}{T}\nabla T.\nabla T\right), \tag{4}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}\cdot\nabla)$ stands for convection derivative, $\rho, \mu, \mu', p, k_1, k_m$, $\mathbf{q}$(u, v, w), denote respectively, the density, viscosity, viscoelasticity, pressure, medium permeability, effective thermal conductivity of porous medium, Darcy velocity vector, D_B is the Brownian diffusion coefficient, D_T is the is the thermophoretic diffusion coefficient, φ is the volume fraction of nanoparticles, ρ_p is the density of nano particles, ρ_f is the density of base fluid, β_T is the uniform temperature gradient , $\beta_{C'}$ and $\beta_{C''}$, are uniform solute gradients, $(\rho c)_f$ is the heat capacity of fluid, $(\rho c)_m$ is the effective heat capacity of porous medium, $(\rho c)_p$ is heat capacity of nanoparticle material, k_m is the effective thermal conductivity of porous medium and we approximate the density of

nanofluid by that of base fluid (i.e., $\rho = \rho_f$) , (Boungiorno [1], Nield and Kuznetsov [8], Sheu [19] and Rana and Chand [16]).
The conservation equations for solute concentrations are

$$\frac{\partial C'}{\partial t} + \mathbf{q}.\nabla C' = D_{S'} \nabla^2 C'. \tag{5}$$

$$\frac{\partial C''}{\partial t} + \mathbf{q}.\nabla C'' = D_{S''} \nabla^2 C''. \tag{6}$$

where $D_{S'}$ and $D_{S''}$ are the solute diffusivities.

We assume that the temperature and volume fraction of the nanoparticles are constant at the boundaries. Then the boundary conditions appropriate to the problem (Nield and Kuznetsov [8]) are

$$w = 0, \quad T = T_{0,} \quad \varphi = \varphi_0 \ , C' = C_0' \ , C'' = C_0'' , \frac{\partial^2 w}{\partial z^2} = 0 \text{ at} \quad z = 0, \tag{7}$$

$$w = 0, \quad T = T_{1,} \ C' = C_1' \ , C'' = C_1'' \ \varphi = \varphi_1 \ , \frac{\partial^2 w}{\partial z^2} = 0 \text{ at} \quad z = 1. \tag{8}$$

We introduce non-dimensional variables as

$$(x^*, y^*, z^*,) = \left(\frac{x, y, z}{d}\right), (u^*, v^*, w^*,) = \left(\frac{u, v, w}{\kappa_m}\right)d, \ t^* = \frac{t\alpha_f}{d^2}, \ p^* = \frac{pd^2}{\mu\alpha_f},$$

$$\varphi^* = \frac{(\varphi - \varphi_0)}{(\varphi_1 - \varphi_0)}, \ T^* = \frac{(T - T_0)}{(T_0 - T_1)}, \ C'^* = \frac{(C' - C_0')}{(C_0' - C_1')}, \ C''^* = \frac{(C'' - C_0'')}{(C_0'' - C_1'')}, \ \alpha_f = \frac{\kappa}{\rho c}$$

There after dropping the dashes (*) for convenience.
Eqs. (1)-(6) in non-dimensional form can be written as

$$\nabla.\mathbf{q} = 0, \tag{9}$$

$$\frac{1}{P_r}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p - \left(1 - F\frac{\partial}{\partial t}\right)\mathbf{q} - Rm\,\hat{e}_z + RaT\,\hat{e}_z - Rn\varphi\,\hat{e}_z - \frac{R_{S'}C'}{Le'}\hat{e}_z - \frac{R_{S''}C''}{Le''}\hat{e}_z, \tag{10}$$

$$\frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}.\nabla\varphi = \frac{1}{Ln}\nabla^2\varphi + \frac{N_A}{Ln}\nabla^2 T, \tag{11}$$

$$\frac{\partial T}{\partial t} + \mathbf{q}.\nabla T = \nabla^2 T + \frac{N_B}{Ln}\nabla\varphi.\nabla T + \frac{N_A N_B}{Ln}\nabla T.\nabla T, \tag{12}$$

$$\frac{\partial C'}{\partial t} + \mathbf{q}.\nabla C' = \frac{1}{Le'}\nabla^2 C'. \tag{13}$$

$$\frac{\partial C''}{\partial t} + \mathbf{q}.\nabla C'' = \frac{1}{Le''}\nabla^2 C''. \tag{14}$$

where

$\mathrm{Pr} = \dfrac{\mu d^2}{\rho \alpha_f k_1}$ is the Darcy-Prandtl number, $Le^{'} = \dfrac{\alpha_f}{D_{S^{'}}}$ is the thermosolutal Lewis number, $Le^{''} = \dfrac{\alpha_f}{D_{S^{''}}}$ is analogous thermosolutal Lewis number, $Ln = \dfrac{\alpha_f}{D_B}$ is the thermo-nanofluid Lewis number, $Ra = \dfrac{\rho g \beta_T k_1 d (T_0 - T_1)}{\mu \alpha_f}$ is the thermal-Darcy Rayleigh Number, $Rs^{'} = \dfrac{\rho g \beta_{C^{'}} k_1 d (C_0^{'} - C_1^{'})}{\mu D_{S^{'}}}$ is the solute Rayleigh Number, $Rs^{''} = \dfrac{\rho g \beta_{C^{''}} k_1 d (C_0^{''} - C_1^{''})}{\mu D_{S^{''}}}$ is the analogous solute Rayleigh Number, $\mathrm{R_m} = \dfrac{(\rho_p \varphi_0 + \rho(1 - \varphi_0)) g k_1 d}{\mu \alpha_f}$ is the basic density Rayleigh number, $\mathrm{R}n = \dfrac{(\rho_p - \rho)(\varphi_1 - \varphi_0) g k_1 d}{\mu \alpha_f}$ is the nanoparticle Rayleigh number, $N_A = \dfrac{D_T (T_0 - T_1)}{D_B T_1 (\varphi_1 - \varphi_0)}$ is the modified diffusivity ratio, $\mathrm{N_B} = \dfrac{\varepsilon (\rho c)_p (\varphi_1 - \varphi_0)}{(\rho c)_f}$ is the modified particle- density ratio and $\mathrm{F} = \dfrac{\mu'}{\mu}$ is the viscoelasticity parameter.

In the spirit of the Oberbeck-Boussinesq approximation, Eq. (12) was linearized by neglecting a term proportional to the product of φ and T.

The dimensionless boundary conditions are

$$w = 0, \quad \mathrm{T} = 1, \mathrm{C}^{'} = 1, \mathrm{C}^{''} = 1, \frac{\partial^2 w}{\partial z^2} = 0, \quad \varphi = 0 \quad \text{at} \quad z = 0\ , \tag{15}$$

$$w = 0,\ T = 0, \mathrm{C}^{'} = 0, \mathrm{C}^{''} = 0, \frac{\partial^2 w}{\partial z^2} = 0, \quad \varphi = 1 \quad \text{at}\ z = 1. \tag{16}$$

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, which are of the form

$$\mathbf{q}(u,v,w) = 0 + q^{\bullet}(u,v,w), T = (1-z) + T^{\bullet}, \mathrm{C}_b^{'} = (1-z) + C^{'\bullet}, \mathrm{C}_b^{''} = (1-z) + C^{''\bullet},$$
$$\varphi = z + \varphi^{\bullet}, p = p_b + p^{\bullet}. \tag{17}$$

Using Eq. (17) into Eqs. (9) – (14), linearizing the resulting equations by neglecting nonlinear terms that are product of prime quantities and dropping the primes ($\bullet$) for convenience, the following equations are obtained

$$\nabla \cdot \mathbf{q} = 0, \tag{18}$$

$$\frac{1}{\Pr}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p - \left(1 - F\frac{\partial}{\partial t}\right)\mathbf{q} + \mathrm{R_D T}\,\hat{e}_z - \mathrm{Rn}\varphi\,\hat{e}_z - \frac{R_{S'}C'}{Le'}\hat{e}_z - \frac{R_{S''}C''}{Le''}\hat{e}_z, \tag{19}$$

$$\frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon}w = \frac{1}{Ln}\nabla^2\varphi + \frac{N_A}{Ln}\nabla^2 T, \tag{20}$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{Ln}\left(\frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z}\right) - \frac{2\mathrm{N}_A N_B}{Ln}\frac{\partial T}{\partial z}. \tag{21}$$

$$\frac{\partial C'}{\partial t} - w = \frac{1}{Le'}\nabla^2 C'. \tag{22}$$

$$\frac{\partial C''}{\partial t} - w = \frac{1}{Le''}\nabla^2 C''. \tag{23}$$

Boundary conditions for Eqs. (30) - (35) are

$$w = 0,\quad \mathrm{T} = 0,\ \mathrm{C'} = 0,\ \mathrm{C''} = 0,\ \frac{\partial^2 w}{\partial z^2} = 0,\quad \varphi = 0 \quad \text{at} \quad z = 0\ , \tag{24}$$

$$w = 0,\ T = 0,\ \mathrm{C'} = 0,\ \mathrm{C''} = 0,\ \frac{\partial^2 w}{\partial z^2} = 0,\qquad \varphi = 0 \qquad \text{at}\ \ z = 1. \tag{25}$$

The parameter *Rm* is not involved in Eqs. (30)-(35) it is just a measure of the basic static pressure gradient.

The eight unknown's *u, v, w, p, T, C', C''* and φ can be reduced to five by operating Eq. (19) with e_z.curl curl, which yields

$$\frac{1}{\Pr}\frac{\partial}{\partial t}\nabla^2 w = -\left(1 - F\frac{\partial}{\partial t}\right)\nabla^2 w + R_D\nabla_H^2 T - Rn\nabla_H^2\varphi - \frac{Rs'}{Le'}\nabla_H^2 C' - \frac{Rs''}{Le''}\nabla_H^2 C'', \tag{26}$$

where $\nabla_H^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$. is the two-dimensional Laplace operator on the horizontal plane and $\zeta = \hat{e}_z.curl\mathbf{q}$ is the z-component of vorticity.

3. Normal modes analysis

We express the disturbances into normal modes of the form

$$[w, T, \mathrm{C'}, \mathrm{C''}, \varphi] = [W(z), \Theta(z), \Gamma(z), \Lambda(z), \Phi(z)]\exp(ilx + imy + \omega t), \tag{27}$$

where l, m are the wave numbers in the *x* and *y* direction, respectively, and ω is the growth rate of the disturbances.

Substituting Eq. (32) into Eqs. (31) and (19) - (23), we obtain the following eigen value problem

$$\left[\frac{\omega}{\Pr} + 1 - \omega F\right](D^2 - a^2)W + a^2 R_D\Theta - a^2 Rn\Phi - \frac{Rs'}{Le'}a^2\Gamma - \frac{Rs''}{Le''}a^2\Lambda = 0, \tag{28}$$

$$W+\left(\frac{1}{Le'}\left(D^2-a^2\right)-\omega\right)\Gamma=0, \tag{29}$$

$$W+\left(\frac{1}{Le''}\left(D^2-a^2\right)-\omega\right)\Lambda=0, \tag{30}$$

$$W+\left(D^2+\frac{N_B}{Ln}D-\frac{2N_AN_B}{Ln}D-a^2-\omega\right)\Theta-\frac{N_B}{Ln}D\Phi=0, \tag{31}$$

$$\frac{1}{\varepsilon}W-\frac{N_A}{Ln}\left(D^2-a^2\right)\Theta-\left(\frac{1}{Ln}\left(D^2-a^2\right)-\omega\right)\Phi=0, \tag{32}$$

$$W=0,\quad D^2W=0, \Gamma=0, \Lambda=0, \Theta=0, \Phi=0 \text{ at } \quad z=0,1 \quad , \tag{33}$$

where $D=\dfrac{d}{dz}$ and $a^2 = l^2+ m^2$ is the dimensionless horizontal wave number.

4. Linear stability analysis and dispersion relation

Considering solutions W, Θ, Γ, Λ and Φ of the form

$$W=W_0\sin(\pi z),\ \Gamma=\Gamma_0\sin(\pi z), \Lambda=\Lambda_0\sin(\pi z), \Theta=\Theta_0\sin(\pi z), \Phi=\Phi_0\sin(\pi z). \tag{34}$$

which satisfy the boundary conditions of Eq. (33) .

Substituting Eq. (35) into Eqs. (28) – (32), integrating each equation from z = 0 to z = 1, and performing some integration by parts, we obtain the following matrix equation

$$\begin{bmatrix} \left(\frac{\omega}{\Pr}+1-\omega F\right)J^2 & -a^2R_D & \frac{a^2Rs'}{Le'} & \frac{a^2Rs''}{Le''} & a^2Rn \\ 1 & 0 & -\left(\frac{J^2}{Le'}+\omega\right) & 0 & 0 \\ 1 & 0 & 0 & -\left(\frac{J^2}{Le''}+\omega\right) & 0 \\ 1 & -\left(J^2+\omega\right) & 0 & 0 & 0 \\ \frac{1}{\varepsilon} & \frac{N_AJ^2}{Ln} & 0 & 0 & \left(\frac{J^2}{Ln}+\omega\right) \end{bmatrix} \begin{bmatrix} W_0 \\ \Gamma_0 \\ \Lambda_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix}=0, \tag{35}$$

where $J^2=\pi^2+a^2$ is the total wave number.

The linear homogeneous system of equations (35) has a non-trivial solution if and only if

$$\begin{vmatrix} \left(\frac{\omega}{\Pr}+1-\omega F\right)J^2 & -a^2R_D & \frac{a^2Rs^{'}}{Le^{'}} & \frac{a^2Rs^{''}}{Le^{''}} & a^2Rn \\ 1 & 0 & -\left(\frac{J^2}{Le^{'}}+\omega\right) & 0 & 0 \\ 1 & 0 & 0 & -\left(\frac{J^2}{Le^{''}}+\omega\right) & 0 \\ 1 & -\left(J^2+\omega\right) & 0 & 0 & 0 \\ \frac{1}{\varepsilon} & \frac{N_AJ^2}{Ln} & 0 & 0 & \left(\frac{J^2}{Ln}+\omega\right) \end{vmatrix} = 0$$

which gives

$$R_D = \frac{J^2}{a^2}\left(\frac{\omega}{\Pr}+1-\omega F\right).\left(\omega+J^2\right)+\left(\omega+J^2\right)\left(\frac{Rs^{'}}{\omega Le^{'}+J^2}+\frac{Rs^{''}}{\omega Le^{''}+J^2}\right)-\frac{\left(\omega+J^2\right)Ln+\varepsilon NaJ^2}{\left(\omega Ln+J^2\right)\varepsilon}Rn \quad . \quad (36)$$

Eq. (36) is the required dispersion relation accounting for the effect of Prandtl number, Darcy number, thermo-solutal Lewis number, analogous thermo-solutal Lewis number, thermo-nanofluid Lewis number, solute Rayleigh Number, analogous solute Rayleigh Number, nanoparticle Rayleigh number, medium porosity and modified diffusivity ratio on the onset of triple diffusive convection in a layer of nanofluid.

To examine the stability of the system, the real part of ω is set to zero and we take $\omega = i\omega_i$ in Eq. (36), then we obtain

$$R_D = \frac{J^2}{a^2}\left(J^2+F-\frac{1}{\Pr}\right)\omega_i^2+\frac{J^4+\omega_i^2Le^{'}}{J^4+\omega_i^2Le^{'2}}Rs^{'}+\frac{J^4+\omega_i^2Le^{''}}{J^4+\omega_i^2Le^{''2}}Rs^{''}-\frac{J^4\left(Ln+\varepsilon N_A\right)+\omega_i^2Ln^2}{\left(J^4+\omega_i^2Ln^2\right)\varepsilon}+i\omega_i\Delta, \quad (37)$$

where

$$\Delta = \frac{J^2}{a^2}\left(1+\frac{J^2}{\Pr}-\frac{FJ^2}{\Pr}\right)-\frac{J^2\left(Le^{'}-1\right)}{J^4+\omega_i^2Le^{'2}}Rs^{'}-\frac{J^2\left(Le^{''}-1\right)}{J^4+\omega_i^2Le^{''2}}Rs^{''}+\frac{J^2\left(1-Ln-\varepsilon N_A\right)Ln}{\left(J^4+\omega_i^2Ln^2\right)\varepsilon}Rn. \quad (38)$$

As R_D is a physical quantity it must be real. Thus it follows from Eq. (38) that either $\omega_i = 0$ (exchange of stability, steady state) or $\Delta = 0$ ($\omega_i \neq 0$, overstability or oscillatory onset).

5. Stationary convection

For stationary convection, putting $\omega = 0$ in equation (36), we obtain

$$\left(Ra\right)_s = \frac{\left(\pi^2 + a^2\right)^2}{a^2} + \mathrm{Rs}' + \mathrm{Rs}'' - \left(\frac{\mathrm{Ln}}{\varepsilon} + \mathrm{N_A}\right)\mathrm{Rn}. \tag{39}$$

Eq. (39) is identical to that obtained by Nield and Kuznetsov [8], Sheu [19], Rana et al. [14-15] and Rana and Chand [16].
In the absence of the solute gradient parameter *Rs''*, Eq. (39) reduces to

$$\left(Ra\right)_s = \frac{\left(\pi^2 + a^2\right)^2}{a^2} + Rs' - \left(\frac{\mathrm{Ln}}{\varepsilon} + \mathrm{N_A}\right)Rn, \tag{40}$$

Equation (40) is same as the results derived by Nield and Kuznetsov [8], Sheu [19], Rana et al. [14-15] and Rana and Chand [16]. The critical cell size at the onset of instability is obtained by minimizing Ra with respect to *a*. Thus, the critical cell size must satisfy

$$\left(\frac{\partial Ra}{\partial a}\right)_{a=a_c} = 0, \tag{41}$$

Equation (41) which gives $a_c = \pi$. (42)

And the corresponding critical thermal Rayleigh number $\left(Ra\right)_c$ on the onset of stationary convection is given by

$$\left(Ra\right)_c = 4\pi^2 + \mathrm{Rs}' + \mathrm{Rs}'' - \left(\frac{\mathrm{Ln}}{\varepsilon} + \mathrm{N_A}\right)\mathrm{Rn}. \tag{43}$$

It is noted that if *Rn* is positive then *Ra* is minimized by a stationary convection. The result given in equation (61) is a good agreement with the result derived by Sheu [19] and Rana and Chand [16].

Since the elastico-viscous parameter F is not present in equation (39), it may be concluded that in the stationary case ($\omega = 0$) Walters' (model B') viscoelastic fluid behaves like an ordinary Newtonian fluid. According to the definition of nanoparticle Rayleigh number Rn corresponds to negative value of Rn for heavy nanoparticle $\rho_p > \rho$. In such cases, values of modified diffusivity ratio N_A are also negative according to the definition of N_A. In the following discussion, we take the values of Rn and N_A negative. Also in Eq. (43) the particle increment parameter N_B does not appear and the diffusivity ratio parameter N_A appears only in association with the nanoparticle Rayleigh number *Rn*. This implies that the nanofluid cross-diffusion terms approach to be dominated by the regular cross-diffusion term.

Now to study the effect of solute Rayleigh number (Rs'), analogous solute Rayleigh number (Rs''), medium porosity (ε), thermo-nanofluid Lewis number (Ln), diffusivity ratio ($\mathrm{N_A}$) and nanoparticle Rayleigh number (Rn) on the stationary convection, we examine the behaviour of $\frac{\partial (Ra)_s}{\partial Rs'}$, $\frac{\partial (Ra)_s}{\partial Rs''}$, $\frac{\partial (Ra)_s}{\partial Ln}$, $\frac{\partial (Ra)_s}{\partial \varepsilon}$, $\frac{\partial (Ra)_s}{\partial N_A}$ and $\frac{\partial (Ra)_s}{\partial Rn}$ analytically.

From Eq. (39), we obtain $\frac{\partial (Ra)_s}{\partial Rs'} = +1,$ (44)

which is positive, therefore, solute Rayleigh number (Rs') inhibits the onset of triple-diffusive stationary convection implying thereby solute Rayleigh number (Rs') has stabilizing effect on the system which is an agreement with the results derived by Nield and Kuznetsov [8], Sheu [19], Rana et al. [14-15] and Rana and Chand [16].

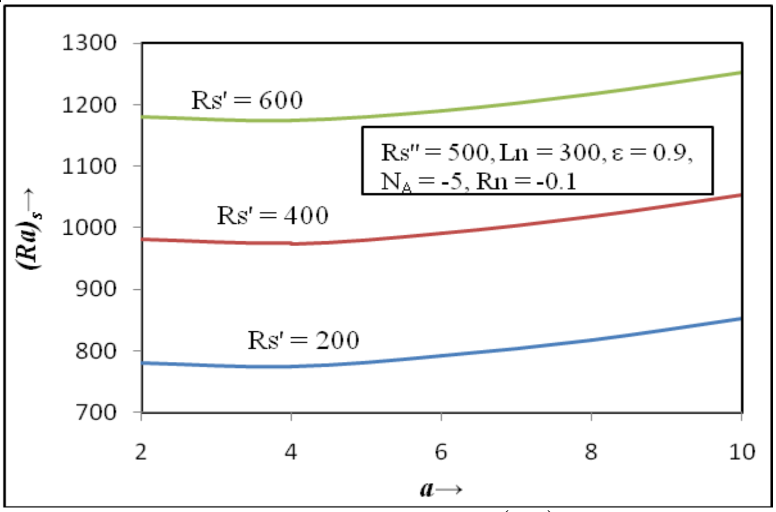

Fig.2. Variation of stationary thermal Rayleigh number $(Ra)_s$ with the wave number *a* for different values of solute Rayleigh number (Rs')

In Fig. 2, the stationary thermal Rayleigh number $(Ra)_s$ is plotted against dimensionless wave number *a* different values of solute Rayleigh number (Rs') as shown. This shows that as Rs' increases, the stationary thermal Rayleigh number $(Ra)_s$ also increases. Thus, solute Rayleigh number (Rs') has stabilizing effect on stationary convection which is in good agreement with the result obtained analytically from Eq. (44).

It is evident from Eq. (39) that $\frac{\partial (Ra)_s}{\partial Rs''} = +1,$ (45)

which is positive, therefore, analogous solute Rayleigh number (Rs'') inhibits the onset of triple-diffusive stationary convection implying thereby solute Rayleigh number (Rs'') has stabilizing effect on the system which is an agreement with the results derived by Kango et al. [5].

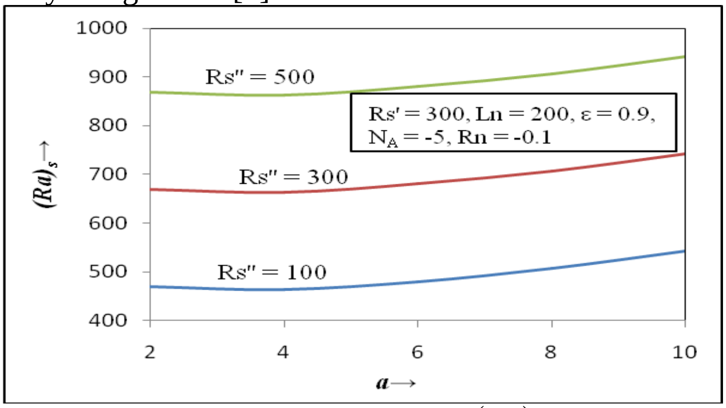

Fig.3. Variation of stationary thermal Rayleigh number $(Ra)_s$ with the wave number *a* for different values of analogous solute Rayleigh number (Rs'').

In Fig. 3, the stationary thermal Rayleigh number $(Ra)_s$ is plotted against dimensionless wave number *a* different values of solute Rayleigh number (Rs'') as shown. This shows that as Rs' increases, the thermal Rayleigh number $(Ra)_s$ also increases. Thus, solute Rayleigh number (Rs'') has stabilizing effect on stationary convection which is in good agreement with the result obtained analytically from Eq. (45).

From Eq. (39), we obtain

$$\frac{\partial (Ra)_s}{\partial Ln} = -\frac{Rn}{\varepsilon}, \tag{46}$$

implying thereby thermo-nanofluid Lewis number (Ln) inhibits the onset of triple-diffusive stationary convection. Thus, thermo-nanofluid Lewis number (Ln) has stabilizing effect on the system if $Rn < 0$ (i.e., bottom heavy arrangement) which is a good agreement with the results derived by Nield and Kuznetsov [8], Sheu [19], Rana et al. [14-15] and Rana and Chand [16].

In Fig. 4, the stationary thermal Rayleigh number $(Ra)_s$ is plotted against dimensionless wave number *a* different values of thermo-nanofluid Lewis number (Ln) as shown. This shows that as *Ln* increases, the thermal Rayleigh number $(Ra)_s$ also increases for bottom-heavy arrangements.

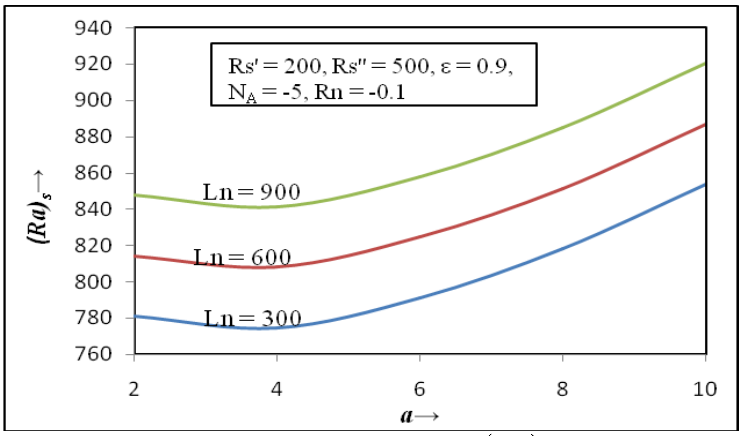

Fig.4. Variation of stationary thermal Rayleigh number $(Ra)_s$ with the wave number *a* for different values of thermo-nanofluid Lewis number (Ln).

Thus, of thermo-nanofluid Lewis number (Ln) has stabilizing effect on stationary convection which is in good agreement with the result obtained analytically from Eq. (46).
From Eq. (39), we obtain

$$\frac{\partial (Ra)_s}{\partial \varepsilon} = \frac{LnRn}{\varepsilon^2}, \tag{47}$$

Thus, medium porosity (ε) has destabilizing effect on the system if $Rn < 0$ (i.e., bottom heavy arrangement) which is a good agreement with the results derived by Sheu [19], Rana et al. [14-15] and Rana and Chand [16]

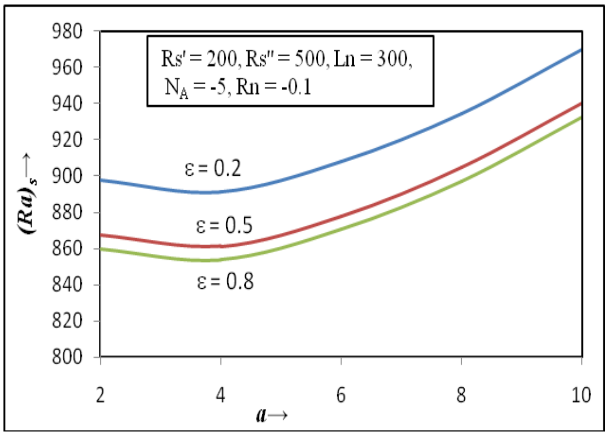

Fig.5. Variation of stationary thermal Rayleigh number $(Ra)_s$ with the wave number *a* for different values of medium porosity (ε).

In Fig. 5, the stationary thermal Rayleigh number $(Ra)_s$ is plotted against dimensionless wave number *a* different values of medium porosity (ε) as shown. This shows that as medium porosity (ε) increases, the thermal Rayleigh number $(Ra)_s$ decreases for bottom-heavy arrangements. Thus, medium porosity (ε) has destabilizing effect on stationary convection which is in good agreement with the result obtained analytically from Eq. (47).

From Eq. (39), we obtain

$$\frac{\partial (Ra)_s}{\partial N_A} = -Rn, \tag{48}$$

implying thereby diffusivity ratio (N_A) inhibits the onset of triple-diffusive stationary convection. Thus, diffusivity ratio (N_A) has stabilizing effect on the system if $Rn < 0$ (i.e., bottom heavy arrangement) which is a good agreement with the results derived by Nield and Kuznetsov [8], Sheu [19], Rana et al. [14-15] and Rana and Chand [16].

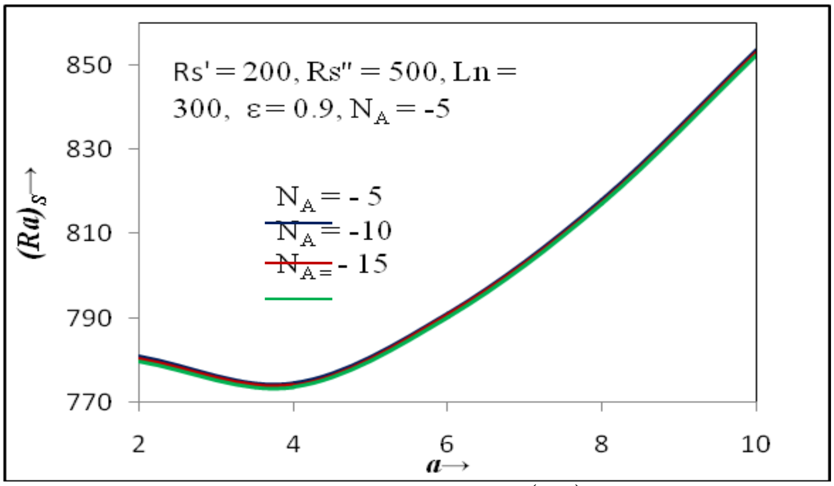

Fig.6. Variation of stationary thermal Rayleigh number $(Ra)_s$ with the wave number *a* for different values of diffusivity ratio (N_A).

In Fig. 6, the stationary thermal Rayleigh number $(Ra)_s$ is plotted against dimensionless wave number *a* different values of diffusivity ratio (N_A) as shown. This shows that as N_A increases slightly, the thermal Rayleigh number $(Ra)_s$ also increases for bottom-heavy arrangements. Thus, diffusivity ratio (N_A) has low stabilizing effect on stationary convection which is in good agreement with the result obtained analytically from Eq. (48).

It is evident from Eq. (39) that

$$\frac{\partial (Ra)_s}{\partial Rn} = -\left(\frac{Ln}{\varepsilon} + N_A\right), \tag{49}$$

which is negative implying thereby nanoparticle Rayleigh number (Rn) hastens the triple-diffusive convection implying thereby nanoparticle Rayleigh number (Rn) has destabilizing effect on the system which is a good agreement with the results derived by Nield and Kuznetsov [8], Sheu [19], Rana et al. [14-15] and Rana and Chand [16]

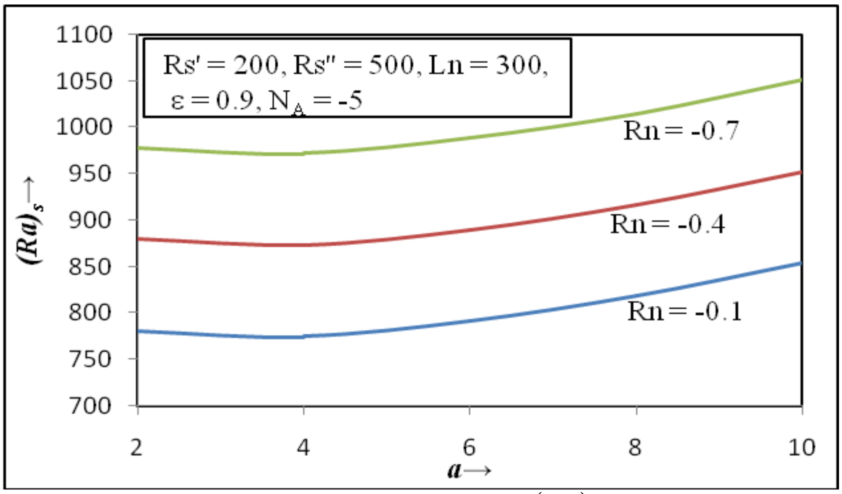

Fig.7. Variation of stationary thermal Rayleigh number $(Ra)_s$ with the wave number *a* for different values of nanoparticle Rayleigh number (Rn).

In Fig. 7, the stationary thermal Rayleigh number $(Ra)_s$ is plotted against dimensionless wave number *a* different values of nanoparticle Rayleigh number (Rn) as shown. This shows that as *Rn* increases, the thermal Rayleigh number $(Ra)_s$ decreases for bottom-heavy arrangements. Thus, nanoparticle Rayleigh number (Rn) has destabilizing effect on stationary convection which is in good agreement with the result obtained analytically from Eq. (49).

It is evident from Eq. (39) that

$$\frac{\partial (Ra)_s}{\partial Rn} = \frac{RnLn}{\varepsilon^2}, \tag{50}$$

which is negative implying thereby nanoparticle Rayleigh number (Rn) hastens the triple-diffusive convection implying thereby nanoparticle Rayleigh number (Rn) has destabilizing effect on the system which is a good agreement with the results derived by Nield and Kuznetsov [8], Sheu [19], Rana et al. [14-15] and Rana and Chand [16].

6. Oscillatory convection

The oscillatory Rayleigh number [obtained by putting $\Delta = 0$ in Eq. (37)] is given by

$$\left(R_D\right)_{osc} = \frac{J^2}{a^2}\left(J^2 + F - \frac{1}{\Pr}\right) + \frac{J^4 + \omega^2 Le'}{J^4 + \omega^2 Le'^2} Rs' + \frac{J^4 + \omega^2 Le''}{J^4 + \omega^2 Le''^2} Rs'' - \frac{J^4\left(Ln + \varepsilon N_A\right) + \omega^2 Ln^2}{\left(J^4 + \omega^2 Ln^2\right)\varepsilon}. \quad (51)$$

The condition $\Delta = 0, \omega_i \neq 0$ gives an expression for the frequency of oscillation ω_i^2 as

$$a_3\left(\omega_i^2\right)^3 + a_2\left(\omega_i^2\right)^2 a_1\left(\omega_i^2\right) + a_0 = 0, \quad (52)$$

Where

$$a_0 = \left(\pi^2 + a^2\right)^4\left(\left(\pi^2 + a^2\right)^3\left(1 + \frac{J^2}{\Pr} - \frac{FJ^2}{Da}\right) - \left(Le' - 1\right)a^2 Rs' - \left(Le'' - 1\right)Rs'' - \frac{\left(1 - \varepsilon N_A - Ln\right)}{\varepsilon} J^2 LnRn\right),$$

$$a_1 = \left(\pi^2 + a^2\right)^2\left(\begin{array}{l}\frac{\left(\pi^2 + a^2\right)^3}{a^2}\left(1 + \frac{J^2}{\Pr} - FJ^2\right)\left(Le'^2 + Le''^2 + Ln^2\right) - \left(Le' - 1\right)\left(Le''^2 + Ln^2\right)Rs' - \\ \left(Le'' - 1\right)\left(Le'^2 + Ln^2\right)Rs'' - \frac{\left(1 - \varepsilon N_A - Ln\right)}{\varepsilon}\left(Le'^2 + Le''^2\right)aLnRn\end{array}\right),$$

$$a_2 = \left(\pi^2 + a^2\right)\left(\frac{\left(\pi^2 + a^2\right)^2}{a^2}\left(1 + \frac{J^2}{\Pr} - FJ^2\right)\left(Le'^2 Le''^2 + Ln^2 Le'^2 + Ln^2 Le'''^2\right)'\right) - \left(Le' - 1\right)Le''^2 Rs'$$
$$- \left(Le'' - 1\right)Le'^2 Rs'' - \frac{\left(1 - \varepsilon N_A - Ln\right)}{\varepsilon} Le'^2 Le''^2 Rn,$$

$$a_3 = \frac{\left(\pi^2 + a^2\right)}{a^2}\left(1 + \frac{J^2}{\Pr} - FJ^2\right)Le'^2 Le''^2 Ln^2.$$

We find the oscillatory neutral solution from Eq. (51) as follows: first find the roots for ω_i^2 of Eq. (45). If there are no positive roots, the oscillatory instability is not possible. If there are positive roots, the critical thermal Rayleigh number for oscillatory convection can be derived numerically by minimizing Eq. (44) with respect to wave number after substituting various values of physical parameters for ω_i^2 of Eq. (52) to determine their effects on the onset of oscillatory convection. Since Ln is of order $10^2 - 10^3$, $1 < N_A < 10$ and so $(1 - \varepsilon N_A - Ln) < 0$. Thus Eq. (51) does not admit positive value of ω_i^2 if $Le^{'}, Le^{''} > 1$. Hence the necessary conditions for the occurrence of oscillatory convection are $Le^{'}, Le^{''} > 1$.

7. Conclusions

The onset of triple-diffusive convection in a layer of viscoelastic Nanofluid heated from below and soluted from below and above has been investigated by using linear stability theory. The main conclusions are:

- For the case of stationary convection, the viscoelastic nanofluid behaves like an ordinary nanofluid.
- The solute Rayleigh number ($Rs^{'}$) and analogous solute Rayleigh number ($Rs^{''}$) have stabilizing effects on the onset of stationary convection for both top-heavy and bottom-heavy arrangements.
- The thermo-nanofluid Lewis number (Ln) has stabilizing effect on the onset of stationary convection for bottom-heavy arrangements.
- The medium porosity (ε) has destabilizing effect on stationary convection.
- The diffusivity ratio (N_A) has very low stabilizing effect on the onset of stationary convection for bottom-heavy arrangements.
- Nanoparticle Rayleigh number (Rn) has destabilizing effect on the onset of stationary convection.
- Necessary conditions for the occurrence of oscillatory convection are $Le^{'}, Le^{''} > 1$.

Acknowledgement

Authors would like to thank the learned referees for their valuable comments and suggestions for the improvement of quality of the paper.

REFERENCES

[1] *J. Buongiorno*, Convective transport in nanofluids, ASME J. of Heat Transfer. 1282006(2006) 240-250.

[2] *S. Choi*, Enhancing thermal conductivity of f luids with nanoparticles, In: D. A. Siginer and H.P. Wang (Eds), Developments and Applications of Non-Newtonian Flows, ASME FED, 231/MD, 66(1995), 99-105.

[3] *R.W. Griffiths*, The influence of a third diffusing component upon the onset of convection, J Fluid Mech. 92(1979), 659-670.

[4] *U. Gupta, P. Aggarwal*, Thermal instability of compressible Walters' (model B') fluid in the presence of hall currents and suspended particles, Thermal Science. 15(2011), 487-500.

[5] *S.K. Kango, G.C. Rana, R. Chand*, Triple-diffusive convection in Walters' (model B') fluid with varying gravity field saturating a porous medium, Studia Geotechnica et Mechanica. XXXV(2013), 45-56.

[6] *A.R. Lopez, L.A. Romero and A. J. Pearlstein*, Effect of rigid boundaries on the onset of convective instability in a triply diffusive fluid layer, Physics of Fluids. A 2(1990), 897-902.

[7] *D.A. Nield, and A.V. Kuznetsov*, Thermal instability in a porous medium layer saturated by a nanofluid, Int. J. Heat Mass Transf. 52(2009), 5796–5801.

[8] *D.A. Nield, and A.V. Kuznetsov*, The onset of double-diffusive convection in a nanofluid layer, Int. J. of Heat and Fluid Flow. 32(2011), 771-776.

[9] *A.J. Pearlstein, R.M. Harris, G. Terrones*, The onset of convective instability in a triply diffusive fluid layer, J Fluid Mech. 202(1989), 443-465.

[10] *G.C. Rana, V. Sharma*, Hydromagnetic thermosolutal instability of Walters' (Model B') rotating fluid permeated with suspended particles in porous medium, Int. J. of Multiphysics. 5(2011), 325-338.

[11] *G.C. Rana, S.K. Kango*, Effect of rotation on thermal instability of Compressible Walters' (Model B') Elastico-Viscous fluid in porous medium, JARAM, 3(2011), 44-57.

[12] *G.C. Rana, S. Kumar*, Thermal instability of Walters' (Model B') Elastico-Viscous rotating fluid permitted with suspended particles and variable gravity field in porous medium, Engineering Trans. 60(2012), 55-68.

[13] *G.C. Rana, R. Chand, S. Kumar*, Thermosolutal convection in compressible Walters' (model B') fluid permeated with suspended particles in a Brinkman porous medium, J. of Computational Multiphase flow. 4(2012), 211-224.

[14] *G.C. Rana, R.C. Thakur, S.K. Kango*, On the onset of thermosolutal instability in a layer of an elastico-viscous nanofluid in porous medium, FME Transactions. 42(2014), 1-9.

[15] *G.C. Rana, R.C. Thakur, S.K. Kango*, On the onset of Double-Diffusive convection in a layer of nanofluid under rotation saturating a porous medium, J. of Porous Media. 17(2014), 657-667.

[16] *G.C. Rana, R. Chand*, Stability analysis of double-diffusive convection of Rivlin-Ericksen elastico-viscous nanofluid saturating a porous medium: a revised model, Forsch Ingenieurwes. 79(2015), 87–95.

[17] *S. Rionero*, Triple diffusive convection in porous media, Acta Mech. 224(2013), 447-458.

[18] V. Sharma, G.C. Rana, Thermal instability of a Walters' (Model B') elastico-viscous fluid in the presence of variable gravity field and rotation in porous medium, J. Non-Equilib. Thermodyn. 26(2001), 31-40.

[19] *L.J. Sheu*, Linear stability of convection in a viscoelastic nanofluid layer, World Academy of Science, Engineering and Technology. 58(2011), 289-295.

[20] *I. S. Shivakumara, J. Lee, M.S. Malashetty, S. Sureshkumara*, Effect of thermal modulation on the onset of thermal convection in Walters' B viscoelastic fluid in a porous medium, Transport in Porous Media. 87(2011), 291-307.

[21] *D.Y. Tzou*, Thermal instability of nanofluids in natural convection, Int. J. of Heat and Mass Transfer. 51(2008), 2967-2979.

[22] *D.Y. Tzou*, Instability of nanofluids in natural convection, ASME J. Heat Transf. 130(2008), 072401.

[23] *D. Yadav, G.S. Agrawal, R. Bhargava*, On the onset of double-diffusive convection in a layer of saturated porous medium with thermal conductivity and viscosity variation. J. of Porous Media. 16(2013), 105-121.