THE CONTROLLABILITY FUNCTION METHOD

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The paper is devoted to the control problem for the movement of an overhead crane with the use of a dynamic model in the form of "trolley - cargo" mechanical system and the driving force as a control parameter. To solve the system of differential equations, which describe the movement of the system taking into account constraints for the control, the controllability function method is applied. The algorithm for solving the problem is described, a program is developed as well as difficulties, which occur while implementing the method, and ways of its solution are marked. Results of constructing the control and system trajectories are also provided as an example of the program work.

Keywords: overhead crane, controllability function method.

1. Introduction

In this paper we consider the control problem for the following mechanical system (a “trolley-cargo” system): an overhead crane, that is, a trolley, which moves along a horizontal rail, and a cargo which is attached to the trolley by means of an inextensible weightless absolutely flexible rope. We use this model as it was described in [1]. As a control we chose the moving force, as was suggested in [1] because choosing velocity or acceleration of the trolley as a control complicates the control mechanism.

2. Description of the method

The linearized equations of the movement for the described system, shown in Fig.1, are given by

$$\begin{cases} m_1\ddot{x}_1 = F(t) - W sign\dot{x}_1 - \dfrac{m_2 g}{l}(x_1 - x_2) \\ m_2\ddot{x}_2 = \dfrac{m_2 g}{l}(x_1 - x_2) \end{cases}$$

where

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1. m_1 and m_2 are the masses of, respectively, the trolley and the cargo;
2. $F(t)$ is the moving force;
3. W is a value of the rolling resistance to the cargo wheels;
4. l is the length of the rope;
5. x_1 and x_2 are the distances from the center of mass of, respectively, the trolley and the cargo to the initial point;
6. $\ddot{x}_1$ and $\ddot{x}_2$ are the accelerations of, respectively, the trolley and the cargo;
7. $\dot{x}_1$ is the velocity of the trolley;
8. g is the gravity constant.

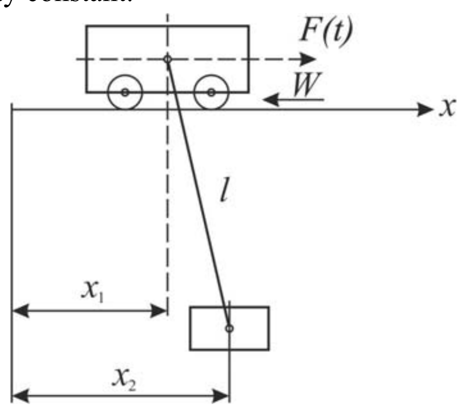

Fig.1 Trolley-cargo model

For this “trolley-cargo” system we researched the following problem: to transfer the system from any given initial position of the trolley and cargo to some given final position of the system, so that the trolley and the cargo stop there, and the cargo is in state of equilibrium. The control which transfers the system from one point to another must be bounded, and the time of the movement must be finite.

The linearized system of differential equations with the control which describes the movement of the overhead crane is in its simplest variation written as follows:

$$\dot{y}(t) = Ay(t) + BU(t) \tag{1}$$

where

1. $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -m_2 g/m_1 l & 0 & m_2 g/m_1 l & 0 \\ 0 & 0 & 0 & 1 \\ g/l & 0 & -g/l & 0 \end{pmatrix}, \ B = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix};$

2. $y = (y_1 \quad y_2 \quad y_3 \quad y_4)^T$ is the vector of coordinates, where y_1 is the distance from the center of mass of the trolley to the initial point, y_2 is the velocity of the trolley, y_3 is the distance from the center of mass of the cargo to the initial point, y_4 is the velocity of cargo;
3. $U(t) = F(t) - W * sign(y_2)/m_1$ is the control function, where $F(t)$ is the moving force applied to the cargo, and W is a value of the rolling resistance to the cargo wheels. We assume that $F(t)$ is bounded, therefore $|U(t)| \le d$. The direction of the resistance force depends on the direction of the trolley's movement.

Using the non-singular change of variables $x = Ly$, where

$$L = \begin{pmatrix} 0 & 0 & l/g & 0 \\ 0 & 0 & 0 & l/g \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

this system is transformed to

$$\dot{x}(t) = A_1 x(t) + B_1 U(t) \tag{2}$$

where

$$A_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -(m_1 + m_2)/m_1 l & 0 \end{pmatrix}, B_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Further we deal with this representation of the original system.
The control problem for the "trolley-cargo system" which is mentioned above leads to the 0-controllability problem for the linear system (2). Namely, the problem is to construct a continuous bounded program control U = U(t) which transfers the system from a given starting point inside some region into zero, such that the time of this transfer is finite.

We research this problem using the controllability function method which was proposed by V.I. Korobov in 1979. We apply the simplest version of this method where controllability function is in fact the time of movement. The algorithm is as follows:

for a given x_0 (x_0 is the initial point of movement)

1. Build matrix

$$N^{-1}(\theta) = \int_0^\theta (1 - t/\theta) e^{-A_1 t} B_1 B_1^T e^{-A_1^T t} dt \tag{3}$$

2. Solve equation

$$2a_0\theta = \left(N(\theta)x_0, x_0\right) \tag{4}$$

with respect to θ and obtain the root θ_0. a_0 is defined according to a theorem of [4].

3. Build the control

$$U(t,x) = 1/2\, B_1 N^{-1}(\theta_0 - t)x \tag{5}$$

4. Solve the Cauchy problem

$$\begin{cases} \dot{x}(t) = A_1 x(t) + B_1 U(t) \\ x(0) = x_0, t \in [0, \theta_0] \end{cases} \tag{6}$$

and obtain the solution $x(t)$.

By this means

$$U(t) = U(t, x(t)) = 1/2\, B_1 N^{-1}(\theta_0 - t)x(t) \tag{7}$$

transfers the system from x_0 into zero for the time θ_0. It can be shown that equation (4) has the unique positive solution, that $N^{-1}(\theta)$ is in fact non-singular and invertible for all $\theta > 0$, and that for sufficiently small a_0 the control (7) is bounded: $\left|U(t)\right| \le d$.

The method by itself is much more general then its variation on the above. Originally the controllability function method was developed for positional synthesis problem. Roughly speaking, if $\theta(x)$ is the solution of (4), then the control $U(x)$ is a feedback control which solves the positional synthesis problem. This method is applicable for some classes of nonlinear control systems $\dot{x} = f(t, x, U)$.

If on step 2 we solve the equation

$$2a_0\theta^{\nu} = \left(N(\theta)x_0, x_0\right) \tag{8}$$

where $\nu \ge 2p + 1$ is a natural number, we get the control with bounded derivatives for the orders up to the p, including p.

The complete description and argumentation of the method, as well as its generalizations can be found in [2, 3, 4].

3. Implementation of the method

When implementating, difficulties appear on the third, and, consequently, fourth step of the algorithm. This happens because the matrix N^{-1} from (3) is ill-conditioned when the parameter θ is small. To obtain the inverse matrix N with sufficient precision we used Taylor series representation for this matrix, replacing it by sufficiently large number of the Taylor series terms. Difficulties on the fourth step of the algorithm arise because the control that is constructed oscillates when t is close to θ_0. This oscillations can be somewhat smoothed by constructing inertial control, using equation (8) instead of (4) when ν is at least 3.

Authors wrote the program for constructing the control and trajectory in Mathematica software. As an example of its work, the system where $m_2 = 1/2\, m_1$ and $l = g$ was considered. For $x_0 = (9 \quad 0 \quad 9 \quad 0)^T$ we get the time of the movement $\theta_0 = 7.849$ and graphics of the trajectory and control have the following look (Fig. 2 and 3):

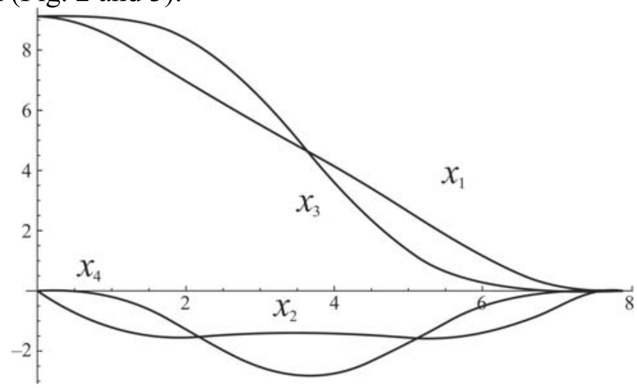

Fig. 2
Components of the trajectory of the system

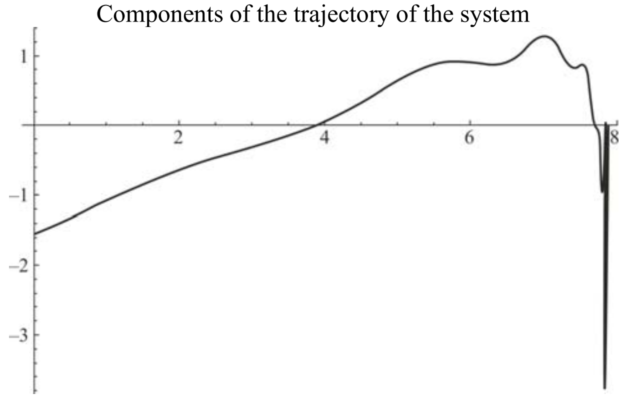

Fig. 3 Control

If we intent to build the inertial control and solve the equation (8) with $\nu = 3$ for the same system and initial point, we get the time of the movement θ_0 = 16.415 and graphics of the trajectory and control are presented in Fig. 4 and 5.

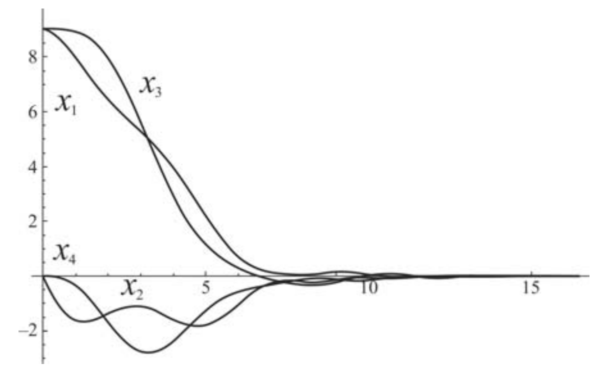

Fig. 4 Components of the trajectory of the system

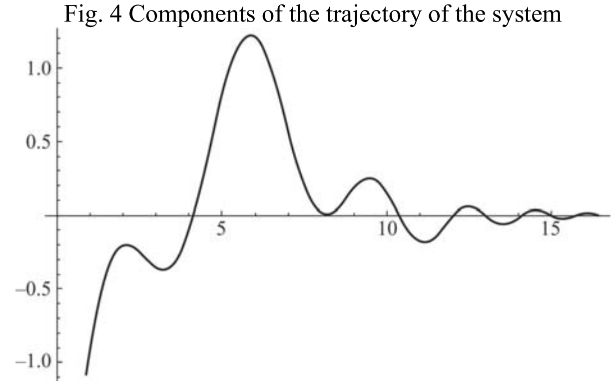

Fig. 5 Control

4. Conclusions

Thus, the controllability function method allows us to effectively create bounded controls that solve the control problem for an overhead crane. Notice that inertial controls behave more regular while increasing the time of movement for the system.

R E F E R E N C E S

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