THE COOLING PROCESS MODELING OF A BAR WITH VARIABLE PROPERTIES IN RELATION TO TEMPERATURE

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The paper presents a numerical method to determine the unstationary temperature field inside a solid body. A model for 2D-heat conduction in a square cross section bar, considering the specific heat and thermal conductivity variable in relation to temperature was developed. The cooling process of the hot bar, in contact with calm air and placed on soil, was studied. The governing equation was solved by the finite difference method. The boundary conditions express the convective and radiative heat transfer on the faces in contact with surrounded air, and conductive heat transfer on the face in contact with soil. By this model, the temperature field can be predicted according to boundary conditions, at different steps of time.

Keywords: numerical modeling, transient 2D-heat conduction, free cooling

1. Introduction

Study of conductive heat transfer, and determination of non-steady state temperature field inside the material, with different boundary conditions, is of particular interest to engineers. Typical applications include the cooling or heating of machines parts such as turbine blades[1] or pistons of reciprocating machines, contact surfaces such as free-boundary solidification [2], and many metallurgical processes such as laser cutting and welding [3]. In the food industry, nestationary temperature field simulation during the sterilization process of canned foods [4], or food frezing [5], is also a matter of great interes. The problem can be solved analytically for simple geometries or boundary conditions [6], or numerically by finite elements or finite differences [7]. Because the regime of heat propagation is non-steady state, the calculus algorithm must generate a solution which to advance over time [8, 9]. In the same time, the field of physically properties of materials is variable because the values of this properties depend by temperature.

In this paper, to determine the temperature field, the authors propose a finite difference numerical method that takes into consideration the dependence

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on temperature of the physical properties of the material. The solution will advance over time depending on boundary conditions imposed on the outline. To test calculation algorithm was considered a homogeneous body with simple geometry and physical properties dependent on temperature, a steel bar with rectangular cross section, heated to a certain initial temperature, uniformly throughout the mass of material. This bar is considered to be resting on the ground or is suspended in the air, far enough away from other objects. On the faces in contact with ambient calm air, the cooling is done by free convection and radiation, and on the face in contact with the soil, by conduction. Because the bar length is considered infinite, it can be assumed that the distribution of the temperature field is the same in any cross-section, and the heat transfer is therefore two-dimensional. For the simulation of the heat transfer process to be close to reality, it was found that the thermal conductivity and specific heat are functions of temperature: $\lambda = \lambda(t)$; $c = c(t)$.

The surrounded calm air temperature and soil temperature was considered constant in time. The heat transfer process inside the body is transient, and therefore the temperature field is variable over time.

2. The transient heat transfer inside the body

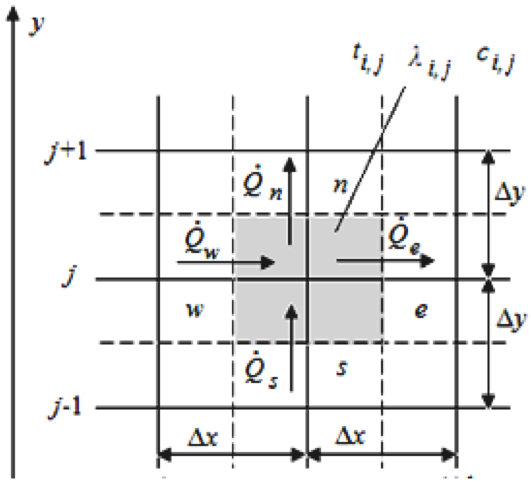

Fig. 1 The grid and the control volume

Equations describing heat transfer will be discretized on a rectangular grid, so that the first and last line to match the body faces [7]. It is considered a control volume element, homogeneous and isotropic, located around a network node, so that it faces to intersect the grid lines midway between two neighboring nodes (Fig. 1).

The thermal balance equation of the volume element, for an elementary interval of time dτ, has expression [9, 10, 11]:

$$\frac{\mathrm{D}t}{\mathrm{D}\tau} = \frac{1}{\rho\, c\, \mathrm{D}V}\left(\delta\dot{Q}_e + \delta\dot{Q}_w + \delta\dot{Q}_n + \delta\dot{Q}_s\right) \qquad (1)$$

where $\delta\dot{Q}_e$, $\delta\dot{Q}_w$, $\delta\dot{Q}_n$, $\delta\dot{Q}_s$ are the elementary heat fluxes through the (*e*), (*w*), (*n*) and (*s*) faces[3] of the volume element, $\mathrm{d}V$ is the size of the volume element and ρ, c are the density and specific heat of the material. This equation is discretized by finite difference around the volume element, by considering the propagation directions: $w \to e$ and $s \to n$ respectively.

According to Fourier law, the module of the heat flux passing through the east (*e*) face of the volume element in $w \to e$ direction has expression [8, 9]:

$$|\dot{Q}_e| = \frac{t_{i,j} - t_e}{\frac{\Delta x}{2\,\Lambda_{i,j}}}\Delta y = \frac{t_e - t_{i+1,j}}{\frac{\Delta x}{2\,\Lambda_{i+1,j}}}\Delta y \quad [\mathrm{W}] \tag{2}$$

where t_e is the temperature on east face, midway between (*i*,*j*) and (*i*+1,*j*) nodes.

From this equation, taking into account that $\dot{Q}_e$ flows out from the volume element ($\dot{Q}_e < 0$), it follows:

$$\dot{Q}_e = \Lambda_{e_{i,j}}\left(t_{i+1,j} - t_{i,j}\right)\frac{\Delta y}{\Delta x} \quad [\mathrm{W}] \tag{3}$$

where:

$$\Lambda_{e_{i,j}} = \frac{2\,\Lambda_{i,j}\,\Lambda_{i+1,j}}{\Lambda_{i,j} + \Lambda_{i+1,j}} \quad [\mathrm{W}\cdot\mathrm{M}^{-1}\cdot\mathrm{K}^{-1}] \tag{4}$$

Similarly, taking into account of the heat transfer sense, it follows for the all other faces:

$$\begin{aligned} \dot{Q}_w &= \Lambda_{w_{i,j}}\left(t_{i-1,j} - t_{i,j}\right)\frac{\Delta y}{\Delta x} \quad [\mathrm{W}] \\ \dot{Q}_n &= \Lambda_{n_{i,j}}\left(t_{i,j+1} - t_{i,j}\right)\frac{\Delta x}{\Delta y} \quad [\mathrm{W}] \\ \dot{Q}_s &= \Lambda_{w_{i,j}}\left(t_{i,j-1} - t_{i,j}\right)\frac{\Delta x}{\Delta y} \quad [\mathrm{W}] \end{aligned} \tag{5}$$

where similarly:

$$\begin{aligned} \Lambda_{w_{i,j}} &= \frac{2\,\Lambda_{i,j}\,\Lambda_{i-1,j}}{\Lambda_{i,j} + \Lambda_{i-1,j}} \quad [\mathrm{W}\cdot\mathrm{M}^{-1}\cdot\mathrm{K}^{-1}] \\ \Lambda_{n_{i,j}} &= \frac{2\,\Lambda_{i,j}\,\Lambda_{i,j+1}}{\Lambda_{i,j} + \Lambda_{i,j+1}} \quad [\mathrm{W}\cdot\mathrm{M}^{-1}\cdot\mathrm{K}^{-1}] \\ \Lambda_{s_{i,j}} &= \frac{2\,\Lambda_{i,j}\,\Lambda_{i,j-1}}{\Lambda_{i,j} + \Lambda_{i,j-1}} \quad [\mathrm{W}\cdot\mathrm{M}^{-1}\cdot\mathrm{K}^{-1}] \end{aligned} \tag{6}$$

[3] The lateral faces of volume element d*V* are denoted as cardinal points: *east (e)*, *west (w)*, *north (n)* and *south (s)*, in relation to the field of coordinates axes (right, left, top, bottom).

By introducing the heat flow expressions in the thermal balance equation (1), for a finite interval of time $\Delta\tau = \tau_{p+1} - \tau_p$ it is obtained:

$$\frac{t_{i,j}^{p+1} - t_{i,j}^{p}}{\Delta\tau} = \frac{1}{\rho\, c_{i,j}^{p}}\left[\frac{\Lambda_{e_{i,j}}^{p}}{\Delta x^2}\left(t_{i+1,j}^{p} - t_{i,j}^{p}\right) + \frac{\Lambda_{w_{i,j}}^{p}}{\Delta x^2}\left(t_{i-1,j}^{p} - t_{i,j}^{p}\right) + \right.$$
$$\left. + \frac{\Lambda_{n_{i,j}}^{p}}{\Delta y^2}\left(t_{i,j+1}^{p} - t_{i,j}^{p}\right) + \frac{\Lambda_{s_{i,j}}^{p}}{\Delta y^2}\left(t_{i,j-1}^{p} - t_{i,j}^{p}\right)\right] \tag{7}$$

Considering a grid with equal steps in the *x* and *y* directions

$$\Delta x = \Delta y = h \tag{8}$$

from the equation (7) it is obtained a recurrence relation between the temperature in the node (*i*, *j*) to the next step of time (*p* + 1), and temperatures in the same node and adjacent nodes, at the current step of time (*p*):

$$t_{i,j}^{p+1} = A_{p_{i,j}}\, t_{i,j}^{p} + A_{e_{i,j}}\, t_{i+1,j}^{p} + A_{w_{i,j}}\, t_{i-1,j}^{p} + A_{n_{i,j}}\, t_{i,j+1}^{p} + A_{s_{i,j}}\, t_{i,j-1}^{p} \tag{9}$$

where the coefficients of equation (10) have the following expressions:

$$A_{e_{i,j}} = \frac{\Delta\tau\, \Lambda_{e_{i,j}}}{\rho\, c_{i,j} h^2};\ A_{w_{i,j}} = \frac{\Delta\tau\, \Lambda_{w_{i,j}}}{\rho\, c_{i,j} h^2};\ A_{n_{i,j}} = \frac{\Delta\tau\, \Lambda_{n_{i,j}}}{\rho\, c_{i,j} h^2};\ A_{s_{i,j}} = \frac{\Delta\tau\, \Lambda_{s_{i,j}}}{\rho\, c_{i,j} h^2}$$
$$A_{p_{i,j}} = 1 - \left(A_{e_{i,j}} + A_{w_{i,j}} + A_{n_{i,j}} + A_{s_{i,j}}\right) \tag{10}$$

The linear system of equations (9), starting from an initial temperature field, determines the solution progress over time. For the stability of the solution, it is necessary to choose the step of time so that, whatever the network node, the coefficient A_p to be positive ($A_{p_{i,j}} \geq 0$) [9, 12] and as a result, is obtained the following condition:

$$\Delta\tau \leq \frac{\rho\, c_{i,j}^{p}\, h^2}{\Lambda_{e_{i,j}}^{p} + \Lambda_{w_{i,j}}^{p} + \Lambda_{n_{i,j}}^{p} + \Lambda_{s_{i,j}}^{p}}\ \ [\mathrm{S}] \tag{11}$$

Each step of the calculation will have to choose the highest value of time step to satisfy condition (11) in all interior nodes. Therefore, the step of time is not constant, but it changes according to the material properties which, in turn, will be changed from one time to another, in relation to the temperature change.

3. Boundary conditions

3.1. Boundary conditions on the surfaces in contact with calm air

On these areas, the heat is transmitted to the external environment by convection and radiation. The convective heat flux transmitted from the front of the body to the external environment should be equal to the conductive heat flux propagated the inside of the control volume near the surface. Denoting the temperature of the body face with $t_{w_{i,j}}$, corresponding to the volume element (*i*, *j*) placed to external face of body, and the ambient temperature with t_a, the thermal balance is:

$$\dot{q} = \frac{t_{i,j} - t_{w_{i,j}}}{\frac{h}{\Lambda_{i,j}}} = (A_c + A_r)\left(t_{w_{i,j}} - t_a\right) \quad [\mathrm{W \cdot M^{-2}}] \tag{12}$$

where α_c, α_r represent the convection, respectively radiation coefficient.

For free convection, the coefficient α_c can be modelled by:

$$A_c = \mathrm{NU}_{m_{i,j}} \frac{\lambda_{am_{i,j}}}{h} = C \cdot \mathrm{RA}^n_{m_{i,j}} \frac{\lambda_{am_{i,j}}}{h} \quad [\mathrm{W \cdot M^{-2} \cdot K^{-1}}] \tag{13}$$

in which $\lambda_{am_{i,j}}$ is the local thermal conductivity of air and $\mathrm{Nu}_{m_{i,j}}, \mathrm{Ra}_{m_{i,j}}$ represent the local Nusselt and respectively Rayleigh number at mean temperature between air and wall: $t_{m_{i,j}} = 0.5\left(t_{w_{i,j}} + t_a\right)$. In the eq. (13), the constant *C* and exponent *n* have values corresponding to the classical equations for the free convection on a plane plate [8, 9, 13].

Based on the Stefan-Boltzmann law, and taking into account the fact that air has not emissive properties, radiation coefficient can be expressed as [8, 9]:

$$A_r = \frac{\varepsilon\, C_0 \left(\frac{T_{w_{i,j}}}{100}\right)^4}{t_{w_{i,j}} - t_a} \quad [\mathrm{W \cdot M^{-2} \cdot K^{-1}}] \tag{14}$$

where $C_0 = 5{,}67$ W·m^{-2}·K^{-4} is the radiation coefficient of the black body and ε is the emissivity coefficient of the surface in contact with air (for simplicity it is assumed constant in respect to temperature).

The equation (12) determines the local temperature of the wall and the therefore, the value of the term from equation (9) corresponding to the wall.

3.2. Boundary conditions on the surface in contact with soil

At this surface, the contact between material and soil is not perfect. The heat is propagating into the soil in conditions of a contact resistance on the separation surface between material and soil and as a result, the heat flux will be given by the expression [8, 9]:

$$\dot{q} = \frac{t_{i,j} - t_s}{\frac{h}{2\Lambda_{i,j}} + \frac{\delta_{ec}}{\Lambda_{ec}} + \frac{\delta_s}{\Lambda_s}} = \frac{t_{i,j} - t_{w_{i,j}}}{\frac{h}{2\Lambda_{i,j}}} \quad [\mathrm{W \cdot M^{-2}}] \tag{15}$$

in which: t_s is the constant temperature of soil at depth δ_s, λ_s is the thermal conductivity of soil and δ_{ec}/λ_{ec} is the equivalent thermal resistance of the imperfect contact zone between material and soil.

The equivalent conductivity of the contact zone between the bottom face and soil is given by the expression [9]:

$$\lambda_{ec} = f_c \frac{2\lambda_{i,j}\Lambda_s}{\Lambda_{i,j} + \Lambda_s} + f_a\Lambda_a \qquad [\mathrm{W \cdot M^{-1} \cdot K^{-1}}] \tag{16}$$

where: f_c represent the percentage of direct contact zone, in total area; f_a represent the percentage of zone with interstices, in total area; λ_a is the conductivity of air from interstices.

From equation (16), it can be calculated the temperature on surface of material:

$$t_{w_{i,j}} = t_{i,j} - \frac{\frac{h}{2\Lambda_{i,j}}}{\frac{h}{2\Lambda_{i,j}} + \frac{\delta_{ec}}{\Lambda_{ec}} + \frac{\delta_s}{\Lambda_s}}\left(t_{i,j} - t_s\right) \qquad [°\mathrm{C}] \tag{17}$$

The equation (17) determines the local temperature of the wall, therefore the value of the term from equation (9) corresponding to the wall.

4. Results

The model achieved has been used to study the cooling process of a steel bar with density $\rho = 8100\ \mathrm{kg \cdot m^{-3}}$ and the square cross section with side lenght of 1 m. It was considered that the bar is taken out of a furnace, where it was heated to initial temperature of 1000 °C, uniformly distributed in the mass of material, and placed: *a) on the ground*, *b) suspended in air*. Ambient air was considered calm and having constant temperature of 20 °C. Starting at a depth of 8 m, it is assumed a constant soil temperature of 10 °C. The specific heat and thermal conductivity of the material were considered to be variable in respect to temperature, according to the expressions [8, 14]:

$$c = 465 + 0{,}423\ t - 0{,}385 \cdot 10^{-4}\ t^2 \quad [\mathrm{J \cdot KG^{-1} \cdot K^{-1}}] \tag{18}$$

respectively:

$$\lambda = 59{,}3\left(1 - 4{,}8 \cdot 10^{-4}\ t^2\right) \quad [\mathrm{W \cdot M^{-1} \cdot K^{-1}}] \tag{19}$$

For calculus of the temperature field to different steps of time, in accordance with the mathematical model developed, it was written and run an originally computer program in FORTRAN 77 language[4]. This program has produced data files containing the temperature values in the nodes of grid, at

[4] Microsoft Developer Studio – Fortran Power Station 4.0

different time steps. Graphics processing of these data sets was performed in MATLAB[5] software.

According to the model described, in Fig. 2 are shown the temperature fields inside the bar, after a period of 1 hour, 4 hours, 8 hours and 10 hours, from initial moment of time, when it is positioned on the ground. The temperature field inside the bar is illustrated by the position of isotherms at certain time point.

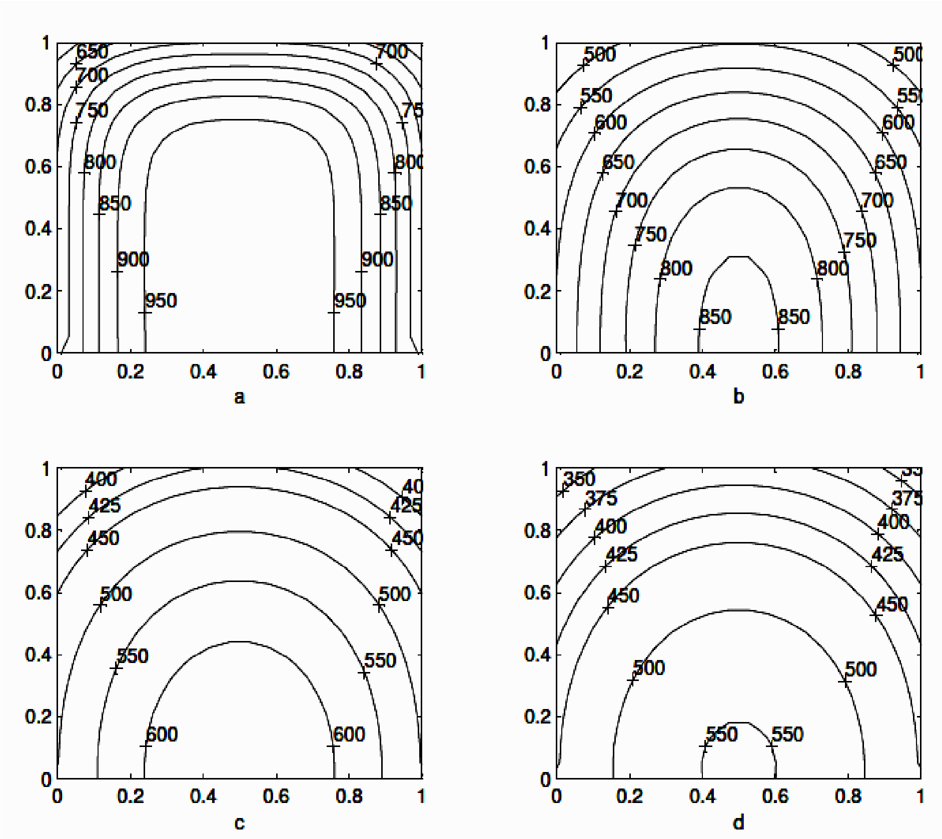

Fig. 2 The temperature field when the bar is seated on the ground:
a) after 1 hour; b) after 4 hours; c) after 8 hours; d) after 10 hours.

The simulation of the temperature field evolution over time, highlights the difference between heat transfer to the ground (to bottom) and heat transfer to the surrounding air. Since the thermal resistance of the soil is much longer than the convective thermal resistance of the boundary layer on the surfaces in contact with the air, the temperature inside the material more rapidly decreases in the

[5] Matlab 6 – Version 6.5.0.180913a Release 13

direction of the faces in contact with air. This explains the characteristic shape of the temperature field.

In Fig. 3 the temperature fields inside the bar are shown, after a period of 1 hour, 4 hours, 8 hours and 10 hours, from initial moment of time, when the bar is suspended in calm air.

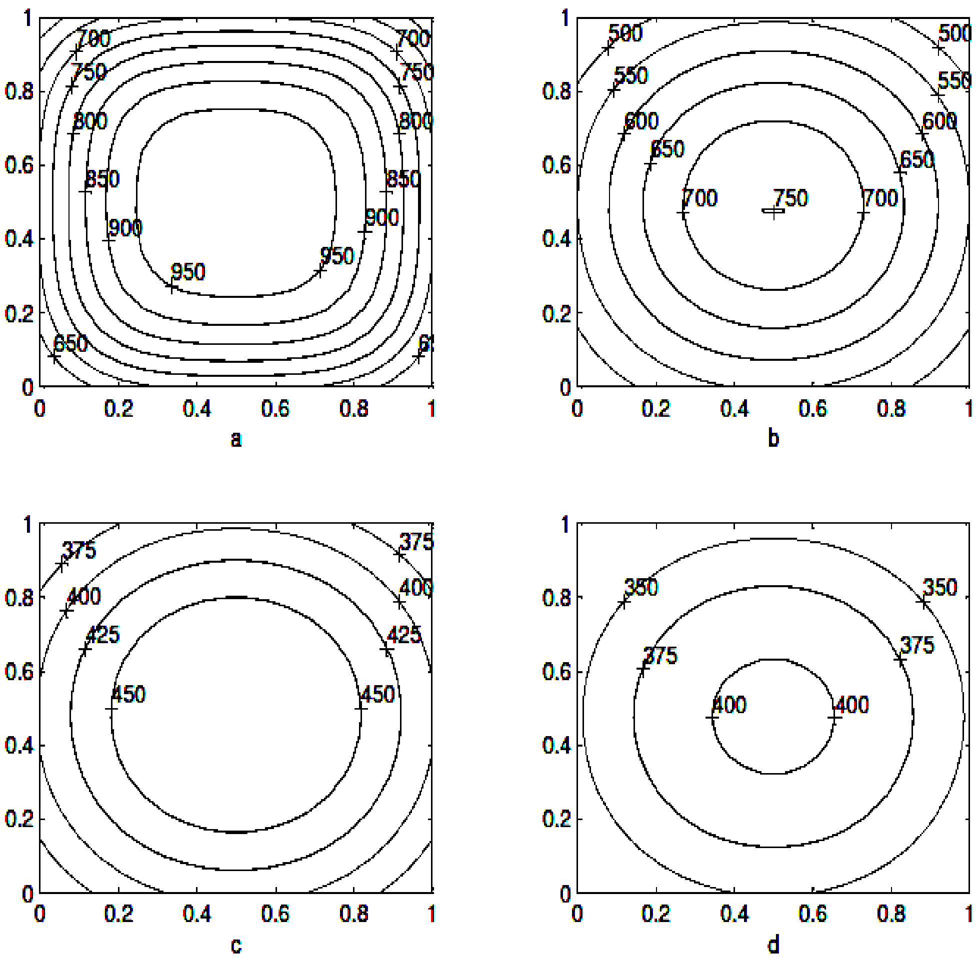

Fig. 3 The temperature field when the bar is suspended in calm air:
a) after 1 hour; b) after 4 hours; c) after 8 hours; d) after 10 hours.

In this case, it can be observed the symmetry of temperature field over time. Obviously, due to identical boundary conditions on all four sides, the cooling rate is the same in any direction.

In Fig. 4 the evolution of temperature over time is shown, along the horizontal axis of the bar.

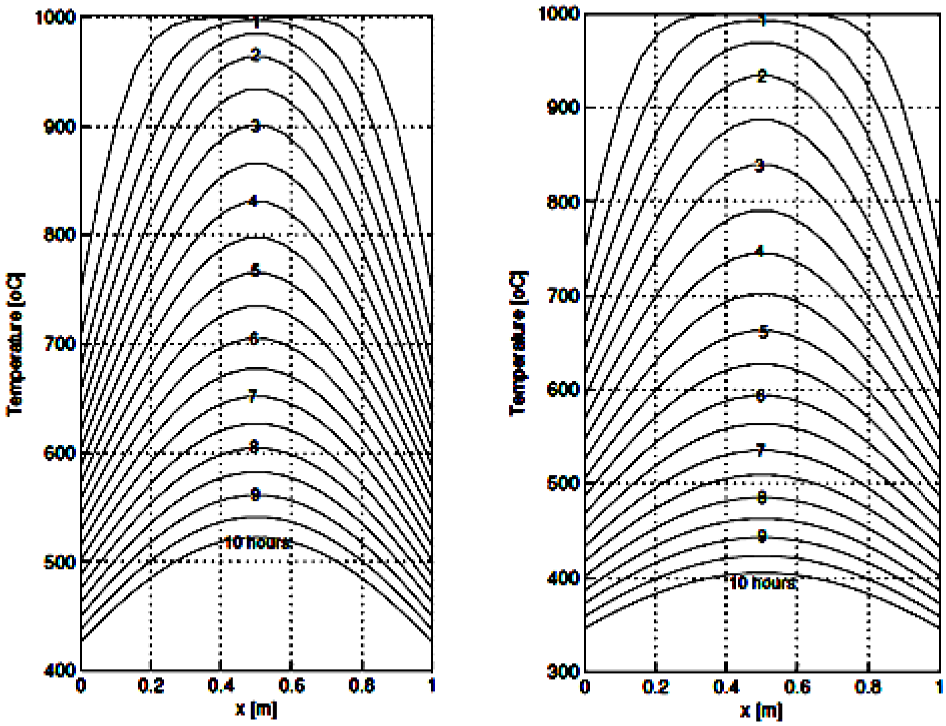

Fig. 4 The temperature evolution along the horizontal axis:
left – the bar seated on ground; *right* – the bar suspended in calm air.

It can be seen that the cooling rate of the body is greater when it is suspended in calm air, compared with case of it is placed on the ground at bottom. It can be observed that after 10 hours, due to higher thermal resistance in heat transfer to the ground, the temperature in the center of the bar placed on the floor it is about 522 ^{o}C compared with approx. 407 ^{o}C if it is suspended in the calm air.

5. Conclusions.

Numerical simulation was performed using a body of simple geometrical shape and a known case of conductive heat transfer, just to test the behavior of mathematical model and numerical algorithm. The results obtained by numerical simulation have highlighted the ability of the mathematical model to behave correctly to changes in boundary conditions.

By considering of the physical properties as functions of temperature, the model leads to results very close to reality. The authors have developed an original algorithm which at every time step changes the physical properties of body, according to the new temperature field. This algorithm can be implemented in any programming language of high level that is capable of generating data files (FORTRAN, C^{++}, PASCAL). They can be processed graphic further into a desired shape.

The developed model can be used to simulate numeric various cooling or warming process in all cases in which the body has a polygonal geometry and the heat transfer from the outer surface to the environment takes place combined by conduction, convection or radiation. This simulation is of great interest for a lot of engineering domains such as heating or cooling processes of the machine parts, metallurgical processes or sterilization processes of canned foods and freezing of foods.

R E F E R E N C E S

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