DYNAMIC DISCRETE SIMULATION MODEL OF AN INVENTORY CONTROL WITH OR WITHOUT ALLOWED SHORTAGES

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This paper shows how to develop a dynamic discrete spreadsheet model of an inventory control in the case of the fixed order quantity, for the finite time horizon, with or without allowed shortages. There is a clear distinction between a discrete controlled object (the law of behavior and control domain) ,a performance criterion and the method used to find an optimal solution. Further, it shows how to derive a performance criterion, including all costs that are considered to be significant by the user. Several respectful papers pertaining inventory control with and without shortages are used to compare and prove accuracy, simplicity and practicality of our approach.

Keywords: Inventory control; Shortages; Finite time horizon; Spreadsheet model.

1. Introduction

The infinite time horizon inventory control models assume that the rate of the annual demand is known and constant over several consecutive years. The finite time horizon inventory control models assume the demand pertains only to the determined time horizon, often shorter than one year period. The number of replenishments obtained by the fixed time horizon inventory models is always an integer number; that is not the case with the infinite time horizon inventory models. This is why the results at the year's level obtained by infinite time or by finite time horizon inventory control models may differ. They are the same only if both of them give an integer as the number of replenishments.

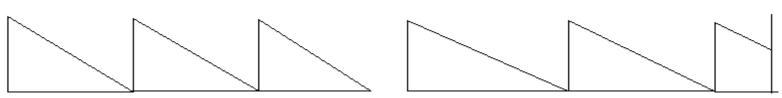

Fig. 1. Stock dynamics over the year: a) integer and b) non-integer number of replenishments

Axsäter [1], Barlow [2], Muller [3], Wild [4], Anderson, Sweeney and Williams [5], in their books and papers dealing with inventory control, describe a classical economic order quantity model in the fixed-order quantity system at the

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finite time horizon and its variants when a demand rate is constant and known, as a starting point for further understanding of inventory dynamics. The study of inventory problems dates back to 1915, when Harris [6], first selected the inventory problem for a mathematical analysis. As the result, the simple but famous EOQ (Economic Order Quantity) formula was established which was also derived, apparently independently, by Wilson [7]. Donaldson [8] came up with a full analytic solution of the inventory replenishment problem with a linear trend in demand over a finite-time horizon. The discrete version of this problem was discussed by Wagner and Whitin [9], Barbosa and Friedman [10]which was further generalized for solutions for various, similar EOQ models. Furthermore, Goyal [11] was the first to develop the EOQ model under the conditions of permissible delay in payments. All these models were developed with the assumption that there are no shortages in inventory.

Deb and Chaudhuri [12] were the first to extend Donaldson's model [8], in order to incorporate shortages in inventory. This extension was further studied by Goyal [13]. Following Donaldson's approach [8], Dave [14] developed an exact replenishment policy for an inventory model taking into account shortages. Sana and Chaudhuri [15] developed EOQ model over a finite-time horizon for perishable items, considering unequal cycle lengths. One method of dealing with EOQ models with time-varying demand and cost over a finite planning horizon is to use discrete Dynamic Programming [16], Wagner and Whitin [9]. Kostic in [17] showed how to model EOQ problem in order to find an optimal number of replenishments in the fixed-order quantity system, as a basic problem of optimal control of the discrete system. The decision environment is deterministic and the time horizon is finite. A discrete system consists of the law of dynamics, control domain, and a performance criterion. It is primarily a simulation model of the inventory dynamics, but the performance criterion enables various other strategies to be compared.

Smith in [18] stated that the spreadsheets are extremely effective in determining the optimum number of distribution facilities, the appropriate mix of transportation modes, production scheduling, inventory optimization, and strategic planning exercises. Vazsonyi [19] holds that the deterministic “what-if” simulation methodology is the most popular decision making tool. Przasnyski in [20] assumes that the spreadsheets have provided a platform for demonstrating the power of simulation in inventory management.

Kostic [17] has shown that modelling inventory problems as a discrete object optimal control is more appropriate for the real-life. He has developed a general approach to inventory models and has shown that all variants of EOQ model applications can be considered the scenarios of the inventory control model, as the model of optimal control of the discrete system.

The traditional EOQ model assumes an infinite time horizon and the number of obtained replenishments is often non-integer (Figure 1b). It is often necessary to make certain approximations in order to use a traditional EOQ model for the finite time inventory problems in practise. It is practically inconvenient to apply 4.7 replenishments and that a replenishment cycle is 77.66 days long. Furthermore, the cost of ordering is linked to the replenishment occurrence, which can be merely integer, but EOQ model often multiplies the ordering costs with the fractional number of replenishments thus giving inaccurate total inventory cost.

This paper is predicated by the paper [17]. As an extension to the paper we have derived a unique model of the inventory control, in the case of the fixed order quantity and finite time horizon with or without allowed shortages. A discrete time system is a more natural way to describe inventory dynamics. The model of discrete system control is both a simulation model of inventory dynamics and an optimization model, which gives an optimal control according to the defined performance criterion.

2. Discrete controlled object: Law of dynamics and Control domain

Following notation will be used for the mathematical relations that describe the discrete object:

Table 1

Variable notations for mathematical relations

t - Discrete time, T - Number of days of the time horizon, D - Item demand for the observed time horizon X_t^1 - Stock at time t, X_t^2 - Shortage at time t, u^1 - Number of replenishments	Y_t^1 - Quantity item received at time t Y_t^2 - Unsatisfied demand at time t Y_t^3 - Demand at time t Y_t^4 - Satisfied shortage at time t u^2 - Percentage of the replenishment quantity which determining allowed shortage

The main characteristic of the inventory control in the case of the fixed order quantity for the finite time horizon is that the replenishment quantity is constant and performed throughout several replenishments which occur at the beginning of the equal portions of the time horizon. The sum of replenishment quantities over the time horizon is equal to the demand in the time horizon (D). In accordance to Kostic [17] this type of the flow is called “Discrete input and continuous output”. Inventory flows may occur with and without allowed shortages. We will develop a model of a discrete controlled object that will encompass both possibilities. Therefore we introduce two flows: one for the dynamics of inventory on hand and the other for the dynamics of shortages that will represent an unsatisfied demand. Both flows consist of the alternating subsequence “action – accumulation –

action”: an input action increases accumulation, and an output action decreases accumulation. Dynamics of accumulations can be expressed as follows:

$$\begin{aligned} &X_0^i = known \qquad , i = 1,2 \\ &X_t^i = X_{t-1}^i + Y_t^i - Y_t^{i+2} \quad , \ t = 1,2,...,T \end{aligned} \tag{1}$$

where X^1 is a state variable pertaining to the flow of the inventory, and X^2 is a state variable pertaining to the flow of shortages. The discrete time t can take only integer values t=0, 1, 2,..T, representing days. T is a number of days over the time horizon. When the replenishment occurs, the level of inventory X_t^1 increases instantaneously. The level X_t^1 decreases in accordance to a daily demand (D/T). There are two possible outcomes when the level X_t^1 meets zero: the first is to get a new replenishment if shortages are not allowed, and the second one is to stop decreasing the level of inventory X_t^1 and start increasing the level of shortages X_t^2 if shortages are allowed. Neither of the two levels could be negative. Y_t^1 represents an input action that increases the inventory on hand. Its value over the time horizon equals zero, which is excepted in the moment when the replenishment occurrs. Denote the number of replenishments as a control variable u^1. The replenishment quantity is u^1-th part of the whole demand for the encompassed time horizon, D/ u^1. The whole time horizon is divided into T time buckets representing days, t=1,2,…,T. If we divide the number of time buckets T with the number of replenishments u^1, the result could be non-integer number, inappropriate to determine the time bucket at which the replenishment will occur. Therefore, we introduce a rule that the replenishment will occur if the inventory on hand threatens to fall below zero if shortages are not allowed, or if shortages exceed allowed level provided they are allowed. In order to deal with the shortages, let us introduce the second control variable u^2 ($0 \le u^2 < 1$) that will represent a percentage of the replenishment quantity, as the highest allowed level of the shortage. If we deal with the inventory without shortages, then the value of the variable u^2 will be zero. Assume that both the initial inventory and initial shortages are zero, $X_0^i = 0$, $i = 1,2$. The first replenishment occurs on the first day. If the shortages are allowed, then the first replenishment quantity should be diminished by the allowed shortage (given as a percentage u^2 of the replenishment quantity D/u^1).

$$Y_1^1 = D \cdot (1 - u^2) / u^1 \tag{2}$$

The ensuing u^1-1 replenishments will occur according to the next mathematical relation:

$$Y_t^1 = \begin{cases} \begin{cases} \begin{cases} \min(D/u^1, D - D\cdot u^2/u^1 - \sum_{tt=0}^{t-1} Y_{tt}^1), \; if \;\; X_{t-1}^2 \geq D\cdot u^2/u^1 \\ 0 \qquad , \; if \;\; X_{t-1}^2 < D\cdot u^2/u^1 \end{cases} & , \; if \;\; u^2 > 0 \\ \begin{cases} \min(D/u^1, D - \sum_{tt=0}^{t-1} Y_{tt}^1), \;\; if \;\; X_{t-1}^1 < D/T \\ 0 \qquad , \; if \;\; X_{t-1}^1 \geq D/T \end{cases} & , \; if \;\; u^2 = 0 \end{cases} & , \; if \;\; D - D\cdot u^2/u^1 > \sum_{tt=0}^{t-1} Y_{tt}^1 \\ 0 & , \;\; if \;\; D - D\cdot u^2/u^1 \leq \sum_{tt=0}^{t-1} Y_{tt}^1 \end{cases}$$

t=2,3,...,T (3)

In this relation a mathematical expression $u^2>0$ corresponds to the situation when the shortages are allowed, and $u^2=0$ corresponds to situation when the shortages are not allowed. If the shortages are allowed then there will be a shortage at the end of the time horizon. Suppose that the shortage is eventually supplied. It will occur the very next day after the end of the time horizon encompassed.

$$Y_{T+1}^1 = D - \sum_{tt=1}^{T} Y_{tt}^1 \tag{4}$$

This relation holds also for the case when the shortages are not allowed or equals the value zero. The depletion of the inventory is consistent with the assumption that the demand over the time horizon is even and equals D/T.

$$Y_t^3 = \min(D/T, \; X_{t-1}^1 + Y_t^1) \tag{5}$$

If the level of inventory on hand X^1 reaches zero and the shortages are allowed, then the recording of the shortages Y^2 occurs according to the next relation.

$$Y_t^2 = \begin{cases} \begin{cases} \begin{cases} D/T - (X_{t-1}^1 + Y_t^1) \; , \; if \;\;\; D/T > (X_{t-1}^1 + Y_t^1) \\ 0 \qquad , \; if \;\;\; D/T \leq (X_{t-1}^1 + Y_t^1) \end{cases} & , \; if \;\; D > \sum_{tt=0}^{t-1} (Y_{tt}^2 + Y_{tt}^3) \\ 0 & , \; if \;\; D \leq \sum_{tt=0}^{t-1} (Y_{tt}^2 + Y_{tt}^3) \end{cases} & , \; if \;\;\; u^2 > 0 \\ 0 & , \;\; if \;\;\; u^2 = 0 \end{cases}$$

(6)

The level of shortages X^2 increases until the new replenishment. With the new replenishment the entire shortage is satisfied i.e. the value of state variable X^2 becomes zero. It means that the value of the flow regulator Y_t^4 is greater than zero only if replenishment occurs.

$$Y_t^4 = \begin{cases} \min(Y_t^1 - Y_t^3, X_{t-1}^2) & , if \;\; Y_t^1 > 0 \\ 0 & , if \;\; Y_t^1 = 0 \end{cases}, \; t = 1,2,...,T \tag{7}$$

The control domain is defined by ensuring non-negativity of the state variables for each t.

$$0 \le X_{t-1}^i + Y_t^i - Y_t^{i+2} \;\;, \; i = 1,2, \; t = 1,2,...,T, T+1 \tag{8}$$

5. Performance criterion $J = \sum f(X_{t=1}, p_t, u)$

The aim of ensuring that the anticipated demand is met can be achieved by keeping stock nonnegative. However, the primary purpose of inventory control is to ensure that the right quantity of the right items is ordered at the right time, according to a known demand, existing constraints and with the objective to minimize the total cost, where the cost is expressed by the equation: Cost = ordering cost + holding cost + shortage cost + purchase cost. This function can be broaden by additional costs according to the real nature of the inventory problem.

Ordering cost includes costs arising from the preparation and dispatch of the order, checking of the goods on delivery, and other clerical support activities. It can be constant (EOQ model) or variable throughout the time horizon, depending (Increasing Delivery Costs – a variation of the Discount model) or not on the ordered quantity. Ordering cost per order Cs is greater than zero only in time t when the order arrives in the stock or when the batch starts.

The cost of holding one unit of an item in stock per day (for instance $20/T a unit per day or as a percentage of the unit cost of the item divided by T, where T is the number of days of the time horizon). It can be constant (EOQ model) or variable throughout the time horizon, independant of the quantity carried in inventory. Holding or carrying cost per one unit Ch per day multiplies a day average inventory. If we retain a classical inventory control model approach, a day average (*dav*) inventory can be calculated as:

$$dav(t) = X_{t-1}^1 + (Y_t^1 - Y_t^4) - Y_t^3 / 2 \;\;, t = 1,2,...,T \tag{9}$$

Shortage cost per one unit Csh per day multiplies a day average shortages. If we retain a classical inventory control model approach, a day average shortage (*dash*) can be calculated as:

$$dash(t) = \begin{cases} X_{t-1}^2 + Y_t^2 / 2 \;, if \;\; Y_t^2 > 0 \\ 0 \;\;, if \;\; Y_t^2 = 0 \end{cases}, t = 1,2,...,T \tag{10}$$

Purchase (unit) cost is the price charged by suppliers for one unit of the item. It can be constant (EOQ model) or variable throughout the time horizon, independant (Quantity Discount model) of the ordered quantity. Purchase (unit) cost Cu multiplies quantity purchased in time t. The general pattern of the performance criterion is

$$J = \sum_{t=1}^{T} [Cs \cdot \begin{Bmatrix} 1, & if\ Y_t^1 > 0 \\ 0, & if\ Y_t^1 = 0 \end{Bmatrix} + Ch \cdot dav(t) + Csh \cdot dash(t) + Cu \cdot Y_t^1] \quad (11)$$

that should be minimized. It is obvious that the value of performance criterion depends on the inflow dynamics Y^1. The function of the performance criterion can contain additional information according to the real decision environment. Values of each partial functions of the performance criterion J over the time horizon T can be presented in separate columns of the spreadsheet. The values of the performance criterion *J* should cumulate values of its partial functions over the time horizon.

7. Discussion and comparison of the results

In this section we investigate the examples of spreadsheet model in the case of EOQ model without allowed shortages. Firstly, we present the dynamic spreadsheet models where the results of simulation are exactly the same as in the static models described in Hesse [21] and Barlow [2], see Table 2.

Table 2

Numerical example of EOQ model without shortages

Problem 1	**Traditional static EOQ model found (Hesse [21])**	**Dynamic discrete spreadsheet model found**
D=500 units per year Cs=$10 per order Ch=$0,0208 per unit per year Cu=$0 per unit T=360 days	EOQ =250 per order N° =2 orders per year Holding cost = $2,60 per year Ordering cost = $20 per year Total Cost = $22,60 per year Cycle time = 0,5 year	EOQ =250 per order u^1 (N°) =2 orders **(decision variable)** Holding cost = $2,60 per period Ordering cost = $20 per period (min) J = $22,60 per period Cycle time = 180 days T (time horizon) = known
Problem 2	**Traditional static EOQ model found (Barlow [2])**	**Dynamic discrete spreadsheet model found**
D=12.000 units per year Cs=$50 per order Ch=$7,5 per unit per year Cu=$25 per unit T=360 days	EOQ =400 per order N° =30 orders per year Holding cost = $1.500 per year Ordering cost = $1.500 per year Purchasing cost = 300.000 per year Total Cost = $303.000 per year	EOQ =400 per order u^1 (N°) =30 orders **(decision variable)** Holding cost = $1.500 per period Ordering cost = $1.500 per period Purchasing cost = 300.000 per period (min) J = $303.000 per period T (time horizon) = known

In previous examples (see Table 2), the deterministic discrete spreadsheet simulation model obtained the same results for ordering, holding, purchasing and total costs as a static model with EOQ formula. The next example of EOQ formula represents the model where the results of simulation are not exactly the same as in the static models described in Anderson et al. [5] - Table 3. In this section we will explain why the results of simulation are different compared against static EOQ formula. In the specific example for Problem 3 (see Table 3), of EOQ model without allowed shortages adjusted with a spreadsheet model, there is only one decision variable - Number of replenishments.

Table 3

Numerical example of EOQ model without shortages

Problem 3	**Traditional static EOQ model found (Anderson et al. [5])**	**Dynamic discrete spreadsheet model found**
D=104.000 units per year	EOQ =1.824,28 per order	EOQ =1.824,56 per order
Cs=$32 per order	N^o =57,01 orders per year	u^1 (N^o) =57 orders **(decision variable)**
Ch=$2 per unit per year	Holding cost = $1.824,28 per year	Holding cost = $2012,31 per period
Cu=$8 per unit	Ordering cost = $1.824,28 per year	Ordering cost = $1.824,00 per period
T=250 days	Total Cost = $3.648,56 per year Cycle time = 4,39 days	J = $3836,59 per period Cycle time = 4,39 days T (time horizon) = known

It is inconvenient to assign a decimal number to the number of orders as (etc. 57,01). The number of orders has to be adopted as an integer number (etc. 57). In the case where the number of orders is a decimal number (see Table 3.), the part of the number after a decimal comma proportionally increases the values of Ordering and Holding costs. It should be noted that Ordering cost exists as a whole number if there is an order, regardless of whether we order the 10^{th} or 100^{th} parts of one order. This fact assumes that parts of Ordering cost in Total cost should not be calculated in the case of orders as decimal number. Ordering costs in a static model are higher than ordering costs in a spreadsheet model for 0,88%, precisely for $0,28 as the ordering cost is proportionally calculated per number of orders (as decimal number), which is impossible in a real situation. As it is shown in Table 3, the Holding cost in a spreadsheet model is different from the Holding cost in a static model (see Table 3.). The main reason for this difference is the way in which the cost is calculated. In the static EOQ model holding cost represents the product of daily average inventory and the cost of holding per unit per day. If inventory dynamics is shown by right-angled triangle (see Figure 1a.), it is clear that all replenishment cycles and appropriate triangles have to be equal in order to calculate total holding costs at the end of the period. However, the question is what happens in the case when the heights of the triangles are not equal, in other words, when the replenishment cycles do not end with the inventory value of zero. In this case the average amount of inventory that is used to calculate the holding cost in a static model in [5] is not precisely defined (see Fig 2.).

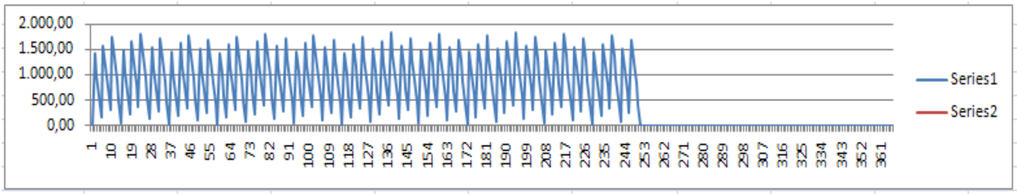


Fig. 2. Stock dynamics over the year in Problem 3 never reaches zero

Table 4. shows a situation when the replenishment cycles do not end with zero inventory value. At the end of the last period of each cycle (see column 4, Table 4.), inventory level never reaches zero. As a result, holding cost varies from cycle to cycle. In this example the total amount of holding cost is higher than the cost of a static model for $188.

Table 4

Comparison of numerical results for Problem 3. where inventory level never reaches zero

1	2	3	4	5
Number of replenishments	**EOQ quantity**	**Demand per each single period in cycles**	**Stock at the last single period in cycle**	**Holding cost at the last single period in cycle**
1	1.824,56	416,00	160,56	2,95
2	1.824,56	416,00	321,12	4,23
.	.	.	.	.
55	1.824,56	416,00	94,88	2,42
56	1.824,56	416,00	255,44	3,71
57	1.824,56	416,00	0,00	1,66
Total:	**104.000,00**			**188,03**

According to this assumption it is appropriate to conclude that the dynamic spreadsheet EOQ model is better for the presentation of holding costs, because the value of cost is taken into account in every single period of time (t) of inventory replenishment cycle. Total Holding Cost value is represented as a sum of individual costs for every discrete time period of the time horizon. Table 5 discusses the case of EOQ model with allowed shortages described in Anderson et al. [5].

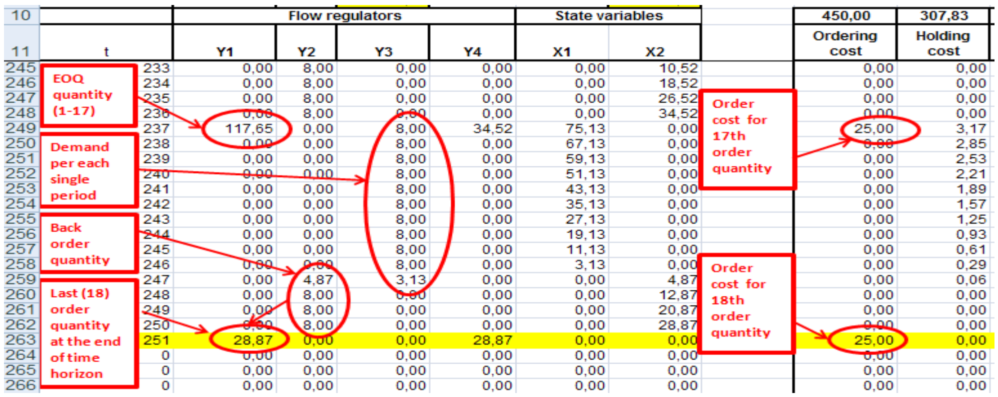
Table 5

Numerical examples of EOQ model with shortages

Problem 4	EOQ model found (Anderson et al.(2003))	Spreadsheet model found
D = 2000 units per year Cs = \$25 per order	EOQ = 115,47 per order N^o = 17,32 orders per year	EOQ = 117,65 per order u^1 = 17 orders per period - **decision variable**
Ch = \$10 per unit per year	Cycle time = 14,43 days	u^2 =25%(Backorder.quan.=28,823) **- decision variable**
Cu = \$50 per unit Csh = \$30 per unit per year	Backorder quantity = 28,87 Holding cost = \$325 per year	Cycle time = 14 days Backorder quantity = 28,82
T = 250 days	Ordering cost = \$433 per year Backord. cost = \$108 per year Total Cost = \$866 per year	Holding cost = \$307,83 per period Ordering cost = \$450 per period Backord. cost = \$137,83 per period Total Cost = \$895,66 per period T (time horizon) = known

In the specified example for Problem 4 of the EOQ model (see Table 5.), with allowed shortages, adjusted with spreadsheet model, there exist two decision variables: the Number of replenishments and the Percentage of the replenishment quantity which determines the value of allowed shortage. The total cost in the static model is lower than the total cost in the spreadsheet model, and the difference is \$30. In the numerical experiments of the EOQ model with shortages (see Table 5.) we can adopt the same assumptions in regard to Ordering and Holding costs as in the case of the EOQ model without shortages (see Table 3 and Table 4.). The basic assumption of the EOQ model with shortages is that each backorder quantity in every cycle must be satisfied at the beginning of the next replenishment cycle. At the end of the time horizon, when there are no any more orders for delivering, the last shortage quantity must be satisfied just in the amount of the height of shortage (see Fig 6.). This fact leads to the occurrence of the 18th order cycle in the time horizon, and also, to an additional ordering cost.

The ordering cost in a static model is lower than the ordering cost in a spreadsheet model for \$17, because of the existing ordering cost for the last 18th order in the spreadsheet simulation model (see Fig 6.). However, the amount of the ordering cost for 17 replenishment cycles should be \$425 (17 orders x \$17), but in the static model EOQ the model ordering cost is \$433, due to the decimal number of orders - 17,32 orders. Decimal number 0,32 is proportionally calculated in Ordering cost, which is impossible in the real-life situation, because the number of orders must be only integer numbers. As shown in Table 5, holding cost in the spreadsheet model is lower than the holding cost in the static model. The main reason for this difference is the way of cost calculation. In the static EOQ model the holding cost represents the product of daily average inventory and the cost of holding per unit per day, and the inventory dynamics is shown by the right-angled triangle (see Figure 1a.), where the heights of the right-angle triangles are equal.



10		Flow regulators				State variables		450,00	307,83
11	t	Y1	Y2	Y3	Y4	X1	X2	Ordering cost	Holding cost
245	233	0,00	8,00	0,00	0,00	0,00	10,52	0,00	0,00
246	234	0,00	8,00	0,00	0,00	0,00	18,52	0,00	0,00
247	235	0,00	8,00	0,00	0,00	0,00	26,52	0,00	0,00
248	236	0,00	8,00	0,00	0,00	0,00	34,52	0,00	0,00
249	237	117,65	0,00	8,00	34,52	75,13	0,00	25,00	3,17
250	238	0,00	0,00	8,00	0,00	67,13	0,00	0,00	2,85
251	239	0,00	0,00	8,00	0,00	59,13	0,00	0,00	2,53
252	240	0,00	0,00	8,00	0,00	51,13	0,00	0,00	2,21
253	241	0,00	0,00	8,00	0,00	43,13	0,00	0,00	1,89
254	242	0,00	0,00	8,00	0,00	35,13	0,00	0,00	1,57
255	243	0,00	0,00	8,00	0,00	27,13	0,00	0,00	1,25
256	244	0,00	0,00	8,00	0,00	19,13	0,00	0,00	0,93
257	245	0,00	0,00	8,00	0,00	11,13	0,00	0,00	0,61
258	246	0,00	0,00	8,00	0,00	3,13	0,00	0,00	0,29
259	247	0,00	4,87	3,13	0,00	0,00	4,87	0,00	0,06
260	248	0,00	8,00	0,00	0,00	0,00	12,87	0,00	0,00
261	249	0,00	8,00	0,00	0,00	0,00	20,87	0,00	0,00
262	250	0,00	8,00	0,00	0,00	0,00	28,87	0,00	0,00
263	251	28,87	0,00	0,00	28,87	0,00	0,00	25,00	0,00
264	0	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
265	0	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
266	0	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

Fig 3. Amount of order quantity for fulfillment of shortage at the end of time horizon

The holding cost may differ from cycle to cycle. The holding cost per cycle is different because the EOQ quantity is not divisible by daily demand (see Column 3, Table 6.). The total holding cost per each cycle varies from one replenishment cycle to another replenishment cycle. It is very important to notice that the replenishment cycles end with inventory value of zero. At the end of the last period of each cycle (see column 4, Table 6.), inventory level reaches zero. This fact results in the lower holding cost because inventory quantity at the end of each cycle does not move into the next cycle, when the stocks start increases. The shortage quantity starts increasing when the inventory quantity reaches zero (e.g. in cycle 1, stock quantity is 0,78 and shortage quantity is 7,22 (i.e.the summary is 8,00 - demand quantity per each single period).

In the case of the EOQ model with shortages it is necessary to pay attention to the Backordering cost. Inventory dynamics of backordering quantities is shown as an inverted right-angled triangle (see Fig 8.). At the start of shortage quantities occurence, after the inventory reaches zero, the first shortage quantity presents the remains of partially satisfied demand (see column 4 and 5, Table 6). This shortage quantity varies from cycle to cycle.

Table 6

Numerical results of spreadsheet model for EOQ model with shortages

1	2	3	4	5	6	7
Number of replenishments	**EOQ quantity**	**Daily demand per each period in cycles**	**Stock quantity in the last period in cycle**	**Shortage quantity in the last period in cycle**	**Holding cost in the last period in cycle**	**Shortage cost in the last period in cycle**
1	117,65	8,00	0,78	7,22	0,02	0,43
2	117,65	8,00	6,42	1,58	0,13	0,09
.	.	.	.	.	.	
15	117,65	8,00	7,84	0,16	0,16	0,01
16	117,65	8,00	5,48	2,52	0,11	0,15
17	117,65	8,00	3,13	4,87	0,06	0,29
Total:	**2.000,00**				**1,38**	**4,01**

The Backordering cost in one replenishment cycle is presented as a value of surface of the inverted right-angle triangle, precisely, as the product of daily average backlog-inventory and the cost of backorders per unit per day. It can be seen that all replenishment cycles and appropriate inverted triangles have to be equal in order to calculate the total holding costs at the end of the period.

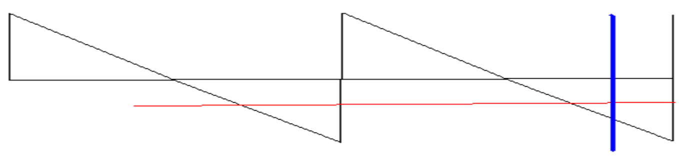


Fig 4. Stock dynamics in the case of backordering in finite time period

Thus, it is clear that this fact may also result with a deviation of Backordering cost when comparing a static model and a dynamic spreadsheet model. A deviation occurs due to a different maximum backlog value in each cycle period. For the same reason triangle areas of backlog are not the same in replenishment cycles of the static and dynamic simulation model (see Fig. 4), also the total cost will be different in a static and dynamic simulation model.

8. Conclusion

The model of inventory control as a discrete system control can be successfully used as a general dynamic model for analyzing inventory dynamics over a finite time horizon in the case of the fixed-order quantity system with or without shortages. The model of inventory control as a discrete system control, developed in a spreadsheet (tables and charts), represents a great tool, both for academics and professionals, for better understanding of dynamics of inventory on the day-to-day basis. This model clearly distinguishes the discrete controlled object, performance criterion, and a method for problem solving. Firstly, this paper gives the mathematical rationale of the discrete object (Law of dynamics and Control domain) representing the dynamics of the inventory stock over the time horizon. Secondly, a user does not have to make cumbersome spreadsheet formulas by s/himself. As it is proved, it is very useful to add a spreadsheet chart depicting stock and shortage dynamics over the time horizon. Thirdly, this paper shows how to define an objective function which will be incorporated into the performance criterion. Subsequently, one can perform ‘‘what if” analyzes or metaheuristics search in order to find the optimal solution which can be simulated and analyzed.

In order to prove superiority over the classical EOQ model, several prominent papers dealing with inventory problems where compared against our approach. We came to the conclusion that our approach might be more convenient for education and training of students and practitioners.

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