THE MAGNITUDE COHERENCE FUNCTION OF ROL/USD-ROL/EUR EXCHANGE RATES

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The magnitude coherence function (MCF) and the magnitude coherence index (MCI) of the exchange rates ROL/USD-ROL/EUR are computed using the estimator of the smoothed periodograms of time length of one quarter. The significance of the result is tested against both the shuffling procedure of the series and the overlapping windows of the estimator. Significant coherences were found at all frequencies with little increase toward the short term cycles of the order of several days. This is considered to be the particular effect of the short run psychological market force which synchronously drives the exchange rates additional to the economic market forces.

Keywords: magnitude coherence function, magnitude coherence index, exchange rate time series.

1. Introduction

Generally, the cross-correlation between signal spectra provides a statement of how common activity between two processes revealed by distinct time series is distributed across frequency. Coherence is measuring the cross-correlations of time series at each frequency. Two factors contribute to a significant value of the coherence, or cross-spectrum at a particular frequency. One is a significant coupling between the two processes at that frequency. The other is a significant concentration of power at that frequency in one or both processes.

In the case of financial events represented by exchange rate time series the power is understood as the squared values of the magnitude of the discrete samples. Due to a proper normalization, the magnitude coherence function (MCF) and the magnitude squared coherence function are largely used as adequate measures to disentangle the medium or short run correlations among distinct processes [1-3]. When smoothing MCF over the entire band, the resulting

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magnitude coherence index (MCI) provides a coarse indicator of the linear relationship between processes [4]. The medium and short terms are in the sense of the usual partition of the trends of the stock exchange markets [5].

2. Theory

By denoting $x(n)$ and $y(n)$ two interfering waves and $X(k)$ and $Y(k)$ the corresponding Fourier images, the power at frequency k is $P(k)$:

$$P_{xy}(k) \propto < \left| X(k) + Y(k) \right|^2 >_\tau \propto P_x(k) + P_y(k) + < 2\,\mathrm{Re}\{X(k) \cdot Y^*(k)\} >_\tau , \qquad (1)$$

where the window length τ is usually the characteristic averaging period of the detector, and the asterisk denotes the complex conjugate. Eq.(1) has the equivalent form

$$P_{xy}(k) = P_x(k) + P_y(k) + 2\sqrt{P_x(k)}\sqrt{P_y(k)} \cdot \frac{< \mathrm{Re}\{X(k) \cdot Y^*(k)\} >_\tau}{\sqrt{P_x(k)}\sqrt{P_y(k)}} . \qquad (1')$$

Since the waves are replaced here with financial time series and the interference is a computational one, the window length is chosen on econometric criteria. The macroeconomic indicators are computed every quarter, therefore the value will be set accordingly to τ=90 days.

The magnitude coherence function (MCF) measures the ratio of the cross-spectrum power to square root powers of the auto-spectra:

$$\gamma(k) = \frac{< 2\,\mathrm{Re}\, X(k) Y^*(k) >_\tau}{\sqrt{P_x(k)}\sqrt{P_y(k)}} . \qquad (2)$$

MCF is a normalization of the cross power spectrum by the product of the two auto-spectra, so that $\gamma(k)$ in Eq.(2) ranges from −1 to +1 for every numeric frequency k. The normalization of the coherence compensates for large values in the cross spectrum that result solely from large power, and thus enhances the influence of coupling. If X and Y are identical processes, then their power spectra are equal to each other and to the cross spectrum. In this case, the coherence is 1 at all frequencies. If X and Y are independent processes, then the cross spectrum approaches 0 for all frequencies and so is the coherence.

While MCF is depending on the numeric frequency k, the magnitude coherence index MCI is defined as the mean value over the frequencies consistent to the window width τ

$$\Gamma = \frac{2}{\tau} \cdot \sum_{k=1}^{k=\tau/2} \gamma(k) , \qquad (3)$$

where MCI is denoted by the capital letter Γ. According to the Shannon-Nyquist condition (see Table 1), the numeric frequency k is related to the period of the cycle T_k (in days) by:

$$T_k = \frac{\tau}{k-1}, \quad k=1,\ldots, 46. \tag{4}$$

Table 1

Time-frequency map

Window length	Sampling time	Freq. resolution	Shortest cycle	Longest cycle
90 days	1 day	(1/90) day^{-1}	2 days	90 days

The estimator $< >_{\tau}$ is computed according to the principles given in [6]. As usual, in the present work the stationary series of the returns, normalized to the standard deviation are used [7].

3. Data

The exchange rates of ROL/EUR and ROL/USD were taken from the site Forex Trading and Exchange Rates Services [8] and correspond to the interval 1 January 1999-31 December 2012, i.e. 5114 values.

4. Method

The estimator "$< >_{\tau}$" is taken using the sliding window technique [9,10] over the full length of the series. The method is implemented with the capabilities of Mathematica software [11].

According to Eq.(4), the main correspondences between the normalized frequency units and the time cycles are given in Table 2 (rounded values). The cycles between 2 and 15 days correspond to the minor cycles and short run trends (high frequencies), while the intermediate cycles and medium run trends range between two weeks and three months [5]. Consequently the short run numeric frequencies are assigned as follows: $8 \le k \le 46$ for the short run, and $2 \le k \le 7$ for the medium run; k=1 stands for zero-frequency and is not of interest in the present study.

Table 2

Time-frequency correspondences

Numeric frequency k	2	4	7	10	16	31	46
Time cycle (days)	90	30	15	10	6	3	2
Type of the cycles	medium			short			

Differing from other techniques [12], every MCF estimate "$< >_{\tau}$" is computed over all possibilities of overlapping, i.e. from no overlapping to almost full overlapping, the ratio of the overlapping parts of the neighboring windows to the length of the window ranging from 0 to 99%. Given the finite length of the series, the advantage of the technique is the coherence estimate does no more depend on the overlapping factor.

In order to draw robust conclusions on the existing of coherence, a statistical test was used by shuffling one term of MCF. The biasing influence of the continuous term at zero frequency was removed. Assuming normal distribution of MCFs with respect to the shuffles, the standard error and the *z*-statistic are used to testing for the statistical significances.

The dependence on the type of the distribution of the coherence margins of the white noise were also evaluated in the case of a large number of shuffled series of synthesized Gaussian white noise. MCF and MCI of the genuine series were compared to the shuffled versions and to the synthesized Gaussian white noise as well.

5. Results and discussions

5.1 MCF and MCI of ROL/USD-ROL/EUR

The stationary series of the returns of the exchange rates are shown in Fig.1.

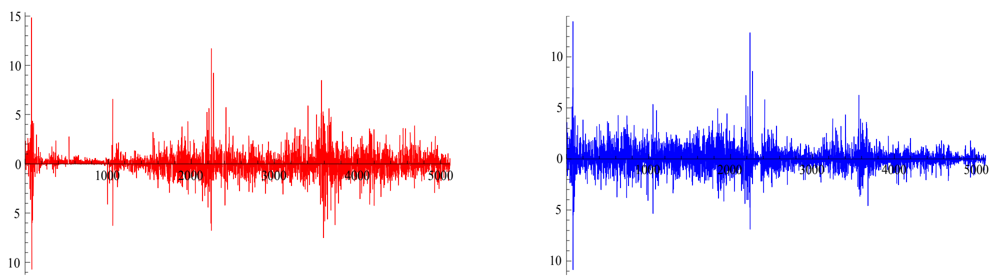

Fig.1 Time series of the normalized returns of ROL/USD (left) and ROL/EUR (right). The abscissae are in days.

MCF of ROL/USD-ROL/EUR is given in Fig.2, left side. For every frequency the following relations hold, which endorse the assumption of some coherence degree among ROL/USD and ROL/EUR:

$$0.42<\gamma(k)<0.78,\ \ k=2,\ldots,46. \quad (5)$$

According to Eq.(3) MCI was found to be

$$\Gamma=0.6262. \quad (5')$$

For a properly confirmation of the coherence a statistical test is mandatory [13].

5.2 MCF and MCI of the shuffles

The challenge is to state if the values of MCF given by (5) and subsequent MCI given by (5′) are likely to arise by hazard (null hypothesis). A hazardous distribution is the one of the MCF values coming from the shuffled exchange rate series. There were found no differences when shuffling ROL/EUR only, ROL/USD only, or both. Hereafter the reference to the results of the shuffled ROL/EUR is presented.

According to the central limit theorem, as the number of shuffles increases, the MCFs of the sampling (shuffled) distribution should approach asymptotically $\mathbf{N}(\mu(k),\sigma_S(k))$, where the estimated mean $\gamma_S(k)$ approaches the mean $\mu(k)$ and the standard error $\sigma_{\gamma_s}(k)$ approaches the standard deviation $\sigma_S(k)$. For a large number of shuffles of the data in the series of ROL/EUR the typical MCF and standard deviations at each frequency are shown in the right side of Fig.2. In computer implementation, as the sums become large, one needs to consider round-off error, arithmetic overflow, and arithmetic underflow. For this reason the formulae of the statistic moments were implemented in the computing software in the running sum form [14].

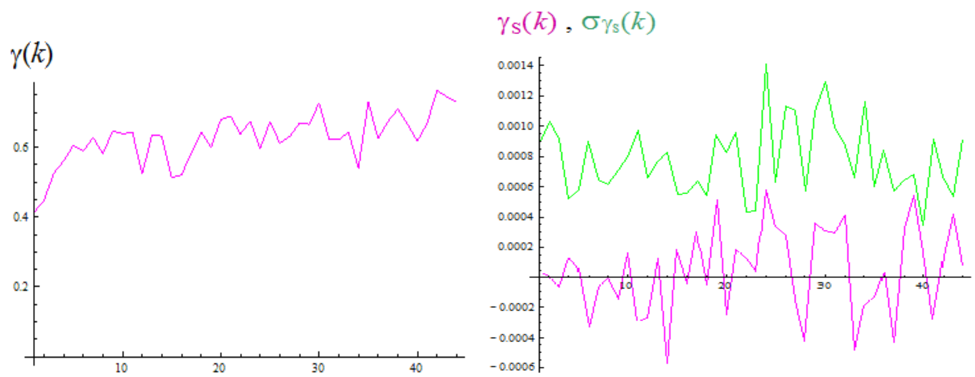

Fig.2 MCF of ROL/USD-ROL/EUR (left), and MCF of ROL/USD-shuffled ROL/EUR (right, 10^5 shuffles). The abscissae are in units of numeric frequency.

One should note the values of MCF are significantly greater in the case of the real series than in the shuffled cases, at any frequency:

$$\frac{\gamma(k)}{|\gamma_S(k)|} > 400\,,\ k=2,\ldots,46. \qquad (6)$$

MCI behaves accordingly, i.e. MCI asymptotically approaches $\mathbf{N}(1.08\cdot 10^{-3},\ 2.19\cdot 10^{-3})$ as the number of shuffles increases significantly. One should remark two things: firstly, a biasing of MCI towards positive residual correlations that characterizes the shuffles – this is the effect of the particular starting moment of the genuine series ROL/EUR –, and secondly, since the mean is significantly smaller than the genuine MCI (see Eq.(5′)), the distribution of the MCI of the shuffles could be approximated by a normal distribution with zero mean.

In order to have a global test irrespective the frequency – there are 45 distinct frequencies –, the MCI normal distribution of the shuffles would be applicable to test the coherence at any frequency. Because the coherence is also negatively defined, the *z*-score for a double-sided test is

$$z = \frac{|\Gamma|}{2.19 \cdot 10^{-3} \cdot \sqrt{45}} \,. \tag{7}$$

From Eq.(7) the interval outside the null hypothesis (no coherence) is rejected if

$$|\Gamma| > 1.45 \cdot 10^{-2}. \tag{8}$$

Eq.(8) applies equally for Γ or any $\gamma(k)$ given by Eqs.(5′) and (5) respectively. In the case of the smallest value $\gamma(2) \cong 0.42$, Eq.(8) the likelihood to accept the null hypothesis is smaller than 10^{-8} [13]. Therefore it is highly unlikely that the MCF and MCI values of the genuine exchange rates ROL/USD-ROL/EUR would be observed under the null hypothesis. The margins defined in (8) are in fact the margins inside the coherence is lacking. For the scope of the present study is therefore useful to define a symmetric noise band centered on zero coherence:

$$-1.45 \cdot 10^{-2} \le \gamma_{\text{noise}}(k) \le 1.45 \cdot 10^{-2}, \; k=2,\ldots 46. \tag{9}$$

The coherence is greater in the short run than in the medium run. There is a slight increasing trend toward the short cycles. This could be explained by the synchronous action of the short and medium range market forces. The psychological market force that drives the short run trends progressively vanish during an interval of the order of two weeks leaving the place to the underlying economic plans that manifest the effects in the range of one month and longer [14].

5.3 Coherence in the case of the white Gaussian noise

The same method was operated onto synthesized series of white Gaussian noise **N**(0,1). A number of 10^5 shuffles were tested for the distribution of the MCFs; the same parameters and computational rules as for the genuine exchange rates apply on the shuffling procedure. The typical results for the mean values and standard deviations are shown in Fig.3.

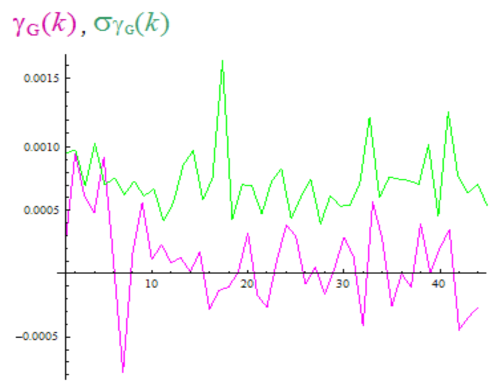

Fig.3. Typical MCF of two synthesized sequences of white Gaussian noise after10^5 shuffles. The abscissa is in units of numeric frequency.

No significant differences were remarked between the Gaussian white noise case and the shuffled exchange rates. This is not surprising since the shuffling destroys time correlations and consequently the coherences at any frequency, while not changing the type of the distribution. Due to the normalization operated by Eq.(2), the type of distribution has no significant influence on the MCF.

To conclude, the characteristics of the distribution of the values in the series do not imprint important effects to the MCF and consequently to the MCI. The results for MCI are summarized in Table 3. The standard deviation of the MCI in the case of ROL/USD-ROL/EUR is not relevant since it is generated by the dependence on the numeric frequency *k*, not by the shuffling.

Table 3

MCI of the real exchange rates as compared to the shuffles and synthesized white Gaussian

	MCI	Standard deviation
ROL/USD-ROL/EUR	$\Gamma=0.6262$	not relevant
Shuffled ROL/EUR	$\Gamma_S=1.08\cdot10^{-3}\cong0$	$\sigma_S=1.45\cdot10^{-2}$
Synthesized white Gaussian	$\Gamma_G\cong0$	$\sigma_G=7.60\cdot10^{-3}$

6. Conclusions

The mutual coherence of ROL/USD and ROL/EUR exchange rate series during 1 January 1999-31 December 2012 has been investigated using the magnitude coherence function and the magnitude coherence index estimated within the window time of one quarter. The window length is chosen in order to match the interval the economic indicators are available and allows to discriminating the short run from the medium run trends that characterize the financial markets.

The time history of ROL/USD and ROL/EUR exhibit linear correlations at all investigated frequencies due to the almost exclusively use of the ROL currency on the Romanian market. USD and EUR account for the trading processes with distinct external markets. The coherence values of MCF and MCI are statistically significant at a confidence level better than 10^{-8} for 10^{5} shuffles. The white noise has little importance either Gaussian or coming from the shuffled exchange rate series.

MCF is greater in the short run than in the medium run. This could be explained by the action of the economic market force in the short and medium run, and by synchronous action of both psychological and economic market forces in the short run.

R E F E R E N C E S

[1] *M.M. Dacorogna, R. Gençay, U.A. Müller, R.B. Olsen, O.V. Pictet*, An Introduction to High-Frequency Finance, Elsevier Inc. 2001.

[2] *A.F.C. Infantosi, D.B. Melges and C.J. Tierra-Criollo*, Use of magnitude-squared coherence to identify the maximum driving response band of the somatosensory evoked potential, Braz. J. Med. Biol. Res, **39**, (12) 1593-1603, 2006.

[3] *E.Wolf*, Radiometric Model for Propagation of Coherence, Optics Letters, **19**, (23), 2024-2026, 1994.

[4] *C.R. Fadragas, R.O. Morales*, Using the Magnitude-Squared Coherence for Determining Order-Chaos Transition in a System Governed by Logistic Equation Dynamics, http://arxiv.org/abs/nlin/0304043v1.

[5] *R.McCurtain*, Stock market posts weekly loss; intermediate cycle at risk, Weekly Review: MAAD & CPFL Analysis, April 7, http://www.futuresmag.com/2013/04/07/stock-market-posts-weekly-loss-intermediate-cycle?t=financials&page=2, 2013.

[6] *P.D. Welch*, The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms, IEEE Transactions on Audio Electroacoustics, AU-15, 70-73, 1967.

[7] *A.B. Schmidt,* Quantitative Finance For Physicists: An introduction, Elsevier Inc., 2005.

[8] http://www.oanda.com/currency/historical-rates.

[9] *S-Y. Wang, M-X. Tang*, Exact confidence interval for magnitude-squared coherence estimates, IEEE Signal Processing Letters, **11**, (3) 326-329, 2004.

[10] *G. Carter*, Confidence bounds for magnitude-squared coherence estimates, IEEE Transactions on Acoustics, Speech, and Signal Processing, **26**, (5), 475-477, 1978.

[11] http://www.wolfram.com #L4701-2665.

[12] *S. Sinha, A. Chatterjee, A. Chakraborti, B.K. Chakrabarti*, Econophysics: An Introduction, Wiley-VCH, 2010.

[13] Engineering Statistics Handbook, http://www.itl.nist.gov/div898/handbook/, 2012.

[14] *B.P. Welford*, Note on a Method for Calculating Corrected Sums of Squares and Products, Technometrics **4**, (3), 419-420, 1962.

[15] *E.I. Scarlat, C.P. Cristescu, C. Stan, L. Preda, A.M. Preda, M. Mihailescu*, Chaotic dynamics in econophysics: modeling the market forces, UPB Sci. Bull. A, **70** (4), 89-95, 2008.