THE EPIPOLAR GEOMETRY AND APPLICATION OF SPHERICAL PANORAMA

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Epipolar geometry is a core issue in computer vision and close-range digital photogrammetry, which could reduce the search range to improve both calculation efficiency and reliability of matching. Currently, epipolar constraint has achieved a wide range of applications based on the traditional pinhole projection images and satellite images. However, panoramic epipolar geometry is different from the other images because of its peculiar geometric modeling. The panoramic epipolar constraint can improve the reliability, accuracy and speed of matching. Nevertheless, this constrain has not been fully applied to the 3D information processing of spherical panorama. The paper focuses on the epipolar geometry for spherical panorama and constructs the epipolar geometry based on the coplanar equation. Then, we derive the mathematical equations and their properties, which provide a theoretical basis for the spherical panoramic epipolar line that could be used in computer vision and the close-range digital photogrammetry. The experiment results show that the epipolar line is effectively validated which can reduce the search range and obtain more useful information.

Keywords: Epipolar geometry; Panorama; Spherical projective geometry

1. Introduction

The epipolar line is the basis of image matching in the fields of digital photogrammetry and computer vision. It can reduce matching search problem to limit the search area from two dimensions to one dimension through epipolar constraint, improving the computational efficiency and reliability. At present, the epipolar constraint is widely used in traditional central projection images and satellite images [1-7], while panoramic images have different geometric mode, which makes the derivation and application of the panoramic epipolar more difficult. So, the epipolar geometry and application of spherical panorama is a research hotspot in digital photogrammetry and computer vision.

Panoramic images have landscape browsing of 360 degrees, which is widely used in street view and digital campus etc. The previous research work mainly focused on the panoramic image stitching [8-9] and panoramic manufacturing [10-11]. Representative works are: Luhmann et al. explored the three-dimensional reconstruction of cylindrical panorama projection [12-14].

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Fangi et al. studied the three-dimensional modeling method under spherical panorama projection [15-17]. They applied a large number of control points to calculate based on the multi-azimuth panoramic images by bundle adjustment. In recent years, many researchers have begun to pay attention to the scalability of the panorama. The panoramic measurement is realized by double spherical panoramic images in literature [18]. The literature [19] realized three-dimensional reconstruction of spherical panoramas using GPS/IMU data and LIDAR elevation information to combine SIFT operator and Fonstner operator. On the work of panoramic epipolar geometry, the literature [12]-[14] and [20] studied and derived the cylindrical panoramic epipolar geometry. The panoramic epipolar equation is studied and deduced as the features of hyperbola and parabola in literature [21]. Literature [22] deduced that the geometric features of spherical panoramic epipolar lines were large circles, which did not been verified and applied.

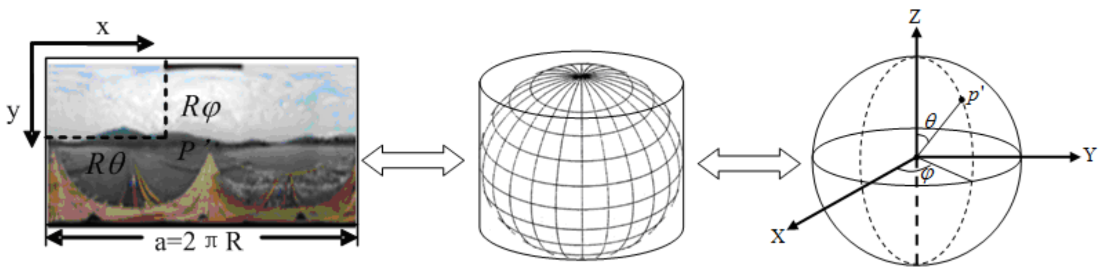
The epipolar constraint can improve the reliability, accuracy and speed of the panorama matching. However, this geometric relationship has not been fully applied in the three-dimensional information processing of spherical panoramic images. For the spherical panoramic images, there is a kind of unique geometric relationship between epipolar lines, which is different from the traditional central projection image.

In this paper, the epipolar line of spherical panoramic images is established based on the coplanar equation by the projection geometry. The mathematical equation and its characteristics are derived, which provides a theoretical basis for the application of epipolar lines in spherical panoramic images, and the experiments show the correctness and feasibility of this spherical panoramic epipolar equation.

2. Spherical panoramic geometric model

Spherical panorama geometry expression is the foundation of the epipolar establishment, and the observation point is located in the centre of a sphere model. The multiple image sequence of outdoor is set to the surface of a sphere through a certain mapping relationship, thus forming the visual observation of 360-degree seamless scenario shown in fig 1. The spherical panoramic image is made up of getting many perspective images to be a rectangle image according to certain algorithm, which is mapped onto the sphere by isometric projection. So, a point P ' on the sphere has a kind of one-to-one mapping relationship with a point p' on the panoramic image.

Fig 1 Spherical panoramic images and virtual sphere texture mapping diagram



a) Panoramic image *b)* Spherical geometry description and the space rectangular coordinate system

Fig 2 The latitude-longitude projection relation between spherical coordinates and the panoramic image coordinate of sphere

The expression of the sphere model is similar to the latitude and longitude grid, and the mapping relation can be described as: Set the length of panoramic image to be *a*; the radius of the sphere model is *R*; the relationship of *R* and *a* is $a = 2\pi R$; and namely the ratio of panoramic length and width is 2:1. Set a point on the coordinates of *p '(x, y)* ; and the corresponding three-dimensional coordinates of the point on the sphere model is *P' (x, y, Z)*. According to the literature [16] - [18] and the mapping shown in fig 2, *p* 'and *P'* coordinates can be expressed as:

$$\begin{cases} x = R \cdot \theta \\ y = R \cdot \varphi \\ R = a / 2\pi \end{cases} \tag{1}$$

$$\begin{cases} X = R \cdot \sin\theta \cdot \sin\varphi \\ Y = R \cdot \cos\theta \cdot \sin\varphi \\ Z = R \cdot \cos\varphi \end{cases} \tag{2}$$

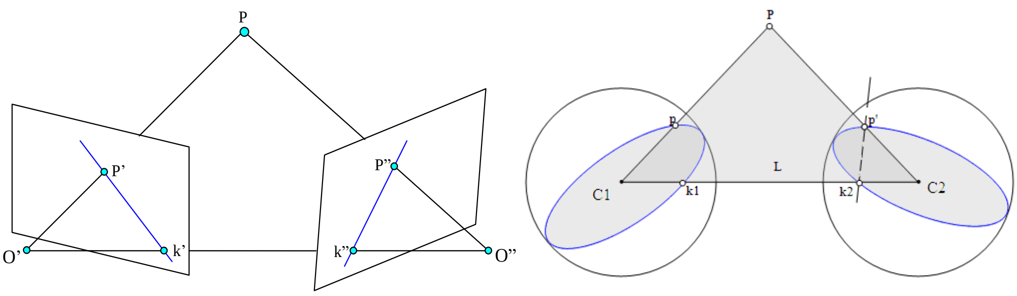
The corresponding relationship between two-dimensional point *p* 'and three-dimensional point *P*' on the sphere can be obtained by simple derivation according to formula (1) and (2).

$$\begin{cases} X = R \cdot \sin\dfrac{x}{R} \cdot \sin\dfrac{y}{R} \\ Y = R \cdot \cos\dfrac{x}{R} \cdot \sin\dfrac{y}{R} \\ Z = R \cdot \cos\dfrac{y}{R} \end{cases} \tag{3}$$

In the formula, the values of angles θ and φ are radians.

3. The epipolar line equation of spherical panoramic image

The geometric relationship between the two images is usually expressed by the coplanar equation or expressed by the epipolar relationship as shown in Fig 3 (a) in the traditional photogrammetry. For the spherical panoramic image, the epipolar line is no longer a straight line, but two spherical large-circle curves as shown in Fig 3 (b). The relative orientation parameters between stereo images can be calculated from the literature [18], and then we can derive the mathematical equations of the epipolar line trajectory corresponding to the feature points. In Fig 3(b), the epipolar plane consisting of a point p and two spherical center points (C1, C2) intersecting with two spheres and the epipolar line can be obtained.



(a) (b)

Fig 3 Traditional photogrammetry epipolar geometry and double spherical projective geometry

3.1 Geometric definition of epipolar line

In Fig 3(b), let C_1 and C_2 be the projection centers of the adjacent two spherical panoramas respectively. Let the vector of the baseline C_1C_2 be $B\ (b_x\ b_x u\ b_x v)^T$. The left panoramic sphere is the reference sphere; and the rotation parameters of the right one relative to the reference sphere is (ϕ, Ω, K). Let P be any point in reality; p is the projection point of P on the left spherical panorama;

set its coordinates in the reference sphere coordinate system to (*X' Y' Z'*). The coordinates of the point C1 point C2 and point p in the virtual sphere are respectively set to $(X_{C1}\ Y_{C1}\ Z_{C1})^T$, $(X_{C2}\ Y_{C2}\ Z_{C2})^T$ and $(X_p\ Y_p\ Z_p)^T$.

3.2 Mathematical model of epipolar line

(1) Solve the coordinates of the points C_1, C_2, and *p* in the right spherical center space rectangular coordinate system.

C_2 is the origin of the virtual sphere; the coordinate is $(0 0\ 0)^T$; the point *p* and point C_1 are rotated by translation and can be converted to the coordinates of the virtual sphere in the space rectangular coordinate system. If the rotation parameter is (ϕ, Ω, K), and then the rotation matrix *R* that rotates to the virtual sphere coordinates with reference to the sphere coordinates is:

$$R = inv\ (R_1) = inv\left(\begin{pmatrix} \cos\Phi & 0 & -\sin\Phi \\ 0 & 1 & 0 \\ \sin\Phi & 0 & \cos\Phi \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Omega & -\sin\Omega \\ 0 & \sin\Omega & \cos\Omega \end{pmatrix} \cdot \begin{pmatrix} \cos K & -\sin K & 0 \\ \sin K & \cos K & 0 \\ 0 & 0 & 1 \end{pmatrix}\right) \tag{4}$$

The point C_1 coordinates in the virtual sphere of the Cartesian coordinate system are:

$$\begin{pmatrix} X_{C1} \\ Y_{C1} \\ Z_{C1} \end{pmatrix} = R_2 \cdot \begin{pmatrix} 0 - b_x \\ 0 - b_x \cdot \mu \\ 0 - b_x \cdot v \end{pmatrix} = -b_x \cdot R_2 \cdot \begin{pmatrix} 1 \\ \mu \\ v \end{pmatrix} \tag{5}$$

The point *p* coordinates are:

$$\begin{pmatrix} X_p \\ Y_p \\ Z_p \end{pmatrix} = R_2 \cdot \begin{pmatrix} X' - b_x \\ Y' - b_x \cdot \mu \\ Z' - b_x \cdot v \end{pmatrix} \tag{6}$$

In formula (6), (X' Y' Z') is given by formula (3).

(2) Find the plane of the epipolar equations for the points C1, C2, and *p*. The general formula for the plane is:

$$AX + BY + CZ + D = 0 \tag{7}$$

The epipolar surface *L* passing through the points p, C1, and C2 also satisfies Expression (7). Substituting the coordinates of the three points into Formula (7) yields the formula for the epipolar surface L:

$$\begin{cases} A = C\dfrac{Z_pY_{C1} - Y_pZ_{C1}}{X_pX_{C1} - X_pY_{C1}} \\ B = C\dfrac{Z_pX_{C1} - X_pZ_{C1}}{Y_pX_{C1} - Y_pY_{C1}} \\ AX + BY + CZ + D = 0 \\ D = 0 \end{cases} \tag{8}$$

Introduce a_1 and a_2 as the new coefficients of the epipolar surface formula *L*, simplifying the epipolar surface formula *L*:

$$\begin{cases} a_1 = \dfrac{Z_pY_{C1} - Y_pZ_{C1}}{X_pX_{C1} - X_pY_{C1}} \\ a_2 = \dfrac{Z_pX_{C1} - X_pZ_{C1}}{Y_pX_{C1} - Y_pY_{C1}} \\ a_1X + a_2Y + Z = 0 \end{cases} \tag{9}$$

In formula (9), a_1 and a_2 are the coefficients of the epipolar surface formula L.

(3) Solve the trajectory equation of the epipolar line.

The virtual sphere spherical formula $X^2 + Y^2 + Z^2 = R^2$ is converted into a function of parameters θ and φ in the polar coordinate system:

$$\begin{cases} X = R \cdot \sin\theta \cdot \sin\varphi \\ Y = R \cdot \cos\theta \cdot \sin\varphi \\ Z = R \cdot \cos\varphi \end{cases} \tag{10}$$

The epipolar line sought is the intersection of the epipolar surface L and the right spherical surface. Therefore, substituting formula (10) into formula (9) can be obtained:

$$(a_1 \cdot \sin\theta + a_2 \cdot \cos\theta) \cdot \sin\varphi + \cos\varphi = 0 \tag{11}$$

Formula (11) is simplified to obtain:

$$\varphi = \arctan\left(-\frac{1}{a_1 \cdot \sin\theta + a_2 \cdot \cos\theta} \right) \tag{12}$$

The trajectory equation formed by the corresponding epipolar line in the virtual sphere is formula (12). The equation takes θ as a parameter; and the value range is 0-2π; the coordinates of C_1 $(Xc_1\ Yc1\ Zc_1)^T$ and $p(Xp\ Yp\ Zp)^T$ can be calculated by formulas (5) and (6) ; the coefficients of a_1 and a_2 can be calculated by substituting formula (9). Formula (12) constitutes the relationship between θ and φ. The equation takes θ as a parameter; the value range is 0-2π; and the value of φ can be determined by θ. The coordinates (*X' Y' Z'*) of the point on the trajectory formed in the virtual sphere with the corresponding epipolar line are obtained according to formula (10).

4. The design and verification epipolar line equation derivation experiment

The theoretical derivation mentioned above of the spherical panoramic epipolar line trajectory is a big circle on a spherical projection. These derivations are performed on the basis that the panoramic image strictly follows the relationship of latitude and longitude projection. So it is necessary to verify to determine whether the points are on the same big circle after the acquisition of the panoramic image. Furthermore, there may be various errors in the process of data acquisition, stitching and relative orientation of the panoramic images, resulting in that the corresponding points are not strictly on the same corresponding line. Therefore, the experiment is designed and verified for these two aspects: (1) whether the epipolar line trajectory of the spherical panoramic is a big circle; (2) statistics and verification of the degree of deviation of the corresponding epipolar line.

As it can be seen from (3), there is an one-to-one correspondence between a feature point on a panoramic image and a three-dimensional point on a spherical panorama. Therefore, the corresponding point after the image matching can be converted into a three-dimensional point on the spherical panorama by the formula (3), it means that the three-dimensional point on the three-dimensional spherical surface can be obtained by the panoramic image matching. Assume that the left spherical panoramic coordinate system is the coordinate system of the entire three-dimensional panoramic system, and the relative orientation parameters can be obtained by more than five groups of corresponding points (p, p') after image matching or manual intervention in [18].

4.1 Experimental design

In order to verify the geometry of epipolar imaging and the deviation of the epipolar line in this paper, the experiment was set up at a park in 2 places; the camera model was Nikon D40; and the fisheye lens model was Sigma 8mm; four photos were taken at 90° intervals in each place and were stitched into 2 spherical panoramic images using commercial software Shanghai Jietu software [23].

During the entire shooting process, the focal length remained unchanged, and one of the panoramic images was selected as a reference image (Fig 4). The design flow is shown in Fig 5 and the specific steps are as follows:

(1) Select five pairs or more corresponding points from two panoramic images to implement relative orientation, and obtain the relative orientation parameters;

(2) Divide the reference map into a 200×200 cell size of 160 areas, and select 100 feature points;

(3) Randomly select 10 feature points from 100 ones, then calculate 10 corresponding epipolar lines according to formula (12) to certify big circle for each epipolar line;

(4) Calculate deviation from distance and error analysis for 100 pairs of feature points with the corresponding epipolar lines.

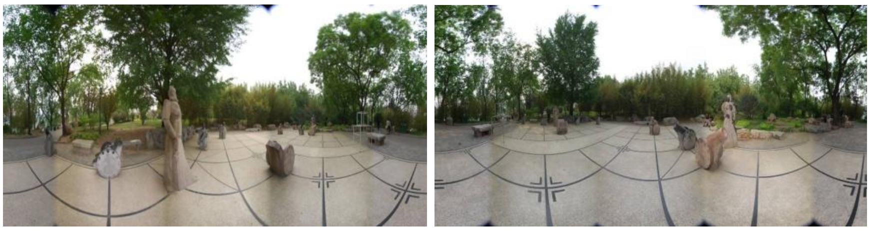


Fig 4 Panoramic stereo pair acquisition

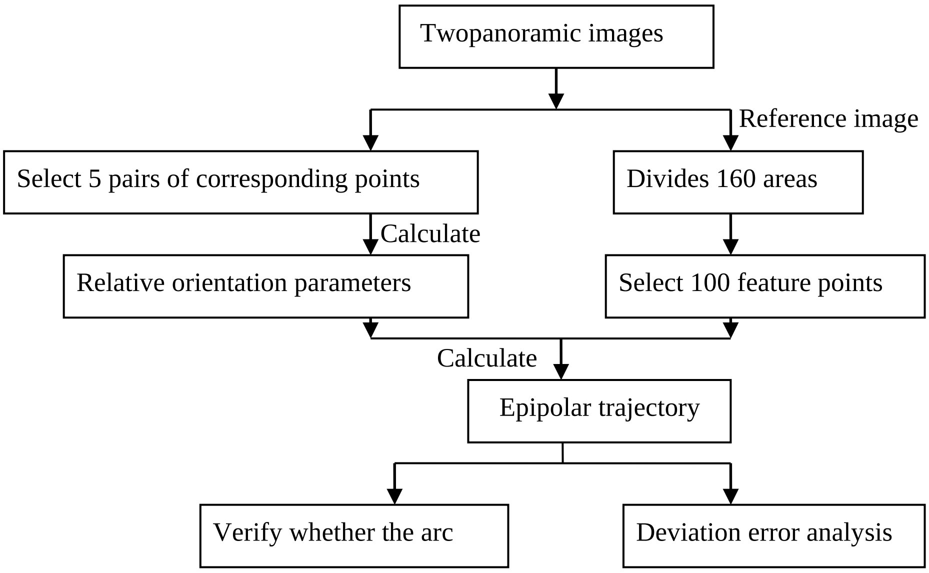


Fig 5.the flowchart of the experimental design

4.2 Experimental verification

(1) Big circle verification

Select any point in the reference image as shown in the black cross wire in Fig 6 (a); calculate corresponding epipolar line according to the relative orientation parameters; and map the point of the epipolar line to the right image.

The nuclear epipolar is arced, seen in Fig 6 (b) namely the red arc. According to the spherical panoramic imaging theory and Fig 3(b), the complete one epipolar line mapped to a spherical panoramic coordinate system should be a large circle with radius R. The width of panoramic image in the experiment was 4000 pixels, and R=4000/(2π)=636.6 pixels according to formula (1).

(a) One feature point (b) Epipolar trajectory

Fig.6 the feature point and the corresponding epipolar line

In this paper, the experiment is verified by inverse calculation. One point is obtained by one degree for a complete epipolar line, thus getting a total of 360 points in fig 7. Then, two points for a difference of 180 degrees are combined to a group (The red point Q and the black point Q ' are collinear in fig 7 (a)). A total of 180 group points, calculating the distance of each set of points, and then verify each distance is two times for the radius, namely the epipolar line is arc shaped.)

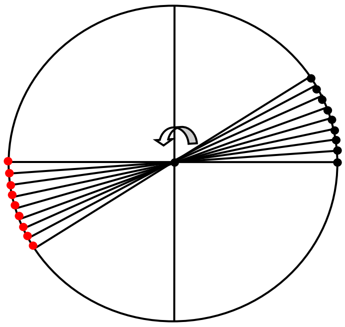
The corresponding epipolar line of any arbitrary point in reference panoramic image is calculated by formula (12). θ is the parameter; and the value range is 0-2π; and the value of φ can be determined by θ.

Since the range of the *arctan* function is in the range of [-π/2, +π/2], according to formula (1), φ can only correspond to the points in the panorama in the range of [0, π]. Therefore, the value of X should be mapped to [0, π]. When φ<0, and then φ=π+φ. The point q on the epipolar line assumes that its coordinate is (θ0, φ0), and the corresponding point q ' of the spherical center corresponding to the difference of 180 degrees is as follows:

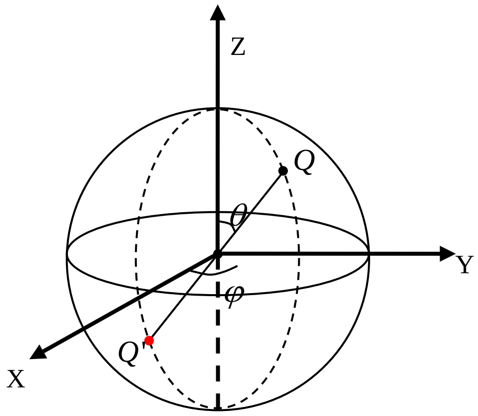
$$\begin{cases} \theta' = \begin{cases} \pi + \theta_0, & \theta_0 \in [0, \pi) \\ \theta_0 - \pi, & \theta_0 \in [\pi, 2\pi) \end{cases} \\ \varphi' = \pi - \varphi_0 \end{cases} \tag{13}$$

Substitute the points q and q ' into formula (2) according to formula (2), the coordinates and the distance of the corresponding spatial point q (fig 7 (b) in the black dot) and q' (fig 7 (b) in the red dot) can be obtained by calculating. The diameter error obtained by selecting 180 groups of points was within 0.001 pixels. Thus, the epipolar line imaging can be verified as a large circular arc with a radius of 636.6 pixels.

According to the principle of spherical panoramic imaging, the paper deduces that the panoramic epipolar trajectory is a big circle on the spherical panorama, which is proved in Experiment 1. The idea is to take 360 points uniformly distributed on the trajectory of the epipolar trajectory, and then divide the angles by 180 degrees into one group, a total of 180 groups, and calculate whether the distance between the 180 groups of the same group is equal to the diameter of the circle, thus being verified. Specific steps are as follows: (1) The epipolar geometry and the three-point coplanar principle is defined in section 3.2 .



(a) Points and groups on the epipolar line



(b) Group of three-dimensional points

Fig.7 The epipolar line of the spherical panorama verification chart

The relationship between the angle parameters (*θ,φ*) of the epipolar line is derived under the premise that the two sphere parameters of the rotation and translation are known (formula (12), it means that the trajectory of the epipolar line on the virtual sphere . (2)The specific relationship of the point angle parameters (*θ,φ*) on the corresponding epipolar at any point is further determined, according to the epipolar trajectory by formula (12) in section 3.2 (see equation (13)). (3) Calculate the trajectory of the epipolar according to formula (13); take a point every one degree in*θ* on the trajectory; take a total of 360 points; and divide the two points whose angle *θ* differs by 180 degrees into a group. In the 180 groups, it is proved that the distance between the two symmetry points of the same group is twice the radius, and the radius has been obtained as 636.6 pixels before.

(2) Deviation error analysis

The reference panoramic image is divided into 160 regions with the same size area of 200 x 200 pixels to calculate the deviation degree of epipolar line as shown in fig 8, and each grid is a black area. 100 feature points are taken from the 160 areas, which are represented by the Red Cross wires.

The corresponding points shall be on the corresponding epipolar lines in theory, however, because of the existence of various errors appear deviation in the actual application as shown in fig 9, the epipolar line should be curve; and because it is the screenshot, therefore, the epipolar line track approximate straight line. Fig 9 (a) is the corresponding feature points (right red crosshair), which almost located on epipolar line. The actual deviation distance of the epipolar line and the corresponding feature points is about 40 pixels shown in fig 9 (b), which can be applied to the panoramic measurement matching as a reference line.

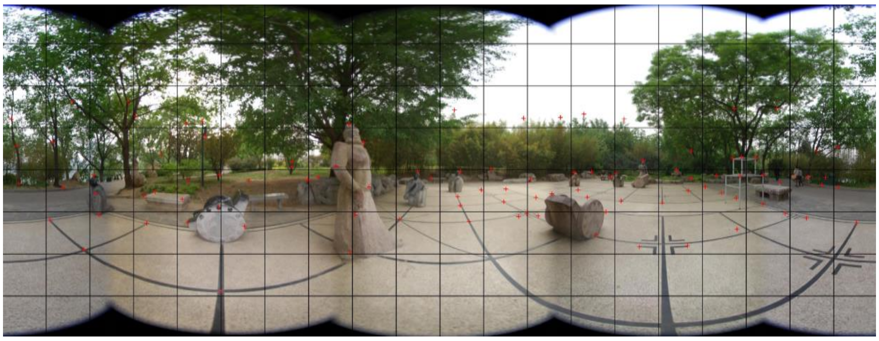
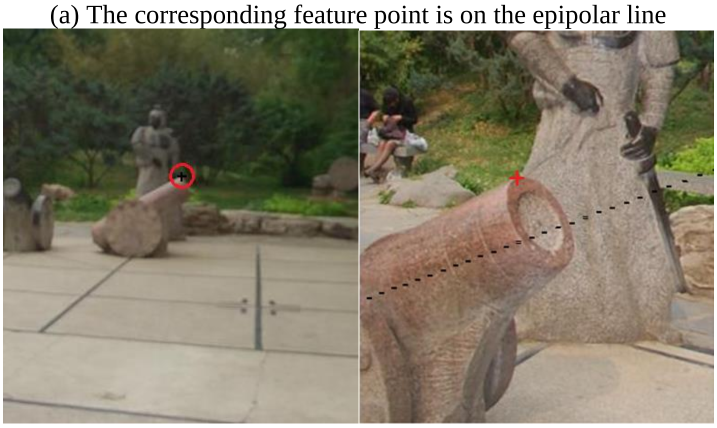


Fig.8 The regional division and distribution of feature points

(a) The corresponding feature point is on the epipolar line



(b) Deviation of the corresponding feature point from the epipolar line

Fig.9 The application of epipolar line and an example of its deviation from the corresponding point (part of the screenshot)

The errors of deviation degree of the above 100 feature corresponding points and their epipolar distance are calculated as shown in Fig. 10.

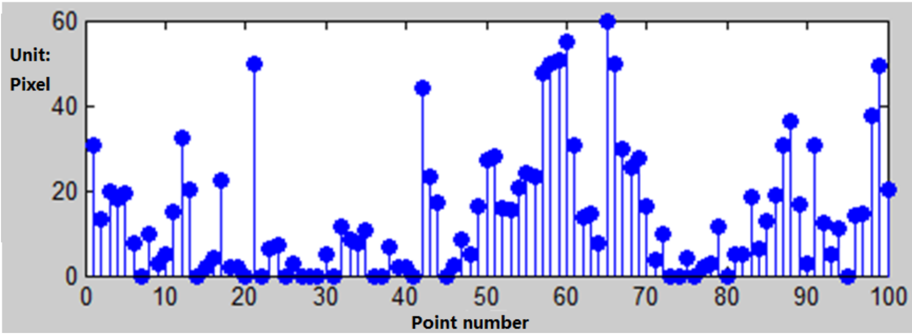


Fig.10 The errors of the feature points deviating from the corresponding epipolar line

The percentage of points that deviate from 30 pixels is 16%, and 84% of the points within 30 pixels in Fig. 10. In Fig. 9, it can be applied to the panoramic measurement matching as a reference line. Deviation from the larger point is almost near the baseline direction, and panoramic measurement near the baseline direction is not stable pointed out in literature [18], even it is difficult to effectively measure, the relevance of both remains to be further research.

The distance between the 100 feature points and the epipolar line is statistically selected, meanwhile the deviation degree and point distribution is obtained in the second experiment, which is applied to the search range of the measurement application. According to close-range photogrammetry and computer vision, the epipolar constraint can reduce the search range of the corresponding point from two-dimensional to one-dimensional. In theory, the corresponding should be on the corresponding epipolar line. However, there is a distortion in the mapping of the sphere to the plane, and there may be errors in the image stitching and the model orientation process. The corresponding t may be offset by the corresponding epipolar line. Specific steps: divide the image into 160 regions and select 100 feature points. According to the epipolar line formula (12) of Section 3.2, the trajectory map of the epipolar line is calculated; the vertical pixel distance of the corresponding feature point from the epipolar line is calculated; the distribution and deviation of the statistical point are obtained; and the degree of deviation as well as the position distribution of the point is obtained. As can be seen from Fig. 10, this degree of deviation is feasible for the feature matching in the secondary measurement applies.

5. Conclusion

This paper deduces the mathematical model of epipolar line under the projection geometry of spherical panoramic image and verifies its geometrical features effectively through experiments.

The corresponding points shall be on the corresponding epipolar lines in theory, but because of the influence of various errors exist, such as the quality of the camera itself, intersection Angle, feature point extraction, image data acquisition, distribution and matching precision, stitching algorithm, the selection of control points, etc., there is a certain distance of deviation for the spherical panorama epipolar line, which still can effectively reduce the search scope. The study results can meet the demand of practical application for matching under human intervention. In the future, we will further study the following aspects:

(1) The problem of epipolar deviation. This paper deduced the spherical projection epipolar line deviation affected by many factors, such as panoramic image acquisition, stitching and the number and distribution of feature points in the process of relative orientation, etc., which needs to further study how the error propagation.

(2) Automation of matching algorithms. The constraint of spherical panoramic epipolar line to some extent reduces the search scope. On this basis, the corresponding algorithm needs to be studied to realize automatic matching of feature points.

Declaration Statements: Availability of data and material

The paper focuses on the epipolar geometry for spherical panorama, and constructs the epipolar geometry based on the coplanar equation. The mathematical equations and their properties are driven for providing a theoretical basis for the spherical panoramic epipolar line, which could be used in computer vision and the close-range digital photogrammetry. And according to the results of the experiment, the epipolar line is effectively validated and can reduce the search range and obtain more useful information.

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