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NON-LINEAR BEHAVIORS IN THE DYNAMICS OF COMPLEX SYSTEMS THROUGH A MULTIFRACTAL HYDRODYNAMIC MODEL

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Some non-linear behaviors in the dynamics of complex systems using the Scale Relativity Theory in the form of Multifractal Hydrodynamic Model are analyzed. By assimilating any complex system to a mathematical multifractal-type object, it is shown that the inversion phenomenon in various mediums (temperature inversion in atmospheric structures, inversion of tension fields at Martensite-Austenite transition etc.) can be "mimed" as multifractal tunnel effects.

Keywords: complex system, multifractal, scale relativity theory, temperature fields inversion, tension fields inversion

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1. Introduction

In a recent paper [1], fractal bistable-type behaviors as transitions in the scale space are obtained. The theoretical model is validated in the case of temperature inversion in the planetary boundary layer. Since such an approach implied the multifractal paradigm of motion, then as a general consequence, the non-differential approach should be well adapted for the field of complex systems, where any real determination is conducted at a finite scale resolution. This implies the development of a new physical theory applied to complex systems for which the motion laws, invariant to spatial and temporal coordinates transformations, are integrated with scale laws, invariant at scale transformations. Such a theory based on the above presented assumptions was first developed in the Scale Relativity Theory [2, 3] with fractal dimension 2 [2] and more recently, in the Scale Relativity Theory with an arbitrary constant fractal dimension [3, 4]. Both theories define the "fractal/multifractal physics models".

In the present paper, some non-linear behaviors in the dynamics of complex systems through a multifractal hydrodynamic model are analyzed. The present results generalize the ones obtained in the previous paper [1]. More precisely, assimilating any complex system to a mathematical multifractal-type object, it is shown that the inversion phenomenon in various mediums (temperature inversion in atmospheric structures, inversion of tension fields at Martensite-Austenite transition etc.) can be "mimed" as multifractal tunnel effects.

2. Non – Differentiability Calibrated on any Complex Systems Dynamics in the Form of the Multifractal Hydrodynamic Model

Let us consider that any complex system can be, both structurally and functionally, assimilated to a mathematical multifractal-type object. In a such conjecture, the complex system dynamics can be described through the Scale Relativity Theory. In this case, the complex system' structural units dynamics occur on continuous but non – differentiable curves – multifractal curves. These dynamics will be described through the scale covariance derivative [3,4]:

$$\frac{\hat{d}}{dt} = \partial_t + \hat{V}^l \partial_l + \frac{1}{4}(dt)^{\left[\frac{2}{f(\alpha)}\right]-1} D^{lp} \partial_l \partial_p, \tag{1}$$

where:

$$\hat{V}^l = V_D^l - V_F^l, \tag{2a}$$

$$D^{lp} = d^{lp} - i\hat{d}^{lp}, \tag{2b}$$

$$d^{lp} = \lambda_+^l \lambda_+^p - \lambda_-^l \lambda_-^p, \tag{2c}$$

$$\hat{d}^{lp} = \lambda_+^l \lambda_+^p + \lambda_-^l \lambda_-^p, \tag{2d}$$

$$\partial_t = \frac{\partial}{\partial t}, \partial_l = \frac{\partial}{\partial x^l}, \partial_l \partial_p = \frac{\partial}{\partial x^l}\frac{\partial}{\partial x^p}, i = \sqrt{-1}, l, p = 1,2,3. \tag{2e}$$

In the above, the variables and parameters that describe the complex system dynamics have the following meaning:

- x^l is the multifractal spatial coordinate;
- t is the non – multifractal time coordinate having the role of an affine parameter of the motion curves;
- dt is the scale resolution;
- $\hat{V}^l$ is the complex velocity;
- V_D^l is the differential velocity independent on the scale resolution;
- V_F^l is the non-differentiable velocity dependent on the scale resolution;
- D^{lp} is the constant tensor associated with the differentiable – non – differentiable transition;
- $\lambda_+^l(\lambda_+^p)$ are constant vectors associated with the backward differentiable – non – differentiable scale transitions;
- $\lambda_-^l(\lambda_-^p)$ are constant vectors associated with the forward differentiable – non – differentiable scale transitions;
- $f(\alpha)$ is the singularity spectrum of order α and α is the singularity index;
- $\alpha = \alpha(D_F)$ where D_F is the fractal dimensions of the motion curves.

Several definitions for fractal dimensions can be found: Kolmogorov fractal dimension, Hausdorff – Besikovitch fractal dimension etc. [5, 6]. By selecting one of these definitions and operating with it in the complex system dynamics, the following condition is imposed: the value of the fractal dimension must be constant and arbitrary for the entirety of the dynamics analysis. We note that, usually, $D_F < 2$ for correlative processes, $D_F > 2$ for non – correlative processes etc. [5, 6]. In such a conjecture, through $f(\alpha)$, it is possible to identify not only the "areas" of the complex system dynamics that are characterized by a certain fractal dimension (monofractal complex system dynamics), but also the number of "areas" for which the fractal dimensions are situated in an interval of values (multifractal complex system dynamics). Moreover, through the singularity spectrum $f(\alpha)$ it is possible to identify classes of universality in the complex system dynamics laws, even when strange or regular attractors have different aspects.

If the complex system dynamics are described through Markov – type stochastic processes (i.e., for multifractalization through Markov – type stochastic processes) [5, 6]:

$$\lambda_+^i \lambda_+^l = \lambda_-^i \lambda_-^l = 2\lambda\delta^{il} \quad i, l = 1, 2, 3, \tag{3}$$

where λ is a specific coefficient associated with the multi-fractal-non-multi-fractal scale transition and δ^{il} is Kronecker's pseudo – tensor, the scale covariant derivative (1) becomes:

$$\frac{d}{dt} = \partial_t + \hat{V}^l \partial_l - i\lambda (dt)^{\left(\frac{2}{f(\alpha)}\right)-1} \partial_l \partial^l. \tag{4}$$

Now, accepting the functionality of the scale covariance principle, which implies applying the operator (1) to the complex velocity fields (2a), in the absence of any external constraint, the motion equations of the complex system structural units dynamics take the following form:

$$\frac{d\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \partial_l \hat{V}^i + \frac{1}{4}(dt)^{\left[\frac{2}{f(\alpha)}\right]-1} D^{lk} \partial_l \partial_k \hat{V}^i = 0. \tag{5}$$

This means that the multi-fractal acceleration, $\partial_t \hat{V}^i$, the multi-fractal convection, $\hat{V}^l \partial_l \hat{V}^i$ and the multifractal dissipation $D^{lk} \partial_l \partial_k \hat{V}^i$ make their balance in every point of any multifractal curve of the complex system' structural units dynamics. Let us note that equation (5) represents a generalization of the first principle of Newton, for complex system dynamics on multifractal manifolds. Particularly, for (3) the motion equation (5) becomes:

$$\frac{\hat{d}\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \partial_l \hat{V}^i - i\lambda (dt)^{\left[\frac{2}{D_F}\right]-1} \partial_l \partial^l \hat{V}^i = 0. \tag{6}$$

Now, separating the complex system' structural units dynamics on scale resolution (differentiable and non – differentiable scale resolutions), (5) becomes:

$$\partial_t V_D^i + V_D^l \partial_l V_D^i - V_F^l \partial_l V_F^i + \frac{1}{4}(dt)^{\left[\frac{2}{f(\alpha)}\right]-1} D^{lk} \partial_l \partial_k V_D^i = 0, \tag{7a}$$

$$\partial_t V_F^i + V_F^l \partial_l V_D^i + V_D^l \partial_l V_F^i - \frac{1}{4}(dt)^{\left[\frac{2}{f(\alpha)}\right]-1} D^{lk} \partial_l \partial_k V_F^i = 0, \tag{7b}$$

while (6) takes the form:

$$\partial_t V_D^i + V_D^l \partial_l V_D^i - \left[V_F^l + \lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial^l \right] \partial_l V_F^i = 0, \tag{8a}$$

$$\partial_t V_F^i + V_D^l \partial_l V_F^i + \left[V_F^l + \lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial^l \right] \partial_l V_D^i = 0. \tag{8b}$$

For the irrotational motions of the complex system' structural units dynamics, the complex velocity fields (2a) take the form:

$$\hat{V}^i = -2i\lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial^i \, ln \, \Psi, \tag{9}$$

where Ψ is the states function. From here, for:

$$\Psi = \sqrt{\rho} e^{is}, \tag{10}$$

where $\sqrt{\rho}$ is the amplitude and s is the phase, the complex velocity fields (9) become explicitly:

$$\hat{V}^i = 2\lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial^i s - i\lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial^i \ln \rho, \tag{11}$$

which enables the definition of the real velocity fields:

$$V_D^i = 2\lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial^i s, \tag{12}$$

$$V_F^i = i\lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial^i \ln \rho. \tag{13}$$

By (12) and (13) and using the mathematical procedures from [3, 4], equations (8) reduce to the multifractal hydrodynamic equations:

$$\partial_t V_D^i + V_D^l \partial_l V_D^i = -\partial^i Q, \tag{14}$$

$$\partial_t \rho + \partial_l \left(\rho V_D^l \right) = 0, \tag{15}$$

with Q the multifractal specific potential:

$$Q = -2\lambda^2 (dt)^{\left[\frac{4}{f(\alpha)}\right]-2} \frac{\partial^l \partial_l \sqrt{\rho}}{\sqrt{\rho}} = -V_F^i V_F^i - \frac{1}{2} \lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial_l V_F^l. \tag{16}$$

Equation (14) corresponds to the multifractal specific momentum conservation law of the complex system dynamics, while equation (15) corresponds to the multifractal state density conservation law of the same dynamics. The multifractal specific potential (16) implies the multifractal specific force:

$$F^i = -\partial^i Q = -2\lambda^2 (dt)^{\left[\frac{4}{f(\alpha)}\right]-2} \partial^i \frac{\partial^l \partial_l \sqrt{\rho}}{\sqrt{\rho}}, \tag{17}$$

which is a measure of the multifractality of the motion curves of the complex system dynamics.

We note that for external constraints, for example the external scalar potential U, equation (5) takes the form:

$$\frac{d\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \partial_l \hat{V}^i + \frac{1}{4}(dt)^{\left[\frac{2}{f(\alpha)}\right]-1} D^{lk} \partial_l \partial_k \hat{V}^i = \partial^i(U) \tag{18}$$

while, for multifractalization through Markov – type stochastic processes, the above written equation becomes:

$$\frac{\hat{d}\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \partial_l \hat{V}^i - i\lambda (dt)^{\left[\frac{2}{D_F}\right]-1} \partial_l \partial^l \hat{V}^i = \partial^i(U) \tag{19}$$

These two previous equations represent a generalization of the second principle of Newton for complex system dynamics on multifractal manifolds.

Taking the above into account, the multifractal hydrodynamic equations take the form

$$\partial_t V_D^i + V_D^l \partial_l V_D^i = -\partial^i (Q + U), \tag{20}$$

$$\partial_t \rho + \partial_l \left(\rho V_D^l\right) = 0. \tag{21}$$

From these equations the following meanings result:

- Through the multifractal specific force, any complex system' structural units are constantly in contact with a multifractal medium;
- The multifractal medium can be assimilated with a multifractal fluid whose dynamics are characterized by the multifractal hydrodynamic model (see Eqs. (14) – (16) or (20), (21));
- Since the velocity field V_F^i is absent from the multifractal states density conservation laws, it induces non-manifest complex system dynamics - it facilitates the transmission of multifractal specific momentum and multifractal energy of focus;
- The "self – aspect" of the multifractal specific momentum transfer in complex system dynamics, the reversibility and existence of eigenstates are guaranteed by the conservation of multifractal energy and multifractal momentum;
- If using the tensor:

$$\hat{\tau}^{il} = 2\lambda^2 (dt)^{\left[\frac{4}{f(\alpha)}\right]-2} \rho \partial^i \partial^l \ln \rho, \tag{22}$$

the equation defining the multifractal specific "forces" deriving from a multifractal specific potential Q can be written in the form of a multifractal equilibrium equation:

$$\rho\partial^i Q = \partial_l \hat{\tau}^{il}. \tag{23}$$

The multifractal tensor $\hat{\tau}^{il}$ can now be written in the form:

$$\hat{\tau}^{il} = \eta\left(\partial_l V_F^i + \partial_i V_F^l\right), \tag{24}$$

with:

$$\eta = \lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}\rho. \tag{25}$$

This is, indeed, a multifractal linear constitutive equation for a multifractal "viscous fluid", offering the reason for an original interpretation of coefficient η as a multifractal dynamic viscosity of the multifractal fluid.

3. Complex System Dynamics Mimed as a Multifractal Tunnel Effect

Let us describe the complex system dynamics based on the following assumptions:

- Any complex system can be assimilated, both structural and functional, to a mathematical object of multifractal type;
- Complex system dynamics can be described through Scale Relativity Theory in the form of multifractal hydrodynamic equations;
- The complex system operates as a multifractal tunnel effect described through the external scalar potential (see Fig. 1)
-

$$U(x) = \begin{cases} 0 & -\infty < x < 0 \\ U_0 & 0 \le x \le a \\ 0 & a < x < +\infty \end{cases}, \tag{26}$$

where U_0 is the multifractal barrier height and a is its width (the characteristics of the complex system).

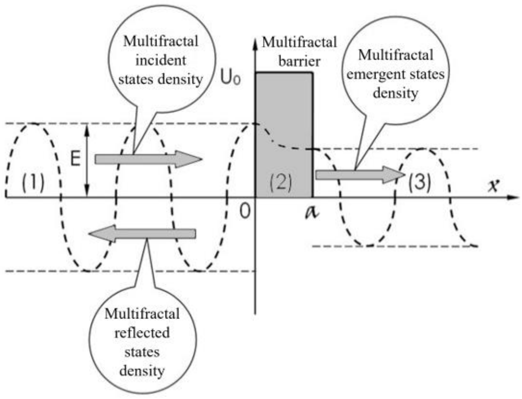


Fig. 1. External scalar potential configuration (multifractal barrier - complex system) for the tunnel effect of the multifractal type.

Then, complex system dynamics are described through the multifractal energy conservation law of the form:

$$Q + U = E, \tag{27}$$

or explicitly:

$$2\lambda^2 (dt)^{\left[\frac{4}{f(\alpha)}\right]-2} \frac{\partial^l \partial_l \sqrt{\rho}}{\sqrt{\rho}} + U = E. \tag{28}$$

In (28), ρ is the multifractal state density, U is the external scalar potential, λ is the specific coefficient associated to multifractal-nonmultifractal transition and E is the multifractal energy constant. We note that the results of (28) are given by means of the functionality of the second principle of Newton applied to (20) on multifractal manifolds.

Considering the one-dimensional case, the equation (28) through the substitution:

$$\sqrt{\rho} = \theta(x), \tag{29}$$

becomes:

$$\partial_{xx}\theta(x) + \frac{1}{2\lambda^2 (dt)^{\left[\frac{4}{f(\alpha)}\right]-2}} (E - U)\theta(x) = 0. \tag{30}$$

In the following, the above equations will be used to mimed complex system dynamics through multifractal tunnel effect (any complex system' structural unit with known energy penetrates a barrier of greater energy than the incident one).

As it is shown in Fig. 1, we distinguish three zones denoted by 1, 2, 3 as:

- 1, the multifractal incidence zone;
- 2, the multifractal barrier;

- 3, the multifractal emergence zone.

In such context, if θ_1, θ_2, and θ_3 are the multifractal functions corresponding to the above mentioned three zones, we have the following equations:

$$\frac{d^2\theta_1}{dx^2} + k^2\theta_1 = 0, -\infty < x < 0 \tag{31a}$$

$$\frac{d^2\theta_2}{dx^2} - q^2\theta_2 = 0, \quad 0 \leq x \leq \mathrm{a} \tag{31b}$$

$$\frac{d^2\theta_3}{dx^2} + k^2\theta_3 = 0, \quad \mathrm{a} < x < +\infty \tag{31c}$$

where:

$$k^2 = \frac{E}{2\lambda^2 (dt)^{(4/f(\alpha))-2}}, \quad q^2 = \frac{U_0 - E}{2\lambda^2 (dt)^{(4/f(\alpha))-2}} \tag{32}$$

Now, through integration, the following solutions of the above equations are obtained:

$$\theta_1(x) = A_1\, e^{ikx} + B_1\, e^{-ikx}, \quad -\infty < x < 0 \tag{33a}$$

$$\theta_2(x) = A_2\, e^{qx} + B_2\, e^{-qx}, \quad 0 \leq x \leq a \tag{33b}$$

$$\theta_3(x) = A_3\, e^{ikx}, \qquad \mathrm{a} < x < +\infty \tag{33c}$$

where A_1, B_1, A_2, B_2, A_3 are constants. We note the following:

- e^{ikx} corresponds to the multifractal incident states density (from $-\infty$) in the multifractal zone 1 and to the multifractal emergent states density (to $+\infty$) in the multifractal zone 3;
- e^{-ikx} corresponds to the multifractal reflected states density which exists only in the multifractal zone 1, passing from $x = 0$ to $x = -\infty$, since in the multifractal zone 3 the external scalar potential is uniform null.

Since the general expression of the multifractal current of the states density in the one-dimensional case has the form [6]:

$$J_x = i\lambda (dt)^{(2/f(\alpha))-1} \left(\theta \frac{d\overline{\theta}}{dx} - \overline{\theta} \frac{d\theta}{dx} \right) \tag{34}$$

then the following currents can be defined:

- the multifractal current density of the multifractal incident states density in the zone 1:

$$J_i = 2\lambda (dt)^{\left(\frac{2}{f(\alpha)}\right)-1} k |A_1|^2 \tag{35}$$

- the multifractal current density of the multifractal emergent states density in the zone 3:

$$J_e = 2\lambda (dt)^{(2/f(\alpha))-1} k |A_3|^2 \tag{36}$$

- the multifractal current density of the multifractal reflected states density:

$$J_r = -2\lambda (dt)^{(2/f(\alpha))-1} |B_1|^2 \tag{37}$$

The above results lead to the possibility of a univocal characterization of multifractal tunnel effect through the multifractal transparency:

$$T = \frac{J_e}{J_i} = \left|\frac{A_3}{A_1}\right|^2 \tag{38}$$

and the multifractal reflectance:

$$R = \frac{J_r}{J_i} = \left|\frac{B_1}{A_1}\right|^2 \tag{39}$$

Imposing now the coupling conditions (in $x = 0$ and $x = a$), both for the functions θ_i and their derivates, i.e.,

$$\theta_1(0) = \theta_2(0) \tag{40a}$$

$$\frac{d\theta_1}{dx}(0) = \frac{d\theta_2}{dx}(0) \tag{40b}$$

$$\theta_2(a) = \theta_3(a) \tag{40c}$$

$$\frac{d\theta_2}{dx}(a) = \frac{d\theta_3}{dx}(a) \tag{40d}$$

the multifractal algebraic system is obtained:

$$A_1 + B_1 = A_2 + B_2 \tag{41a}$$

$$ik(A_1 - B_1) = q(A_2 - B_2) \tag{41b}$$

$$e^{qa}A_2 + e^{-qa}B_2 = e^{iqa}A_3 \tag{41c}$$

$$q(e^{qa}A_2 - e^{-qa}B_2) = ike^{iqa}A_3 \tag{41d}$$

Following the same mathematical procedure from [7], the multifractal transparency takes the form:

$$T = \frac{4q^2k^2}{4q^2k^2 + (q^2 + k^2)^2 \mathrm{sh}^2(qa)} \tag{42}$$

while the multifractal reflectance becomes:

$$R = \frac{(k^2 + q^2)^2}{(q^2 - k^2)^2 + 4q^2k^2 \cdot \mathrm{cth}^2(qa)} \tag{43}$$

Moreover, in the old notations (32), it is obtained:

$$R = \frac{U_0^2 \mathrm{sh}^2 \left\{ \left[\frac{(U_0 - E)}{2\lambda^2 (dt)^{(4/f(\alpha))-2}} \right]^{1/2} a \right\}}{U_0^2 \mathrm{sh}^2 \left\{ \left[\frac{(U_0 - E)}{2\lambda^2 (dt)^{(4/f(\alpha))-2}} \right]^{1/2} a \right\} + 4E(U_0 - E)} \tag{44}$$

$$T = \frac{4E(U_0 - E)}{U_0^2 \mathrm{sh}^2 \left\{ \left[\frac{(U_0 - E)}{2\lambda^2 (dt)^{(4/f(\alpha))-2}} \right]^{1/2} a \right\} + 4E(U_0 - E)} \tag{45}$$

For graphical dependencies it is preferable to use the dimensionless coordinate system:

$$X = ka = \left[\frac{E}{2\lambda^2 (dt)^{\left(\frac{4}{f(\alpha)}\right)-2}} \right]^{\frac{1}{2}} a \tag{46a}$$

$$Y = qa = \left[\frac{(U_0 - E)}{2\lambda^2 (dt)^{(4/f(\alpha))-2}}\right]^{\frac{1}{2}} a \tag{46b}$$

Then, the multifractal transparency and multifractal reflectance become:

$$R = \frac{(X^2 + Y^2)^2}{(Y^2 - X^2)^2 + 4X^2Y^2 \mathrm{cth}^2(Y)} \tag{47}$$

$$T = \frac{4X^2Y^2}{4X^2Y^2 + (X^2 + Y^2)^2 \mathrm{sh}^2(Y)} \tag{48}$$

The 3D variations of the multifractal transparency T on the dimensionless coordinates X and Y are depicted in Figs. 2 (a, b).

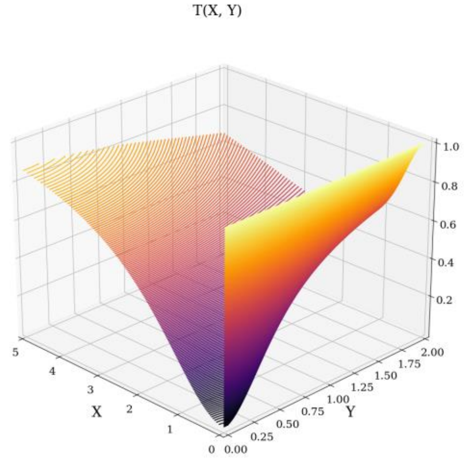
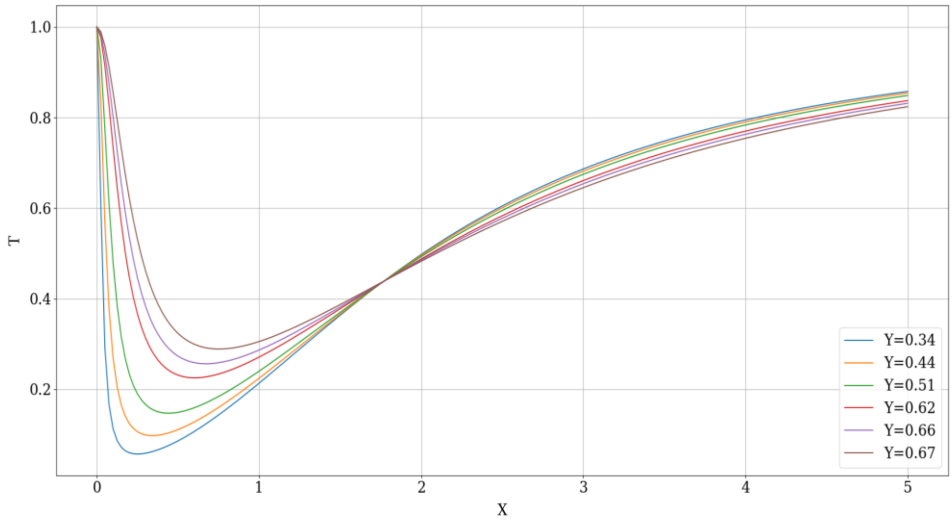
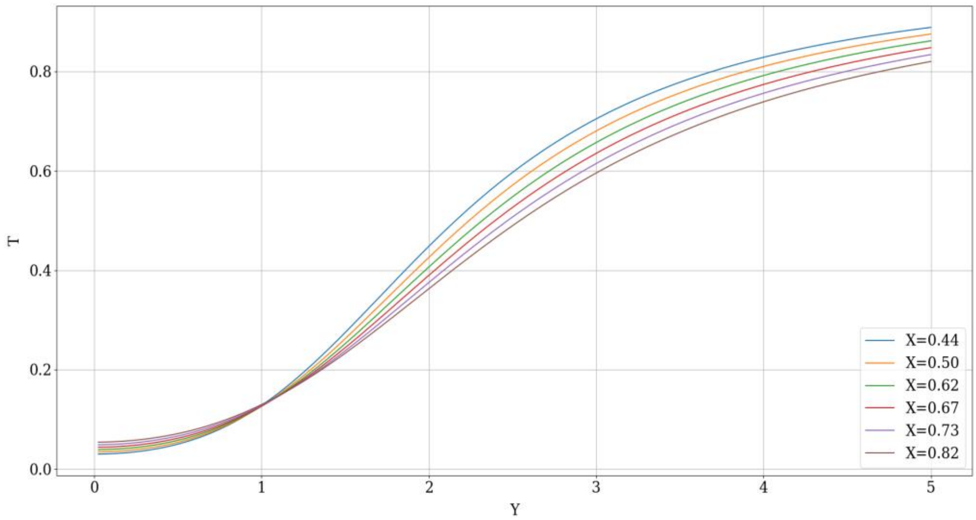


Fig. 2. The 3D variations of the multifractal transparency T of dimensionless coordinates X and Y: the dependence T = T (X, Y)

The 2D variations of the multifractal transparency T on the dimensionless coordinates X and Y are depicted in Figs. 3 (a, b).



(a)



(b)

Fig. 3a, b. The 2D variations of the multifractal transparency T of dimensionless coordinates X and Y: a) the dependence T = T (X, Y=constant); b) the dependence T = T (X=constant, Y)

In Figs. 4 the 3D variation of the multifractal reflectance R on the dimensionless coordinates X and Y are given.

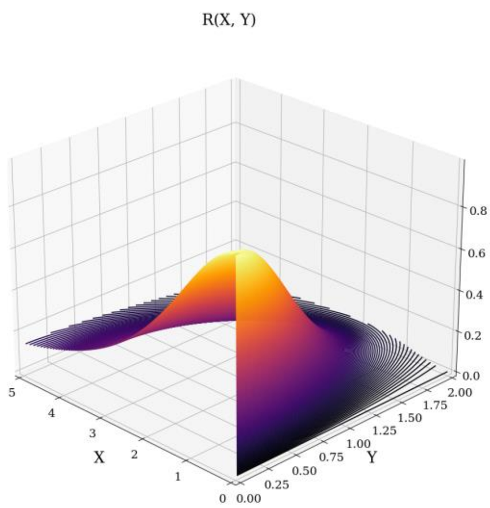
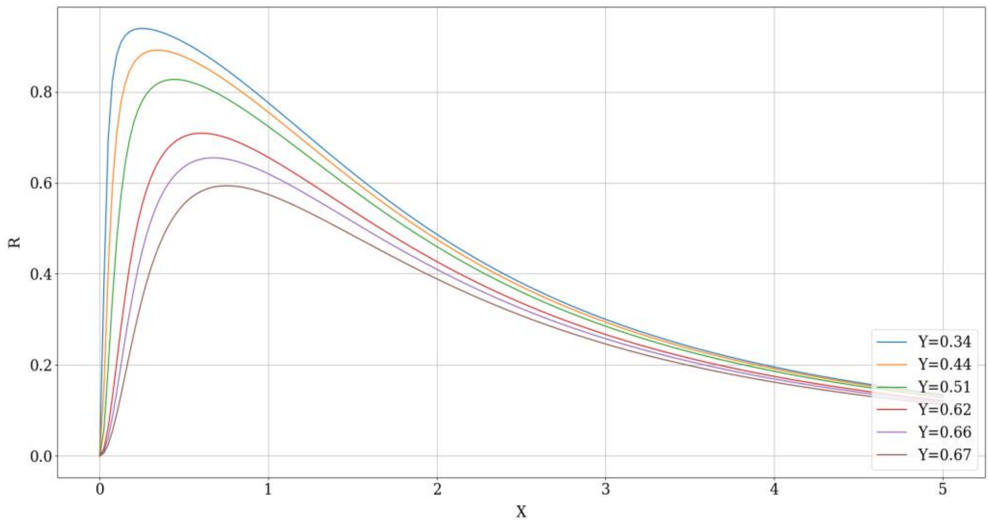
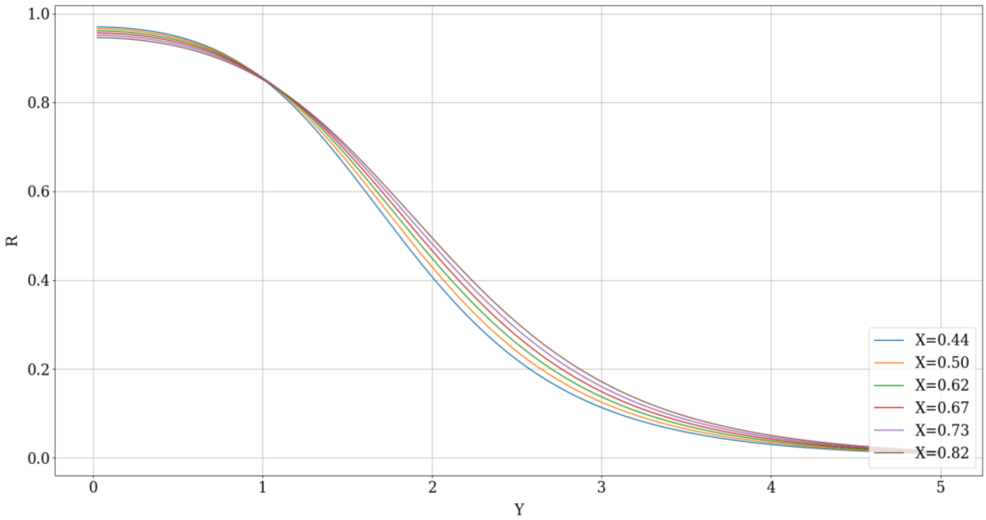


Fig. 4. The variation of the multifractal reflectance R of dimensionless coordinates X and Y: the dependence R = R (X, Y)

In Figs. 5 (a, b) the 2D variations of the multifractal reflectance R on the dimensionless coordinates X and Y are given.



(a)



(b)

Fig. 5a, b. The 2D variations of the multifractal reflectance R of dimensionless coordinates X and Y: a) the dependence R = R (X, Y=constant); b) the dependence R = R (X=constant, Y)

The dependence of T on X involves minimal and asymptotic increases of the multifractal transparency, while the dependence of T on Y involves only asymptotic increases of the multifractal transparency. The dependence of R on X involves maximal and asymptotic decreases of the multifractal reflectance, while the dependence of R on Y involves only asymptotic decreases of the multifractal reflectance.

4. Examples of various fields inversion

In such a frame, since X is proportional with a minimal dimension relevant to the complex system, namely the potential barrier width a, and T has a proportionality relation with the complex system field variable, Fig. 3a can be transformed into Fig. 6, that presents the inversion of the field variable.

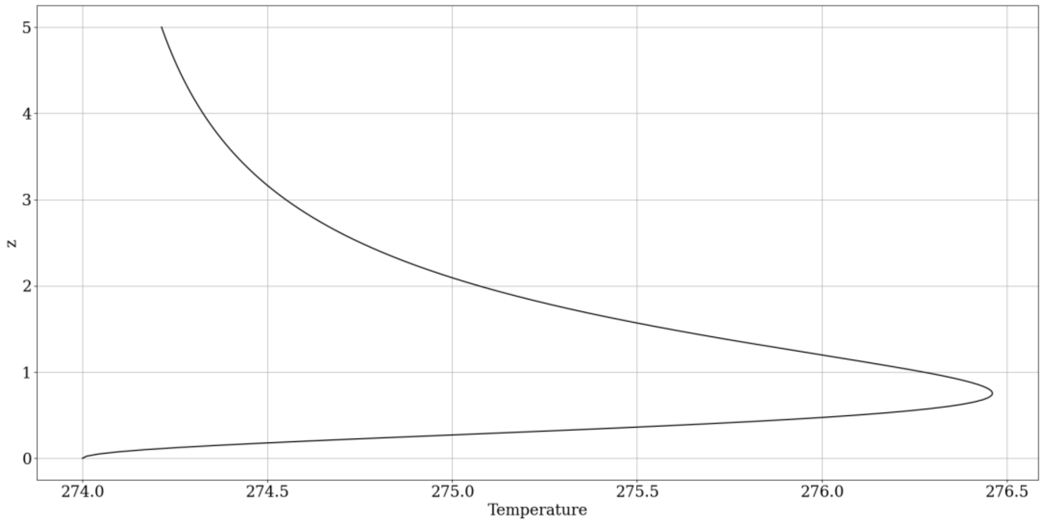


Fig. 6. The inversion of the field variable in arbitrary unity z=z(T)

In the following, let us present an experimental situation in which inversion occurs.

Since X is proportional with a minimal dimension relevant to the planetary boundary layer (PBL), namely the potential barrier width a, and T has a proportionality relation with the PBL temperature, temperature inversion occurs.

In Figs. 7 and 8 two cases of atmospheric temperature inversion are shown. For more details, see [1].

Other situations in which such inversions can occur are phase transitions in shape memory materials (more precisely, at Martensite-Austenite phase transition). Shape memory alloys (SMA in short) exhibit a series of properties which are very different compared to regular metallic materials. One of their main characteristic is the ability to change their geometric shape when subjected to an increase from low to high temperature. In certain conditions, this shape change can be reversible, such that the material can “memorize” two geometric shapes: the high temperature shape (the “warm” shape) and the low temperature shape (the “cold” shape). These transformations occur due to an effect known as shape memory effect (SME in short). Moreover, through SME, the material is able to produce mechanical work when shifting from the “cold” shape to the “warm” one [9]. For example, in the case of CuAlZn alloys, double inversions can be highlighted in the displacement – force diagrams. Such a diagram is given in Fig. 9. For more details, see [8].

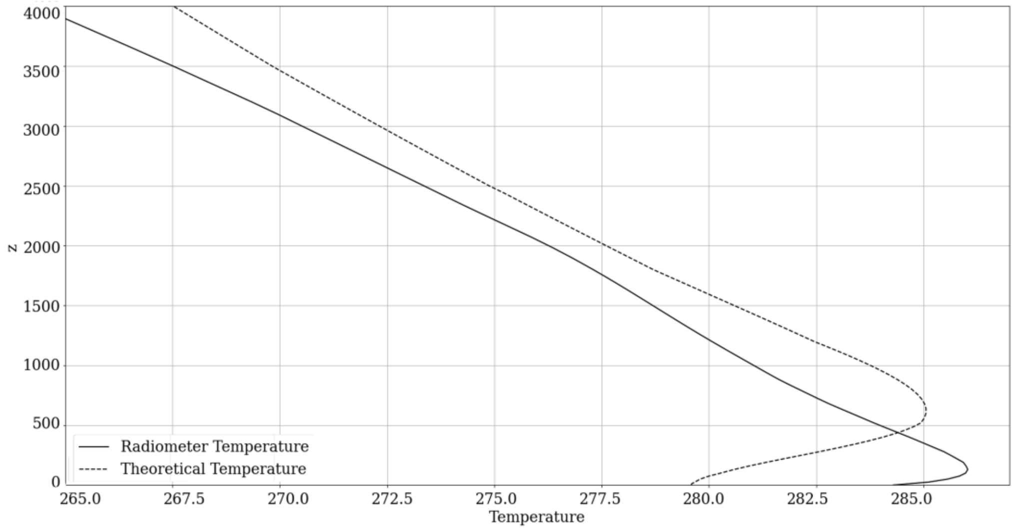


Fig. 7. Profile of atmospheric temperature; radiometer data and theoretical model data; Galati, Romania, 07/05/2022; straight line: radiometer temperature; dotted line: theoretical temperature.

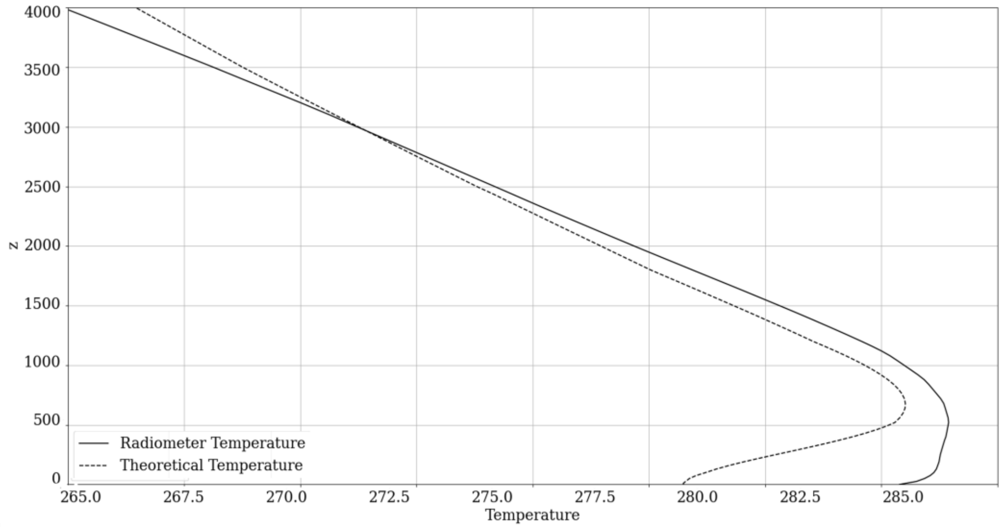


Fig. 8. Profile of atmospheric temperature; radiometer data and theoretical model data; Galati, Romania, 08/05/2022; straight line: radiometer temperature; dotted line: theoretical temperature.

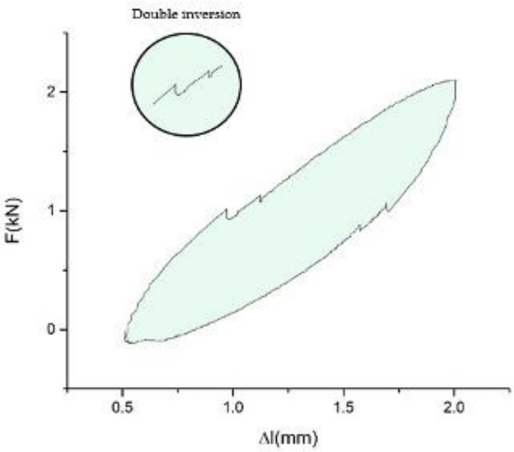


Fig. 9. Force displacement diagram for a CuAlZn alloy (68.95% Cu, 20.04% Al, 10.90% Zn). Double inversion

In the end, it is reminded that the use of multifractal models in the description on complex system dynamics can explain a large range of physical phenomena, from a microscopic to a macroscopic scale, for example the ones specified in [10-27].

5. Conclusions

The multifractal models presented here, in the mathematical description on complex system dynamics, can explain a large range of physical circumstances, from a microscopic to a macroscopic scale, more precisely from the microcosm to the macrocosm.

In the present paper, utilizing multifractal hydrodynamic model, some inversion phenomena in various reference mediums are successfully analyzed. In other words, both the temperature field inversion in atmospheric structures and inversion of mechanical tension fields at Martensite-Austenite transition can be "mimed" as author of multifractal tunnel effects. For example, in the case of CuAlZn alloys, double inversions were highlighted in the displacement – force diagrams, which fully justifies the advanced theoretical consequences.

The obtained experimental results are in good agreement with the theoretical ones developed in the models introduced in this study.

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