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THE STATIONARY MOTION OF A BOGIE ALONG A CIRCULAR CURVE

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În lucrare se propune o nouă abordare privind rezolvarea ecuaţiilor de echilibru ale mişcării permanente a unui boghiu într-o curbă circulară, abordare bazată pe definirea poziţiilor limită secantă şi coardă. Prin aceasta se rezolvă problema nedeterminării poziţiei boghiului datorită jocului în cale. Sunt calculate forţele centrifuge necompensate corespunzătoare acestor poziţii limită şi, pe această bază, se stabileşte poziţia boghiului în funcţie de forţa centrifugă necompensată care acţionează efectiv. Este analizat cazul unui boghiu cu conducere elastică a osiilor şi se pune în evidenţă influenţa razei curbei şi vitezei, precum şi a ampatamentului şi elasticităţilor conducerii osiilor asupra regimului staţionar.

This paper proposes a new approach of solving the equilibrium equations of the stationary motion of a bogie in a circular curve, based on the secant and chord limit positions. Thus, the issue of the undetermined position of the bogie due to the clearance in the track is solved. The unbalanced centrifugal forces corresponding to the two limit positions are calculated and then the bogie position is determined depending on the unbalanced centrifugal force applied on the bogie. The case of a bogie with elastic steering of the wheelsets is analyzed and the influence of the curve radius, speed, the bogie base and the elasticity of the steering of the wheelsets on the curving behaviour is pointed out.

Key words: bogie, curve, stationary motion, limit positions

1. Introduction

Curves are a critical area of any railway system, since almost every challenge in vehicle/track interaction is a greater test in curves. Derailment of trains occurs more frequently in curves or in switches and crossing, which may be considered a special case of the curved track [1 - 4]. The reprofiling interval of wheels is usually prescribed by the rapid wear that occurs in both treads and flanges in curves. Vehicles are more sensitive to lateral irregularities in track geometry because they follow the high rail in a curve more closely and thereby affect the passenger ride. On the other hand, the rail life is generally much shorter in curves; rail fastenings break off from greater movement of the rails, sleeper skew and, thereby, narrow the gauge etc. [5-7].

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By studying the curving of the vehicles the best technical improvements of the construction [8-11] can be found and the traction [12, 13] and braking [14] can be optimized.

Solving the issues of the curving simulation is a difficult task, due to the nonlinearities of the wheelset/rails interface: the wheelset/rails clearance, the geometry of the contact between the wheels and rails and the friction coefficient [1, 15, 16].

This work proposes a new approach of the equilibrium equations of the stationary motion of a bogie along a circular curve based on the concept of the limit positions. These limit positions are defined as the geometric contact between the rear wheelset of the bogie and the inner rail – *the secant limit position*, and the outer rail – *the chord limit position*; there is no leading force acting on the rear wheelset. Starting from the values of the unbalanced centrifugal force corresponding to the two limit positions, the bogie position may be found (secant, free or chord position), depending on the effective unbalanced centrifugal force acting on bogie. Here, this method is applied to point out the basic features of the stationary motion of a bogie along a circular curve.

2. The equilibrium equations

The mechanical model of a two-axle bogie in stationary motion with constant velocity V along a circular curve of radius R is presented in Fig. 1. The wheelsets are linked to a body frame of the bogie by means of linear springs of stiffness k_x, k_y in the longitudinal and lateral direction, respectively. The bogie base is $2a$, the transversal base of the suspension $2b$ and the distance between the rolling circles of the wheels is $2e$.

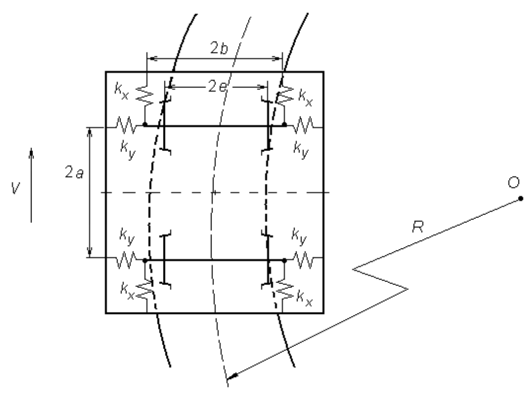

Fig. 1. Mechanical model of the bogie in curve.

The position of each wheelset is determined by the displacements y_1 and y_2 respectively of the wheelset centre in respect to its local reference moving frame and the attack angles α_1 and α_2 with respect to the radial position (Fig. 2). Also, the position of the body frame of the bogie is given by the lateral displacement y_b of the body frame centre and the rotation α_b.

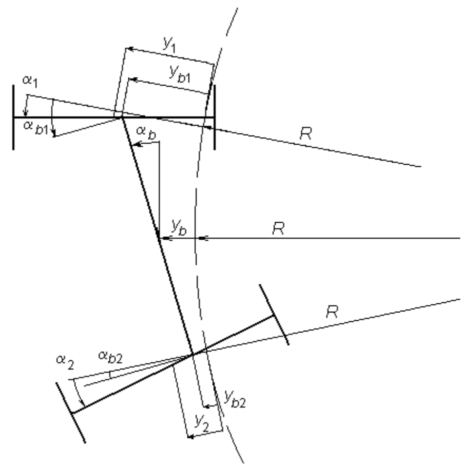

Fig. 2. Coordinates of the bogie.

It is supposed that the attacking wheel of the leading wheelset is in contact with the flange of the high rail. The position of the rear wheelset may be in contact with the high rail or the low rail or even between both rails, depending on the equilibrium position of the bogie. The three cases are known as the so-called *the chord position*, *the secant position* and *the free position*, respectively. The effect of the wheel flange is replaced by a guidance roller that introduces the leading forces P_1 and P_2, corresponding to the two wheelsets. This hypothesis has been adopted by Heummann [17] and recommended by Sebeşan [1], and others.

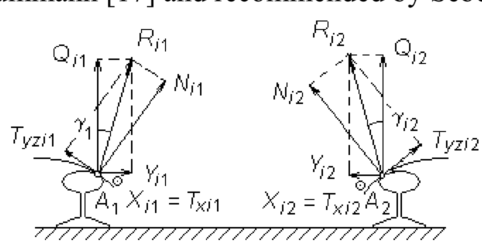

Fig. 3. Contact forces.

On the rolling surfaces the contact forces act (Fig. 3): the normal force N_{ij} and the friction force T_{ij}, where the index i = 1, 2 stands for the leading or rear wheelset and the index j = 1, 2 stands for the outer or inner rail. The friction force has the components T_{xij} and T_{yzij}. The components of the resultants of the contact forces are

$$X_{ij} = T_{xij} \,; \tag{1}$$

$$Y_{ij} = \mp N_{ij} \sin\gamma_{ij} + T_{yzij} \cos\gamma_{ij} ; \tag{2}$$

$$Q_{ij} = N_{ij} \cos\gamma_{ij} \pm T_{yzij} \sin\gamma_{ij} ; \tag{3}$$

where γ_{ij} stands for the contact angle.

The friction forces may be calculated using the nonlinear formula provided by Chartet [18]

$$T_{xij} = -\frac{\kappa \nu_{xij} N_{ij}}{\sqrt{1+(\kappa \nu_{ij}/\mu)^2}} ; \; T_{yzij} = -\frac{\kappa \nu_{yzij} N_{ij}}{\sqrt{1+(\kappa \nu_{ij}/\mu)^2}} , \tag{4}$$

where κ stands for the creepage coefficient, μ is the coefficient of adherence, ν_{ij} is the creepage of the wheel '*ij*' and ν_{xij} and ν_{yzij} are its components

$$\nu_{ij} = \sqrt{\nu_{xij}^2 + \nu_{yzij}^2} \; . \tag{5}$$

The creepage components result from the kinematics of the wheelset

$$\nu_{xij} = \pm\left[\frac{e}{R} - \frac{\Delta r_{ij}(y_i)}{r}\right]; \; \nu_{yzij} = -\frac{\alpha_i}{\cos\gamma_{ij}(y_i)} , \tag{6}$$

where

$$\Delta r_{i1}(y_i) = r_{i1}(y_i) - r \; ; \; \Delta r_{i2}(y_i) = r - r_{i2}(y_i) \, , \tag{7}$$

with $r_{ij}(y_i)$ - the radius of the rolling circles depending on the wheelset displacement and r - the wheel radius when the wheelset takes the central place between rails. To calculate $\Delta r_{ij}(y_i)$, the contact curve method may be applied [1]. In fact, this method allows to solve the issue of the contact between the wheels and rails, including the contact angles $\gamma_{ij}(y_i)$.

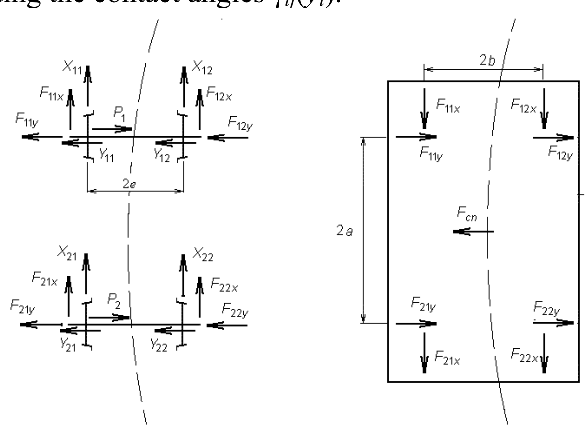

Fig. 4. Forces acting on the wheelsets and the frame of the bogie.

Fig. 4 shows the forces acting on the wheelsets and the bogie frame for the chord position. It distinguishes the elastic forces between the wheelsets and the bogie frame $F_{ijx,y}$, the contact forces X_{ij} and Y_{ij}, the unbalanced centrifugal force

F_{cn} and the leading forces P_1 and P_2. It has to be noticed that the leading force P_2 satisfies the following conditions

- for the chord position $P_2 > 0$;
- for the free position $P_2 = 0$;
- for the secant position $P_2 < 0$.

The equations of equilibrium for the chord position of the bogie may be written as follows

$$\sum_{j=1}^{2}(Y_{1j} + F_{1j}) - P_1 = 0\,; \tag{8}$$

$$\sum_{j=1}^{2}(Y_{2j} + F_{2j}) - P_2 = 0\,; \tag{9}$$

$$\sum_{j=1}^{2}(-1)^{j+1}(eX_{1j} - bF_{1j}) = 0\,; \tag{10}$$

$$\sum_{j=1}^{2}(-1)^{j+1}(eX_{2j} - bF_{2j}) = 0\,; \tag{11}$$

$$F_{cn} - \sum_{i=1}^{2}\sum_{j=1}^{2} F_{ijy} = 0\,; \tag{12}$$

$$\sum_{i=1}^{2}\sum_{j=1}^{2}\left[(-1)^{j+1} bF_{ijx} + (-1)^{i+1} aF_{ijy}\right] = 0\,. \tag{13}$$

Likewise, the above equations represent the equilibrium condition for the secant position, but, in this case, P_2 will have negative sign. If the bogie takes the free position, then Eq. (9) writes

$$\sum_{j=1}^{2}(Y_{2j} + F_{2j}) = 0\,. \tag{14}$$

The elastic forces depend on the stiffness k_x and k_y, and the relative position between bogie frame and the wheelsets

$$F_{i1x} = -F_{i2x} = bk_x(\alpha_{bi} - \alpha_i)\,,\ i = 1, 2; \tag{15}$$

$$F_{i1y} = F_{i2y} = k_y(y_{bi} - y_i)\ \ i = 1, 2, \tag{16}$$

where

$$\alpha_{bi} = \alpha_b \pm \frac{a}{R};\ y_{bi} = y_b + \frac{a^2}{2R} \pm a\alpha_b\,. \tag{17}$$

The unbalanced centrifugal force depends on the curve radius R, the super-elevation of the track h and the bogie velocity V

$$F_{cn} = Mg\left(\frac{V^2}{gR} - \frac{h}{2e}\right), \tag{18}$$

where M is the 1/2 vehicle mass and g stands for the acceleration of gravity.

When the bogie velocity equals

$$V = V_0 = \sqrt{\frac{ghR}{2e}} \ , \qquad (19)$$

the unbalanced centrifugal force is zero and the bogie velocity is *the equilibrium velocity*.

When the velocity is lower than the equilibrium velocity, the unbalanced centrifugal force acts into the inner rail and its magnitude may be calculated using

$$F_{cn} = Mg\frac{E}{2e}, \qquad (20)$$

where E is the so-called *excess of super-elevation*

$$E = h - 2\frac{eV^2}{gR} > 0 \text{ with } V < V_0. \qquad (21)$$

When the velocity is higher than the equilibrium velocity, the unbalanced centrifugal force acts into the outer rail and Eq. (18) reads

$$F_{cn} = Mg\frac{I}{2e}, \qquad (22)$$

where I stands for the so-called *insufficiency of super-elevation*

$$I = 2\frac{eV^2}{gR} - h > 0 \text{ with } V > V_0. \qquad (23)$$

The unbalanced centrifugal force is the cause of a load transfer but this may be usually neglected; hence, corroborating the fact that the contact angle on the rolling surface is a small angle, Eq. (3) becomes

$$Q_{ij} = N_{ij}\cos\gamma_{ij} \pm T_{yzij}\sin\gamma_{ij} \cong N_{ij} \cong Q_0, \qquad (24)$$

where Q_0 is the static load on wheel.

By inserting the forces expressions in the equilibrium equations, a set of nonlinear equations is obtained:

$$\mathbf{Aq} = \mathbf{B}, \qquad (25)$$

where **q** is the column vector of the bogie displacements and leading force(s), **A** is a matrix depending on the wheelsets displacements and **B** is the column vector of the free terms, including the unbalanced centrifugal force.

Considering the wheelset/track clearance 2σ, we have the following cases:

- the bogie takes place in the secant position

$$y_1 = \sigma, \ y_2 = -\sigma \ \mathbf{q} = [\alpha_1 \quad \alpha_2 \quad y_b \quad \alpha_b \quad P_1 \quad P_2]^{\mathrm{T}}; \qquad (26)$$

- the bogie occupies the free position

$$y_1 = \sigma, \ P_2 = 0, \ \mathbf{q} = [\alpha_1 \quad y_2 \quad \alpha_2 \quad y_b \quad \alpha_b \quad P_1]^{\mathrm{T}}, \qquad (27)$$

with $-\sigma < y_2 < \sigma$;

- the bogie is in the chord position

$$y_1 = \sigma,\ y_2 = \sigma,\ \mathbf{q} = [\alpha_1 \quad \alpha_2 \quad y_b \quad \alpha_b \quad P_1 \quad P_2]^{\mathrm{T}}. \qquad (28)$$

The structure of the matrix **A** and the column vector **B** changes, according to the three cases.

The nonlinear equation (25) may be solved following an iterative method.

It has to be noticed that the issue of the bogie curving is nonlinear, due to the clearance of the wheelset that modifies the shape of the equilibrium equations. Moreover, for every case, the equilibrium equations are nonlinear due to wheels/rails contact, including the geometry and the friction coefficient.

A value of the unbalanced centrifugal force being given, the bogie position cannot be directly determined, due to the nonlinearity of the wheelset/track clearance. To overcome this shortcoming, the following procedure should be taken into consideration. There are two limit positions, named *the limit secant position* when the rear wheelset and the inner rail are in contact and *the limit chord position* when the wheelset and the outer rail are in contact; the leading force P_2 is zero for both. The bogie touches the limit positions for two particular values of the unbalanced centrifugal force, F_{cns} and F_{cnc}.

These values may be calculated starting from the equations (8) and (10-14) corresponding to the free position of the bogie rewritten in the matrix form (25), where the column vector **q** is as follows

- for the limit secant position

$$y_1 = \sigma,\ y_2 = -\sigma, P_2 = 0,\ \mathbf{q} = [\alpha_1 \quad \alpha_2 \quad y_b \quad \alpha_b \quad P_1 \quad F_{cns}]^{\mathrm{T}}; \qquad (29)$$

- for the chord limit position

$$y_1 = \sigma,\ y_2 = \sigma, P_2 = 0,\ \mathbf{q} = [\alpha_1 \quad \alpha_2 \quad y_b \quad \alpha_b \quad P_1 \quad F_{cnc}]^{\mathrm{T}}. \qquad (30)$$

When the two limit values of the unbalanced centrifugal force are calculated, the bogie position is determined by the correlations below:

- the bogie is in the secant position for $F_{cn} < F_{cns}$
- the bogie is in the free position for $F_{cns} < F_{cn} < F_{cnc}$
- the bogie is in the chord position for $F_{cnc} < F_{cn}$.

For a particular F_{cn} force, one may choose the adequate equilibrium equation according to the correlation above. Then, solving this equation iteratively, the position of each part of the bogie and the leading force(s) are obtained.

3. Numerical application

Next, the stationary motion of a particular two-axle bogie along a curved circular track is numerically analysed using the equilibrium equations and the method presented in previous section. The physical parameters of the bogie taken into account are as follows: M = 20000 kg, k_x = 40 MN/m, k_y = 10 MN/m, 2a = 2.56 m, 2r = 0.89 m, Q_o = 49 kN, μ = 0.36, κ = 195. The CFR S78 wheel profile is considered for the bogie wheelsets. The track has the UIC 60 rails and the super-elevation h = 150 mm for any radius of curvature. Also, the admissible value of the *insufficiency of super-elevation* is I_{adm} = 70 mm.

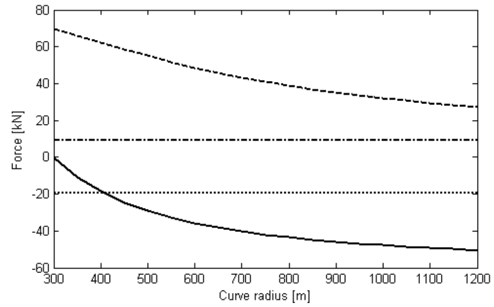

Fig. 5. Unbalanced centrifugal forces at the limit positions and extreme values: ─, F_{cns} ; --- F_{cnc}; ……, F_{cnmin} ; - · - · -, F_{cnmax};

Fig. 5 presents the unbalanced centrifugal forces at the limit positions versus the curve radius. Also, the min/max values of the unbalanced centrifugal force acting on the bogie are displayed in order to determine the curving position of the bogie. The min/max values of the unbalanced centrifugal force (F_{cnmin}/ F_{cnmax}) are obtained taking $E = h$ (V = 0) and $I = I_{\text{adm}}$ ($V = V_{\text{adm}}$) in Eqs. (20) and (23), respectively. One observes that the bogie takes the free position for all speeds when the curve radius is higher than 390 m. For a low speed, the bogie occupies the secant position for the curves of radius lower than 390 m.

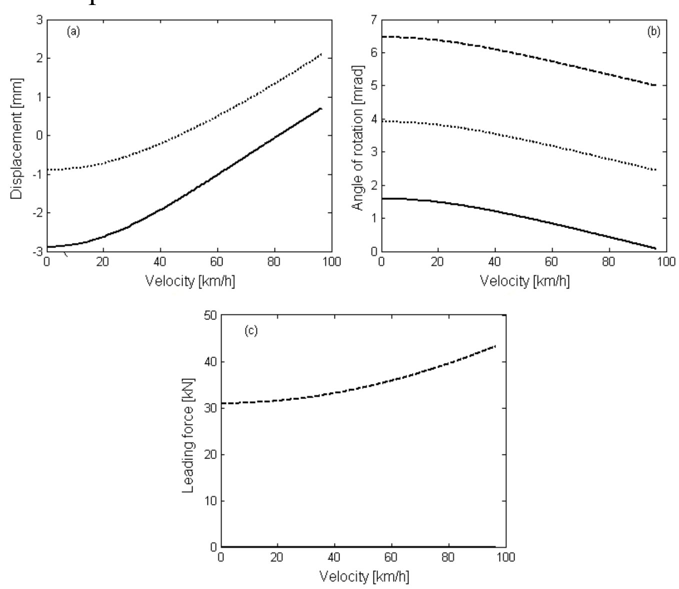

Fig. 6. Influence of velocity at R = 500 m: (a) ─, rear wheelset displacement; ……, displacement of the bogie frame; (b) ---, attack angle of the leading wheelset; ─, attack angle of the rear wheelset; ……, rotation of the bogie frame; (c) ---, P_1; ─, P_2.

Fig. 6 shows the influence of the bogie velocity upon the curving behaviour when the bogie runs along a 500 m radius curve. The maximum velocity corresponding to the insufficiency of super-elevation of 70 mm is 96.5 km/h. The bogie moves in the free position for any value of velocity, according to the preceding results. When the velocity increases, the rear wheelset and the bogie frame move to the outer rail due to the unbalanced centrifugal force that increases as the velocity increases (fig. 6 (a)). The attack angles of the wheelsets and the rotation angle of the bogie frame decrease (fig. 6 (b)). In fact, the bogie exhibits the tendency to take place in the chord position but it does not touch this position. The leading force increases as long as the velocity increases and its increasing rate becomes higher when the velocity is higher than 30-40 km/h.

Fig. 7 diplays the influence of the curve radius upon the stationary motion of the bogie when the insufficiency of super-elevation takes the admisible value. It is noticed that the attack angles of both wheelsets and the leading force increases at small radiuses.

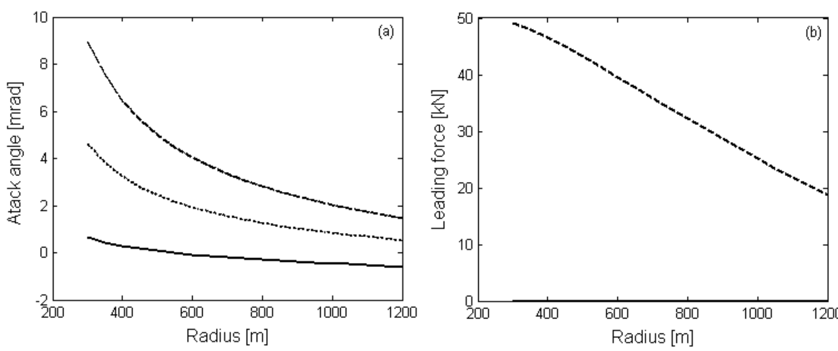

Fig. 7. Influence of curve radius ($I = I_{adm}$): (a) ---, attack angle of the leading wheelset; —, attact angle of the rear wheelset; ⋯⋯, rotation of the bogie frame; (c) ---, P_1; —, P_2.

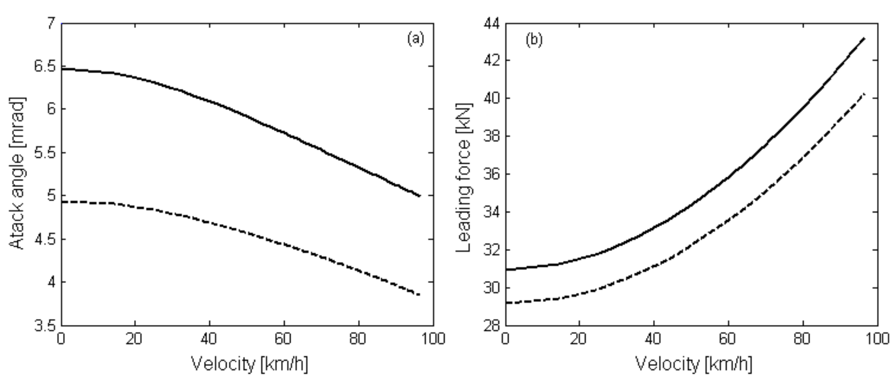

Fig. 8. Influence of the bogie base (R = 500 m): (a) attack angle of the leading wheelset; (b) leading force; —, $2a$ = 2,56 m; ⋯⋯, $2a$ = 2 m.

The attack angle of the leading wheelset and the leading force calculated for two bogie base values, 2.56 m and 2.00 m respectively, are presented in Figure 8. The numerical simulation takes into consideration a radius of 500 m and velocities from 0 to 96.5 km/h. When the bogie base is smaller, the attack angle and the leading force of the leading wheelset are lower. It seems that the tendency of the two parameters are poorly influenced by the bogie velocity.

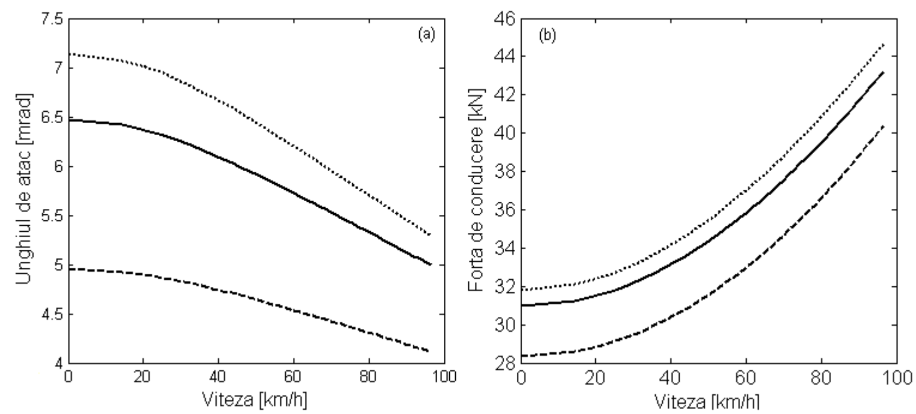

Fig. 9. Influence of longitudinal stiffness (R = 500 m): (a) attack angle of the leading wheelset; (b) leading force; ······, bogie with rigid wheelsets; —, k_x = 40 MN/m, k_y = 6 MN/m; ---, k_x = 6 MN/m, k_y = 6 MN/m.

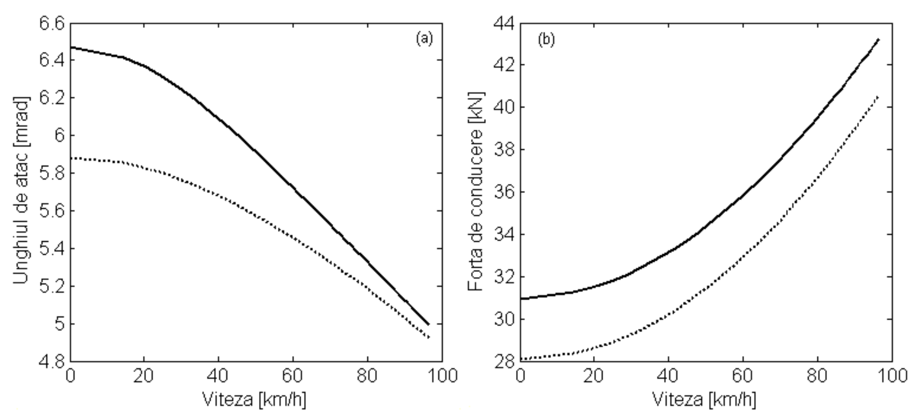

Fig. 10. Influence of transversale stiffness (R = 500 m): (a) attack angle of the leading wheelset; (b) leading force; —, k_x = 40 MN/m, k_y = 6 MN/m; ······, k_x = 40 MN/m, k_y = 1 MN/m.

Fig. 9 shows the influence of the longitudinal stiffness, upon considering the circulation in a 500 m radius curve. The attack angle of the leading wheelset and the leading force are presented for the following wheelsets steering: bogie with rigid wheelsets, bogie with elastic steering – the reference option k_x = 40 MN/m, k_y = 6 MN/m and bogie with a very elastic longitudinal steering k_x = 6 MN/m, k_y = 6 MN/m. It is evident that the larger attack angles and leading forces derive for the bogie with rigid wheelset. The lower the longitudinal stiffness of the bogie steering system, the lower the attack angle and the leading force.

Similar results show in case of the influence of the lateral stiffness of the wheelsets steering, as is figure 10. The numerical simulation refers to the reference option and to the instance when the transversal leading is more elastic (k_x = 40 MN/m, k_y = 1 MN/m). The greater the elasticity of the bogie transversal leading, the smaller the attack angle and the leading force.

6. Conclusions

The study of the stationary motion of a bogie along a circular curve represents an essential theoretical issue, with real applications in terms of safety against derailment, wear of treads, stability of track, etc.

In order to solve the issue of the motion along a curve, it is required to take into account the nonlinearities, due to the wheelset/rails interface that changes the equilibrium equations in dependence with the wheelset/rails clearance, the geometry of the contact between the wheels and rails and the friction coefficient depending on the creepage. This work proposes a new approach of the equilibrium equations of the stationary motion of a bogie along a circular curve based on the concept of the limit positions. To this end, the secant limit position and the chord limit position are defined and the corresponding unbalanced centrifugal forces are calculated. Starting from these, the position of the bogie may be found, depending on the effective unbalanced centrifugal force acting on bogie.

The method suggested here has been used to examine the main issues related to the two-axle bogie in stationary motion with constant velocity V along a circular curve of radius R. The higher the speed, the closer the bogie tends to get to the chord position, and the higher the leading force and the lower attack angle of the leading wheelset. The bogie performance of moving along the curve improves while crossing curves of long radii, by lowering the wheelbase or raising the elasticity of the wheelsets steering.

It is a must to highlight the fact that the setting of parameters of a bogie is a compromise between matching the contradictory requirements imposed by the curving and the stability of the rolling movement.

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