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BOUNDS INVOLVING GAUSS'S HYPERGEOMETRIC FUNCTIONS VIA (p, h)-CONVEXITY

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In this paper, we consider the class of (p, h)-convex functions. We establish some new estimates for trapezoidal and midpoint type inequalities via differentiable (p, h)-convex functions. We also discuss some special cases which can be deduced from our main results.

Keywords: Convex functions; p-convex functions; Hermite-Hadamard inequalities

MSC 2000: 26D15,26A51.

1. Introduction and Preliminaries

Recently much attention has been given to the theory of convexity due to its great importance in other fields of pure and applied sciences. Another reason which makes theory of convexity more fascinating is its strong relationship with the theory of inequalities. In past few years a number of new generalizations of the classical convexity have been proposed, see [3, 4, 12]. Resultantly many classical inequalities which were obtained via convex functions have also been generalized for new classes of convex functions, see [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. The motivation of this article is to establish some new estimates for trapezoidal and mid-point inequalities via (p, h)-convex functions. We also discuss special cases which can be deduced from our main results. We expect that the ideas and techniques used in the paper may motivate further research in the field.
Now we recall some previously known concepts.

Definition 1.1 ([12]). *An interval $I \subset \mathbb{R}$ is said to be a p-convex set if*

$$M_p(x, y; t) = [tx^p + (1-t)y^p]^{\frac{1}{p}} \in I$$

for all $x, y \in I, t \in [0, 1]$, where $p = 2k+1$ or $p = \frac{n}{m}, n = 2r+1, m = 2t+1$ and $k, r, t \in \mathbb{N}$.

Definition 1.2 ([12]). *Let I be a p-convex set. A function $f : I \to \mathbb{R}$ is said to be p-convex function or belongs to the class $PC(I)$, if*

$$f(M_p(x, y; t)) \leq tf(x) + (1-t)f(y), \forall x, y \in I, t \in [0, 1].$$

It is obvious that for $p = 1$ Definition 1.2 reduces to the definition for classical convex functions.
Note that for $p = -1$, we have the definition of harmonically convex functions.

Definition 1.3 ([4]). *A function $f : I \subset \mathbb{R} \setminus \{0\} \to \mathbb{R}$ is said to be harmonically convex function, if*

$$f\left(\frac{xy}{(1-t)x + ty}\right) \leq tf(x) + (1-t)f(y), \forall x, y \in I, t \in [0, 1].$$

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Also note that for $t = \frac{1}{2}$ in Definition 1.2, we have Jensen p-convex functions or mid p-convex functions.

$$f(M_p(x, y; 1/2)) \leq \frac{f(x) + f(y)}{2}, \forall x, y \in I, t \in [0, 1].$$

Recently Fang et al. [3] introduced a new class of convex functions, which is called as (p, h)-convex functions.

Definition 1.4 ([3]). *Let $h : [0, 1] \to \mathbb{R}$ be a non-negative function. A function $f : I \to \mathbb{R}$ is said to be (p, h)-convex function, if*

$$f(M_p(x, y; t)) \leq h(t)f(x) + h(1 - t)f(y), \forall x, y \in I, t \in [0, 1].$$

Note that for $h(t) = t$, we have the class of p-convex functions.
When $h(t) = t^s$, then we have the class of Breckner type of (p, s)-convex functions, which is defined as:

Definition 1.5. *A function $f : I \to \mathbb{R}$ is said to be Breckner type of (p, s)-convex function, if*

$$f(M_p(x, y; t)) \leq t^s f(x) + (1 - t)^s f(y), \forall x, y \in I, t \in [0, 1], s(0, 1].$$

When $h(t) = t^{-s}$, then we have the class of Godunova-Levin type of (p, s) functions, which is defined as:

Definition 1.6. *A function $f : I \to \mathbb{R}$ is said to be Godunova-Levin type of (p, s) function, if*

$$f(M_p(x, y; t)) \leq t^{-s} f(x) + (1 - t)^{-s} f(y), \forall x, y \in I, t \in (0, 1), s[0, 1].$$

When $h(t) = 1$, then we have the class of (p, P)-functions, which is defined as:

Definition 1.7. *A function $f : I \to \mathbb{R}$ is said to be (p, P) function, if*

$$f(M_p(x, y; t)) \leq f(x) + f(y), \forall x, y \in I, t \in (0, 1).$$

The following results play an important role in establishing our main results.

Lemma 1.1 ([11]). *Let $f : I = [a, b] \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function on I^0 (the interior of I) with $a < b$. If $f' \in L[a, b]$, then, we have*

$$\begin{aligned} R_f(a, b; p) =& \frac{f(a) + f(b)}{2} - \frac{p}{b^p - a^p} \int_a^b \frac{f(x)}{x^{1-p}} \mathrm{d}x \\ =& \frac{b^p - a^p}{2p} \int_0^1 [ta^p + (1 - t)b^p]^{1-\frac{1}{p}} (1 - 2t) f'([ta^p + (1 - t)b^p]^{\frac{1}{p}}) \mathrm{d}t. \end{aligned}$$

Lemma 1.2 ([11]). *Let $f : I = [a, b] \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function on I^0 with $a < b$. If $f' \in L[a, b]$, then, we have*

$$\begin{aligned} L_f(a, b; p) =& \frac{p}{b^p - a^p} \int_a^b \frac{f(x)}{x^{1-p}} \mathrm{d}x - f\Big(\Big[\frac{a^p + b^p}{2}\Big]^{\frac{1}{p}}\Big) \\ =& \frac{b^p - a^p}{p} \int_0^1 [ta^p + (1 - t)b^p]^{1-\frac{1}{p}} \vartheta(t) f'([ta^p + (1 - t)b^p]^{\frac{1}{p}}) \mathrm{d}t, \end{aligned}$$

where

$$\vartheta(t) = \begin{cases} t, & [0, \frac{1}{2}), \\ t - 1, & [\frac{1}{2}, 1]. \end{cases}$$

For the reader's convenience we recall here the definitions of the Gamma function

$$\Gamma(x)=\int_0^\infty e^{-x}t^{x-1}\mathrm{d}t,\quad x>0,$$

and the Beta function

$$\mathrm{B}(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}\mathrm{d}t=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)},\quad x>0,y>0.$$

The integral form of the hypergeometric function is

$${}_2F_1(x,y;c;z)=\frac{1}{\mathrm{B}(y,c-y)}\int_0^1 t^{y-1}(1-t)^{c-y-1}(1-zt)^{-x}\mathrm{d}t$$

for $|z|<1, c>y>0$.

2. **Main Results**

In this section, we derive our main results.

Theorem 2.1. *Let $f:I=[a,b]\subset\mathbb{R}\to\mathbb{R}$ be a differentiable function on I^0 with $0<a<b$ and $f'\in L[a,b]$. If $|f'|$ is (p,h)-convex function, then, we have*

$$|R_f(a,b;p)|\le\frac{b^p-a^p}{2p}\left(\theta_1\left|f'(a)\right|+\theta_2\left|f'(b)\right|\right),$$

where

$$\theta_1=\int_0^1\frac{|1-2t|\,h(t)}{[ta^p+(1-t)b^p]^{\frac{1}{p}-1}}\mathrm{d}t, \tag{2.1}$$

and

$$\theta_2=\int_0^1\frac{|1-2t|\,h(1-t)}{[ta^p+(1-t)b^p]^{\frac{1}{p}-1}}\mathrm{d}t. \tag{2.2}$$

Proof. Using Lemma 1.1 and the fact that $|f'|$ is (p,h)-convex function, we have

$$\begin{aligned}
&|R_f(a,b;p)|\\
&=\left|\frac{b^p-a^p}{2p}\int_0^1[ta^p+(1-t)b^p]^{1-\frac{1}{p}}(1-2t)f'([ta^p+(1-t)b^p]^{\frac{1}{p}})\mathrm{d}t\right|\\
&\le\frac{b^p-a^p}{2p}\int_0^1[ta^p+(1-t)b^p]^{1-\frac{1}{p}}|1-2t||f'([ta^p+(1-t)b^p]^{\frac{1}{p}})|\mathrm{d}t\\
&\le\frac{b^p-a^p}{2p}\int_0^1[ta^p+(1-t)b^p]^{1-\frac{1}{p}}|1-2t|[h(t)|f'(a)|+h(1-t)|f'(b)|]\mathrm{d}t\\
&=\frac{b^p-a^p}{2p}\left(\theta_1\left|f'(a)\right|+\theta_2\left|f'(b)\right|\right).
\end{aligned}$$

This completes the proof. □

Now, we discuss some special cases for Theorem 2.1.

I. If $h(t)=t$ in Theorem 2.1, then, we have Theorem 3.1 [11].
II. If $h(t)=t^s$ in Theorem 2.1, the, we have a new result for Breckner type of (p,s)-convex functions.

Corollary 2.1. *Let $f : I = [a, b] \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function on I^0 with $0 < a < b$ and $f' \in L[a, b]$. If $|f'|$ is Breckner type of (p, s)-convex function, then, we have*

$$|R_f(a, b; p)| \leq \frac{b^p - a^p}{2p} \left(\theta_3 \left|f'(a)\right| + \theta_4 \left|f'(b)\right|\right),$$

where

$$\begin{aligned}\theta_3 &= \int_0^1 \frac{|1-2t|\, t^s}{[ta^p + (1-t)b^p]^{\frac{1}{p}-1}} \mathrm{d}t \\ &= b^{p-1}\left[\frac{2}{s+2}\, {}_2F_1\left(\frac{1}{p} - 1, s+2; s+3; 1 - \frac{a^p}{b^p}\right)\right. \\ &\quad - \frac{1}{s+1}\, {}_2F_1\left(\frac{1}{p} - 1, s+1; s+2; 1 - \frac{a^p}{b^p}\right) \\ &\quad \left. + \frac{2^{-s}}{(s+1)(s+2)}\, {}_2F_1\left(\frac{1}{p} - 1, s+1; s+3; \frac{1}{2}\left(1 - \frac{a^p}{b^p}\right)\right)\right], \end{aligned} \tag{2.3}$$

and

$$\begin{aligned}\theta_4 &= \int_0^1 \frac{|1-2t|\, (1-t)^s}{[ta^p + (1-t)b^p]^{\frac{1}{p}-1}} \mathrm{d}t \\ &= b^{p-1}\left[\frac{2}{(s+1)(s+2)}\, {}_2F_1\left(\frac{1}{p} - 1, 2; s+3; 1 - \frac{a^p}{b^p}\right)\right. \\ &\quad \left. - \frac{1}{s+1}\, {}_2F_1\left(\frac{1}{p} - 1, 1; s+2; 1 - \frac{a^p}{b^p}\right) + \frac{1}{2}\, {}_2F_1\left(\frac{1}{p} - 1, 1; 3; \frac{1}{2}\left(1 - \frac{a^p}{b^p}\right)\right)\right]. \end{aligned} \tag{2.4}$$

III. If $h(t) = t^{-s}$ in Theorem 2.1, the, we have a new result for Godunova-Levin type of (p, s)-convex functions.

Corollary 2.2. *Let $f : I = [a, b] \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function on I^0 with $0 < a < b$ and $f' \in L[a, b]$. If $|f'|$ is Godunova-Levin type of $(p.s)$-convex function, then, we have*

$$|R_f(a, b; p)| \leq \frac{b^p - a^p}{2p} \left(\theta_5 \left|f'(a)\right| + \theta_6 \left|f'(b)\right|\right),$$

where

$$\begin{aligned}\theta_5 &= \int_0^1 \frac{|1-2t|\, t^{-s}}{[ta^p + (1-t)b^p]^{\frac{1}{p}-1}} \mathrm{d}t \\ &= b^{p-1}\left[\frac{2}{2-s}\, {}_2F_1\left(\frac{1}{p} - 1, 2-s; 3-s; 1 - \frac{a^p}{b^p}\right)\right. \\ &\quad - \frac{1}{1-s}\, {}_2F_1\left(\frac{1}{p} - 1, 1-s; 2-s; 1 - \frac{a^p}{b^p}\right) \\ &\quad \left. + \frac{2^{-s}}{(1-s)(2-s)}\, {}_2F_1\left(\frac{1}{p} - 1, 1-s; 3-s; \frac{1}{2}\left(1 - \frac{a^p}{b^p}\right)\right)\right], \end{aligned} \tag{2.5}$$

and

$$\begin{aligned}\theta_6 &= \int_0^1 \frac{|1-2t|\,(1-t)^{-s}}{[ta^p+(1-t)b^p]^{\frac{1}{p}-1}}\mathrm{d}t \\ &= b^{p-1}\Bigg[\frac{2}{(1-s)(2-s)}\,{}_2F_1\left(\frac{1}{p}-1,2;3-s;1-\frac{a^p}{b^p}\right) \\ &\quad -\frac{1}{1-s}\,{}_2F_1\left(\frac{1}{p}-1,1;2-s;1-\frac{a^p}{b^p}\right)+\frac{1}{2}\,{}_2F_1\left(\frac{1}{p}-1,1;3;\frac{1}{2}\left(1-\frac{a^p}{b^p}\right)\right)\Bigg]. \qquad (2.6)\end{aligned}$$

IV. If $h(t)=1$ in Theorem 2.1, the, we have a new result for (p,P)-functions.

Corollary 2.3. *Let $f:I=[a,b]\subset\mathbb{R}\to\mathbb{R}$ be a differentiable function on I^0 with $0<a<b$ and $f'\in L[a,b]$. If $|f'|$ is (p,P)-function, then, we have*

$$|R_f(a,b;p)|\leq\frac{(b^p-a^p)\theta_7}{2p}\left(|f'(a)|+|f'(b)|\right),$$

where

$$\begin{aligned}\theta_7 &= \int_0^1 \frac{|1-2t|}{[ta^p+(1-t)b^p]^{\frac{1}{p}-1}}\mathrm{d}t \\ &= b^{p-1}\Bigg[{}_2F_1\left(\frac{1}{p}-1,2;3;1-\frac{a^p}{b^p}\right) \\ &\quad -\,{}_2F_1\left(\frac{1}{p}-1,1;2;1-\frac{a^p}{b^p}\right)+\,{}_2F_1\left(\frac{1}{p}-1,1;3;\frac{1}{2}\left(1-\frac{a^p}{b^p}\right)\right)\Bigg]. \qquad (2.7)\end{aligned}$$

Theorem 2.2. *Let $f:I=[a,b]\subset\mathbb{R}\to\mathbb{R}$ be a differentiable function on I^0 with $0<a<b$ and $f'\in L[a,b]$. If $|f'|^q$ is (p,h)-convex function, where $q\geq1$, then, we have*

$$|R_f(a,b;p)|\leq b^{1-p}\cdot\frac{b^p-a^p}{2p}\theta_7^{1-\frac{1}{q}}\left\{\theta_1\left|f'(a)\right|^q+\theta_2\left|f'(b)\right|^q\right\}^{\frac{1}{q}}.$$

where θ_1, θ_2 and θ_7 are given by (2.1), (2.2) and (2.7).

Proof. Using Lemma 1.1, the fact that $|f'|$ is (p,h)-convex function and power mean's inequality, we have

$$
\begin{aligned}
&|R_f(a,b;p)|\\
&= \left|\frac{b^p-a^p}{2p}\int_0^1 [ta^p+(1-t)b^p]^{1-\frac{1}{p}}(1-2t)f'([ta^p+(1-t)b^p]^{\frac{1}{p}})\mathrm{d}t\right|\\
&\leq \frac{b^p-a^p}{2p}\left(\int\limits_0^1 [ta^p+(1-t)b^p]^{1-\frac{1}{p}}|1-2t|\mathrm{d}t\right)^{1-\frac{1}{q}}\\
&\quad\times\left(\int\limits_0^1 [ta^p+(1-t)b^p]^{1-\frac{1}{p}}|1-2t||f'([ta^p+(1-t)b^p]^{\frac{1}{p}})|^q\mathrm{d}t\right)^{\frac{1}{q}}\\
&\leq \frac{b^p-a^p}{2p}\left(\int\limits_0^1 [ta^p+(1-t)b^p]^{1-\frac{1}{p}}|1-2t|\mathrm{d}t\right)^{1-\frac{1}{q}}\\
&\quad\times\left(\int\limits_0^1 [ta^p+(1-t)b^p]^{1-\frac{1}{p}}|1-2t|[h(t)|f'(a)|^q+h(1-t)|f'(b)|^q]\mathrm{d}t\right)^{\frac{1}{q}}\\
&= b^{p-1}\cdot\frac{b^p-a^p}{2p}\theta_7^{1-\frac{1}{q}}\left\{\theta_1\left|f'(a)\right|^q+\theta_2\left|f'(b)\right|^q\right\}^{\frac{1}{q}}.
\end{aligned}
$$

This completes the proof. □

Now, we discuss some special cases for Theorem 2.2.

I. If $h(t)=t$ in Theorem 2.2, then, we have Theorem 3.2 [11].

II. If $h(t)=t^s$ in Theorem 2.2, the, we have a new result for Breckner type of (p,s)-convex functions.

Corollary 2.4. *Let $f: I=[a,b]\subset\mathbb{R}\to\mathbb{R}$ be a differentiable function on I^0 with $0<a<b$ and $f'\in L[a,b]$. If $|f'|^q$ is Breckner type of (p,s)-convex function, where $q\geq 1$, then, we have*

$$|R_f(a,b;p)|\leq b^{1-p}\cdot\frac{b^p-a^p}{2p}\theta_7^{1-\frac{1}{q}}\left\{\theta_3\left|f'(a)\right|^q+\theta_4\left|f'(b)\right|^q\right\}^{\frac{1}{q}}.$$

where θ_3,θ_4, θ_7 are given by (2.3) and (2.4) and (2.7) respectively.

III. If $h(t)=t^s$ in Theorem 2.2, the, we have a new result for Breckner type of (p,s)-convex functions.

Corollary 2.5. *Let $f: I=[a,b]\subset\mathbb{R}\to\mathbb{R}$ be a differentiable function on I^0 with $0<a<b$ and $f'\in L[a,b]$. If $|f'|^q$ is Godunova-Levin type of (p,s)-convex function, where $q\geq 1$, then, we have*

$$|R_f(a,b;p)|\leq b^{1-p}\cdot\frac{b^p-a^p}{2p}\theta_7^{1-\frac{1}{q}}\left\{\theta_5\left|f'(a)\right|^q+\theta_6\left|f'(b)\right|^q\right\}^{\frac{1}{q}}.$$

where θ_5,θ_6, θ_7 are given by (2.5) and (2.6) and (2.7) respectively.

IV. If $h(t)=1$ in Theorem 2.2, then, we have a new result for (p,P)-functions.

Corollary 2.6. *Let $f: I=[a,b]\subset\mathbb{R}\to\mathbb{R}$ be a differentiable function on I^0 with $0<a<b$ and $f'\in L[a,b]$. If $|f'|^q$ is (p,P)-function, where $q\geq 1$, then, we have*

$$|R_f(a,b;p)|\leq b^{1-p}\cdot\frac{b^p-a^p}{2p}\theta_7\left\{\left|f'(a)\right|^q+\left|f'(b)\right|^q\right\}^{\frac{1}{q}}.$$

where θ_7 is given by (2.7).

Theorem 2.3. *Let $f : I = [a,b] \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function on I^0 with $0 < a < b$ and $f' \in L[a,b]$. If $|f'|$ is (p,h)-convex function, then, we have*

$$\begin{aligned} &|L_f(a,b;p)| \\ &\leq \frac{b^p - a^p}{p}\Big[\{\phi_1 + \phi_2\}|f'(a)| + \{\phi_3 + \phi_4\}|f'(b)|\Big], \end{aligned}$$

where

$$\begin{aligned} \phi_1 &= \int_0^{\frac{1}{2}} t[ta^p + (1-t)b^p]^{1-\frac{1}{p}} h(t)\mathrm{d}t, \\ \phi_2 &= \int_{\frac{1}{2}}^1 (1-t)[ta^p + (1-t)b^p]^{1-\frac{1}{p}} h(t)\mathrm{d}t, \\ \phi_3 &= \int_0^{\frac{1}{2}} t[ta^p + (1-t)b^p]^{1-\frac{1}{p}} h(1-t)\mathrm{d}t, \end{aligned}$$

and

$$\phi_4 = \int_{\frac{1}{2}}^1 (1-t)[ta^p + (1-t)b^p]^{1-\frac{1}{p}} h(1-t)\mathrm{d}t,$$

respectively.

Proof. Using Lemma 1.2 and the fact that $|f'|$ is (p,h)-convex function, we have

$$\begin{aligned} &|L_f(a,b;p)| \\ &= \Bigg|\frac{b^p - a^p}{p} \int_0^{\frac{1}{2}} t[ta^p + (1-t)b^p]^{1-\frac{1}{p}} f'([ta^p + (1-t)b^p]^{\frac{1}{p}})\mathrm{d}t \\ &\quad + \frac{b^p - a^p}{p} \int_{\frac{1}{2}}^1 (t-1)[ta^p + (1-t)b^p]^{1-\frac{1}{p}} f'([ta^p + (1-t)b^p]^{\frac{1}{p}})\mathrm{d}t\Bigg| \\ &\leq \frac{b^p - a^p}{p}\Bigg[\int_0^{\frac{1}{2}} t[ta^p + (1-t)b^p]^{1-\frac{1}{p}} |f'([ta^p + (1-t)b^p]^{\frac{1}{p}})|\mathrm{d}t \\ &\quad + \int_{\frac{1}{2}}^1 (1-t)[ta^p + (1-t)b^p]^{1-\frac{1}{p}} |f'([ta^p + (1-t)b^p]^{\frac{1}{p}})|\mathrm{d}t\Bigg] \\ &\leq \frac{b^p - a^p}{p}\Bigg[\int_0^{\frac{1}{2}} t[ta^p + (1-t)b^p]^{1-\frac{1}{p}} [h(t)|f'(a)| + h(1-t)|f'(b)|]\mathrm{d}t \\ &\quad + \int_{\frac{1}{2}}^1 (1-t)[ta^p + (1-t)b^p]^{1-\frac{1}{p}} [h(t)|f'(a)| + h(1-t)|f'(b)|]\mathrm{d}t\Bigg]. \end{aligned}$$

This completes the proof. □

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REFERENCES

1. S. S. Dragomir, R. P. Agarwal, Two Inequalities for Differentiable Mappings and Applications to Special Means of Real Numbers and to Trapezoidal Formula, Appl. Math. Lett. 11, 91–95, (1998).
2. S. S. Dragomir, C. E. M. Pearce, Selected topics on Hermite-Hadamard inequalities and applications, Victoria University, 2000.
3. Z. B. Fang, R. Shi, On the (p, h)-convex function and some integral inequalities, J. Inequal. Appl. 2014, 2014:45.
4. I. Iscan, Hermite-Hadamard type inequalities for harmonically convex functions, Hacet. J. Math. Stat., 43(6), 935-942, (2014).
5. U. S. Kirmaci, Inequalities for differentiable mappings and applications to special means of real numbers and to midpoint formula, Appl. Math. Comput. 147, 137–146, (2004).
6. M. V. Mihai, New Hermite-Hadamard type inequalities obtained via Riemann-Liouville fractional calculus, Analele Uni. Oradea Fasc. Matematica, 127-132, (2013).
7. M. V. Mihai, F. C. Mitroi, Hermite-Hadamard type inequalities obtained via Riemann-Liouville fractional calculus, accepted to Acta Mathematica Universitatis Comenianae, Slovacia.
8. M. V. Mihai, New inequalities for co-ordinated convex functions via Riemann-Liouville fractional calculus, Tamkang J. Math., 45(3), 285-296, (2014).
9. M. V. Mihai, M. A. Noor, K. I. Noor, M. U. Awan, Some integral inequalities for harmonic h-convex functions involving hypergeometric functions, Appl. Math. Computation, 252 (2015) 257262,
10. M. A. Noor, G. Cristescu, M. U. Awan, Generalized fractional Hermite-Hadamard inequalities for twice differentiable s-convex functions, Filomat, 29(4), (2015), 807-815.
11. M. A. Noor, K. I. Noor, M. V. Mihai, M. U. Awan, Hermite-Hadamard inequalities for differentiable p-convex functions using hypergeometric functions, preprint, (2015).
12. K. S. Zhang, J. P. Wan, p-convex functions and their properties. Pure Appl. Math. 23(1), 130-133 (2007).