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THE ANALYSIS OF AN HYDROMAGNETIC VARIABLE – VISCOSITY CHANNEL FLOW WITH NON – UNIFORM WALL TEMPERATURE

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This paper investigates the influence of magnetic intensity on the inherent irreversibility of a variable--viscosity channel flow with varying wall temperatures. The hydromagnetic fluid flow is considered to be steady, incompressible with variable viscosity that is a linear function of temperature. The approximate solutions are obtained seeking asymptotic solutions for the fluid motion and heat transfer as well as the rate of entropy generation and other thermophysical aspects of the flow regime with respect to the magnetic influence are presented and discussed.

Keywords: Hydromagnetic, variable—viscosity, non--uniform wall temperature and entropy generation.

1. Introduction

Studies involving hydromagnetic fluid flow have been extensively carried out in the past few decades because of its numerous applications and relevance in geophysics, engineering, industry and technology. For example, the study in [1] highlighted the applications which include the design of electromagnetic pumps, liquid-metal cooling of nuclear reactor, etc. Relevant studies involving hydromagnetic fluid flow have been extensively carried out by eminent researchers. To mention few, [2] analyzed states of hydromagnetic natural convection and mass transfer flow of viscous reactive incompressible and electrically conducting fluid. Also, [3] employed laws of thermodynamics to examine the fluid flow and thermal decomposition in hydromagnetic variable viscosity Couette flow in a rotating system with hall current. In addition to that, [4] explored the numerical analysis of a fluid flow through a rotating rectangular straight duct under the impact of magnetic field.

However, extensive survey with experimental, theoretical and numerical approaches on magnetic fluid under some thermophysical properties. For example, [5] examined the effects of thermal radiation and magnetic field of a high viscous fluid with temperature-dependent viscosity and thermal conductivity on hydromagnetic Couette flow through a porous channel and [6] investigated hydromagnetic effects on the flow of couple stress fluid through a porous channel with walls suction and injection. Other significant studies on the impact of

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magnetic strength over the flow arena are considered in [7] – [14] where attention was focused on the influence of the entropy generation rates, thermal stability analysis and effects of internal heat generation on a reactive hydromagnetic fluid flow through a channel.

Meanwhile, other physical properties of the fluid flow have also been considered by some researchers without the influence or impact of magnetic field intensity, for example, [15] and [16] respectively studied the flow of a variable viscosity fluid through an ordinary plane and parallel porous channels with non-uniform wall temperature without any consideration for the magnetic intensity of the fluid flow. Other studies related to variable viscosity reactive fluid flow on the second law analysis and thermal criticality were investigated, to mention few, in [17] – [19] without considering the impact of magnetic strength.

The earlier investigations mentioned in [1] – [14] and others reported in [20] – [23] significantly showed the importance of magnetic physical properties and cannot be totally neglected in a fluid flow because of its application and relevance in industries, especially in medical technology and bio-engineering, for instance, [24] mentioned that magnetic fluid is famous in the field of medicine where magnetic devices have been successfully used for cell separation, focused transportation of drugs in cells, hyperthermia and reduction of bleeding, cancer treatment, etc. Motivated by the impact of magnetic strength in industrial, engineering and medical applications, hence, this study intends to extend the studies in [15] and [16] by investigating the effect of magnetic intensity on the irreversibility flow of a variable--viscosity fluid through a channel with non-uniform wall temperature. This effect is shown on the fluid motion and heat transfer as well as on the entropy generation rate and other thermophysical properties of the flow regime.

2. Problem Formulation

Consider the steady flow of an incompressible and electrically conducting fluid flowing in one direction through a channel of width (a) and length (L), with a non-uniform wall temperature under the action of a constant pressure gradient with the influence of a transverse magnetic field strength (B_o) as illustrated in [16] shown below in figure 1.

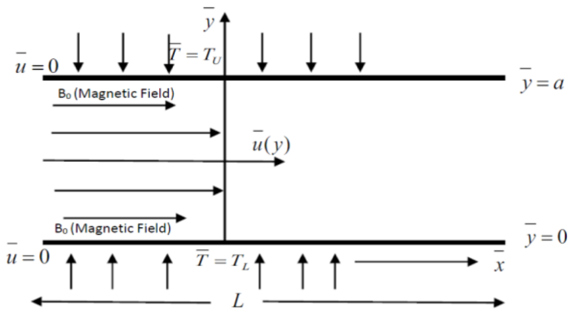

Fig. 1: Schematic diagram of the problem

The temperature dependent viscosity ($\bar{\mu}$) as mentioned in [15] and 16] can be shown to be

$$\bar{\mu} = \mu_0 \left[1 - \beta\left(\bar{T} - T_L\right)\right] \tag{1}$$

here, μ_0 represents the fluid dynamic viscosity at lower temperature (T_L), also β stands for viscosity – variation parameter and T is the fluid temperature.

Disregarding the consumption of the reactant, the continuity, momentum and energy equations governing the problem in dimensionless form may be written in Cartesian coordinate (x, y) as in [13, 15, 16];

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2}$$

$$\rho\left[\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right] = -\frac{\partial \bar{p}}{\partial \bar{x}} + 2\frac{\partial}{\partial \bar{x}}\left(\bar{\mu}\frac{\partial \bar{u}}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{y}}\left[\bar{\mu}\left(\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}}\right)\right] - \sigma B_0^2 \bar{u} \tag{3}$$

$$\rho\left[\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right] = -\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\partial}{\partial \bar{y}}\left(\bar{\mu}\frac{\partial \bar{v}}{\partial \bar{y}}\right) + \frac{\partial}{\partial \bar{x}}\left[\bar{\mu}\left(\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}}\right)\right] \tag{4}$$

$$\rho c_p\left[\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right] = k\left[\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right] + \bar{\mu}\left[2\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^2 + 2\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right] + \sigma B_0^2 \bar{u}^2 \tag{5}$$

In equations (2) – (5), u and v respectively represent the axial and normal velocities, ρ stands for fluid density, k for thermal conductivity, p is the pressure, σ for electrical conductivity and c_p is the specific heat at constant pressure. It is worthy to mention that the bar on each variable represents the non dimensionless form. However, the last term in equations (3) and (5) represents the magnetic intensity influence on the flow regime as in [6, 12 – 14].

We have employed the following non dimensional quantities in equations (1) – (5):

$$y = \frac{\bar{y}}{\varepsilon L}, x = \frac{\bar{x}}{L}, u = \frac{\bar{u}}{U}, v = \frac{\bar{v}}{\varepsilon U}, \varepsilon = \frac{a}{L}, \mu = \frac{\bar{\mu}}{\mu_0}, T = \frac{\bar{T} - T_L}{T_u - T_L},$$
$$p = \frac{\varepsilon^2 L \bar{p}}{\mu_0 U}, \alpha = \beta\left(T_u - T_L\right), Br = \frac{\mu_0 U^2}{k\left(T_u - T_l\right)}, -\frac{\partial p}{\partial x} = G, Pe = \frac{\rho C_p U L}{k}, \tag{6}$$
$$\mathrm{Re} = \frac{\rho U L}{\mu_0} \text{ and } H^2 = \frac{\sigma B_0^2 a^2}{\mu_0}$$

where G is the pressure gradient, T_u is the upper wall temperature, ε is the channel aspect ratio, U is the velocity scale and α represents viscosity -

variation parameter. Also, *Br*, *Re*, *Pe* and *H* are respectively numbers for Brinkman, Reynolds, Peclet and Hartmann.

Therefore, the governing boundary value problem equations (1) – (5) with the introduction of (6) become the following in dimensionless form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$\varepsilon^2 \operatorname{Re}\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+2\varepsilon^2\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\varepsilon^2\frac{\partial v}{\partial x}\right)\right]-H^2u \tag{8}$$

$$\varepsilon^4 \operatorname{Re}\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial y}+2\varepsilon^2\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+\varepsilon^2\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\varepsilon^2\frac{\partial v}{\partial x}\right)\right] \tag{9}$$

$$\varepsilon^2 Pe\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=\varepsilon^2\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+Br\left[\mu\phi+H^2u^2\right] \tag{10}$$

such that

$$\phi=2\varepsilon^2\left(\frac{\partial u}{\partial x}\right)^2+2\varepsilon^2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\varepsilon^2\frac{\partial v}{\partial x}\right)^2 \tag{11}$$

Based on the fact that the channel aspect ratio is narrowly small such that $0<\varepsilon<<1$, then, it is assumed that the lubrication approximation based on an asymptotic simplification of the governing equations (7) – (11) is reduced as in [15] and [16] such that the governing equations become:

$$G+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-H^2u=0 \tag{12}$$

$$-\frac{\partial p}{\partial y}+o(\varepsilon^2)=0 \tag{13}$$

$$\frac{\partial^2 T}{\partial y^2}+Br\left[\mu\left(\frac{\partial u}{\partial y}\right)^2+H^2u^2\right]=0 \tag{14}$$

Here, $\mu=1-\alpha T$ with the following boundary conditions at the upper wall of the channel as: $u=0,\quad T=1\quad$ at $y=1$ (15)

and at the lower wall of the channel as: $u=0,\quad T=0\quad$ at $y=0$ (16)

In actual sense, the boundary conditions (15) and (16) evidently signify that both walls are fixed with different wall temperatures.

3. Method of Solution

Based on the previous assumptions described in [15] and [16] that the fluid viscosity is extremely small, we thereby seek asymptotic solutions for both the fluid motion and heat transfer under the influence of magnetic intensity as follows:

$$u = u_0 + \alpha u_1, \qquad T = T_0 + \alpha T_1 \tag{17}$$

such that if (17) is substituted into equations (12) – (14) then, the following equations are obtained in respective order as follows:
Order zero (α^0)

$$\frac{\partial^2 u_0}{\partial y^2} - H^2 u_0 + G = 0, \qquad \frac{\partial^2 T_0}{\partial y^2} + Br\left[\left(\frac{\partial u_0}{\partial y}\right)^2 + H^2 u_0{}^2\right] = 0 \tag{18}$$

subjected to the boundary conditions

$$u_0 = 0, \quad T_0 = 1 \text{ at } y = 1 \tag{19}$$

$$\text{and} \quad u_0 = 0, \quad T_0 = 0 \text{ at } y = 0 \tag{20}$$

Also, for order zero (α^1)

$$\frac{\partial^2 u_1}{\partial y^2} - T_0 \frac{\partial^2 u_0}{\partial y^2} - \frac{\partial T_0}{\partial y}\frac{\partial u_0}{\partial y} - H^2 u_1 = 0, \tag{21}$$

$$\frac{\partial^2 T_1}{\partial y^2} + Br\left[2\frac{\partial u_0}{\partial y}\frac{\partial u_1}{\partial y} - T_0\left(\frac{\partial u_0}{\partial y}\right)^2 + 2H^2 u_0 u_1\right] = 0 \tag{22}$$

with the boundary conditions

$$u_1 = 0, \quad T_1 = 0 \text{ at } y = 1 \text{ and } \quad u_1 = 0, \quad T_1 = 0 \text{ at } y = 0 \tag{23}$$

We now solve equations (18) - (23) with appropriate boundary conditions using Mathematica software package to get the desired solutions of fluid motion and temperature in equation (17) for further discussion in next sections.

4. Entropy Generation Analysis

Flow and heat transfer processes between two parallel plates are irreversible. The account of irreversible and disorderliness arise due to exchange of energy and momentum within the flow regime, thus resulting in entropy generation. It is also a way of providing another variable that can be used to describe the state of a system along with pressure, volume and temperature. Following [12 – 16, 25], the general equation for the rate of entropy generation under the impact of magnetic field is expressed as:

$$S^m = \frac{k}{T_L^2}\left(\frac{\partial \overline{T}}{\partial \overline{y}}\right)^2 + \frac{\overline{\mu}}{T_L}\left(\frac{\partial \overline{u}}{\partial \overline{y}}\right)^2 + \frac{\sigma B_0{}^2 \overline{u}^2}{T_L} \tag{24}$$

The first term in (24) represents the irreversibility due to heat transfer, the second term is the entropy generation due to viscous dissipation and the third term is the local entropy generation due to the effects of magnetic intensity. The dimensionless form of (24) using the existing quantities in (6) further gives the following:

$$N_s = \frac{S^m a^2 T_l^2}{k(T_u - T_L)^2} = \left(\frac{\partial T}{\partial y}\right)^2 + \frac{Br}{\Omega}\left[\mu\left(\frac{\partial u}{\partial y}\right)^2 + H^2 u^2\right] \tag{25}$$

where Ω stands for the wall temperature difference parameter.

For easy computations, we assigned the first term, N_1 and other terms as N_2 such that:

$$N_1 = \left(\frac{\partial T}{\partial y}\right)^2, \quad N_2 = \frac{Br}{\Omega}\left[\mu\left(\frac{\partial u}{\partial y}\right)^2 + H^2 u^2\right] \tag{26}$$

Here, N_1 is used to represent the irreversibility due to heat transfer and N_2 is the entropy generation rate due to the compound impacts of viscous dissipation and magnetic strength.

However, it is essential to understand the supremacy of heat transfer irreversibility over fluid friction, as a result of that, we used (26) to defined irreversibility distribution ratio (ϕ) such that

$$\phi = \frac{N_2}{N_1} \tag{27}$$

Relation (27) shows that heat transfer has dominion when $0 < \phi < 1$ and fluid friction has dominion when $\phi > 1$. But, when the rate of entropy production of heat transfer is equal to that of fluid friction, it implies that, $\phi = 1$. Moreover, as an alternative to irreversibility parameter, the Bejan number (*Be*) which shows the contribution of both the heat transfer and fluid friction to entropy generation rate is defined as

$$Be = \frac{N_1}{N_s} = \frac{1}{1+\phi} \qquad \text{where} \qquad 0 \le Be \le 1. \tag{28}$$

The expressions from (26) to (28) can be determined from the solutions in (17) using Mathematica software package as well.

5. Discussion of Results

This section discussed the effects of the magnetic intensity together with other important flow parameters on the hydromagnetic variable-viscosity fluid flow with non-uniform wall temperature. These are presented in table and graphs using solutions from (17) with code on Mathematica software.

Table 1 showed the computation of the rate of entropy distribution using (26 – 28). From the table, it is observed that heat transfer dominates at the center channel of the fluid flow while fluid friction dominates at both lower and upper walls of the channel but more noticeable towards the upper wall of the channel. Also, the contribution of both heat transfer and fluid friction to entropy generation are almost equal at both ends of the wall channel as $Be \cong 0.5$. This is further illustrated in Figs. 8 to 16.

Table 1:

Computation of the entropy Analysis

$H = 1, G = 1, Br = 10, \alpha = 0.1, \Omega = 0.1$				
y	$N_1 = \left(\frac{\partial T}{\partial y}\right)^2$	$N_2 = \frac{Br}{\Omega}\left[\mu\left(\frac{\partial u}{\partial y}\right)^2 + H^2u^2\right]$	$\phi = \frac{N_1}{N_2}$	$Be = \frac{1}{1+\phi}$
0.0	1.95429925	2.19140018	1.12132274	0.47140399
0.1	1.48861731	1.40565713	0.94427031	0.51433177
0.2	1.23124017	0.83649339	0.67939092	0.59545398
0.3	1.09549953	0.45038277	0.41112091	0.82169502
0.4	1.02870453	0.22322533	0.21699655	0.70865649
0.5	0.99454917	0.14128854	0.14206290	0.87560851
0.6	0.96314811	0.20179727	0.20951842	0.82677534
0.7	0.90624054	0.41355517	0.45634150	0.68665213
0.8	0.79694731	0.79779867	1.00106827	0.49973307
0.9	0.61547329	1.38936781	2.25739738	0.30699355
1.0	0.36472370	2.23817057	6.13661948	0.14012237

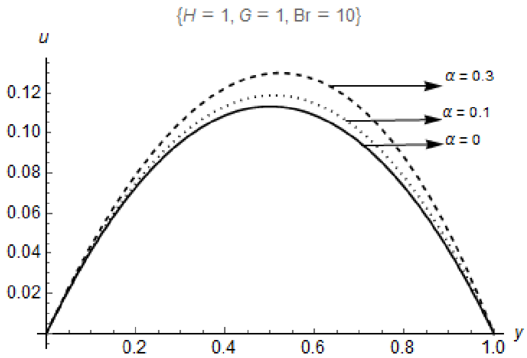

Fig. 2: Velocity profile with change in α

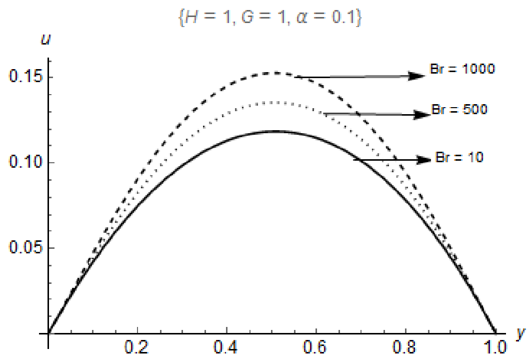

Fig. 3: Velocity profile with change in *Br*

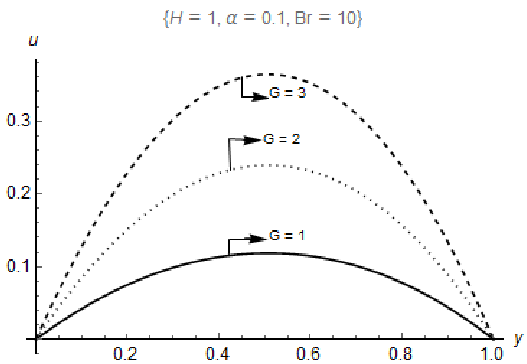

Fig. 4: Velocity profile with change in *G*

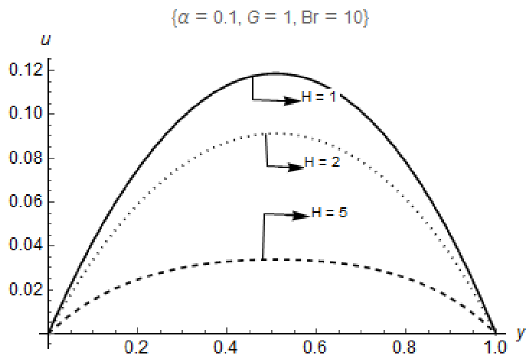

Fig. 5: Velocity profile with change in H

Figures 2–5 displayed respectively the velocity profiles for variations in viscosity-variation parameter (α), Brinkman number (Br), pressure gradient (G) and magnetic intensity parameter (H). On a general note, the maximum velocity is obtained at the centerline of the channel and minimum at the upper and lower walls of the flow channel. It is noticed that the fluid motion increases with rising values of viscosity-variation parameter (α) in Fig. 2, Brinkman number (Br) in Fig. 3 and pressure gradient (G) in Fig. 4 while the reversed is observed in Fig. 5 where an increasing values of magnetic intensity parameter (H) bring a reduction in the fluid velocity. This is due to the presence of magnetic strength which has retarding ability on the speed of the fluid.

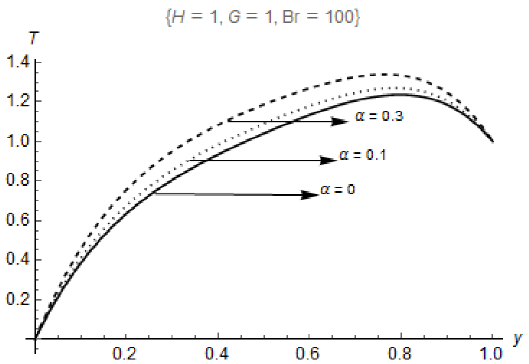

Fig. 6: Temperature profile with change in α

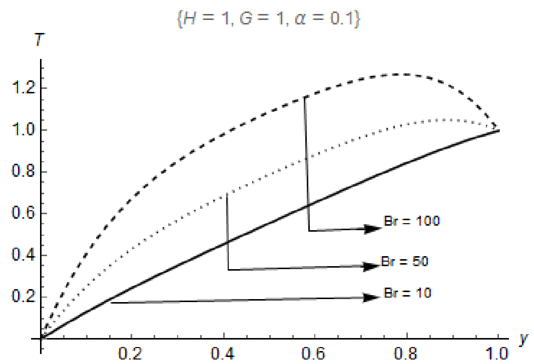

Fig. 7: Velocity profile with change in Br

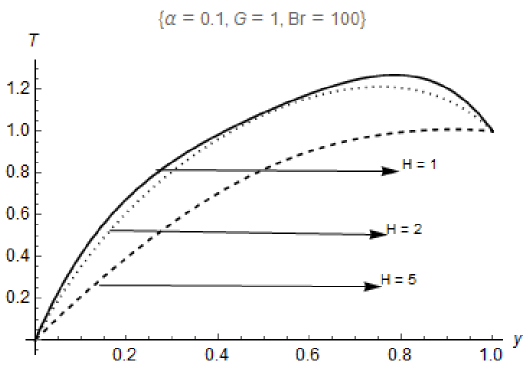

Fig. 8: Temperature profile with change in H

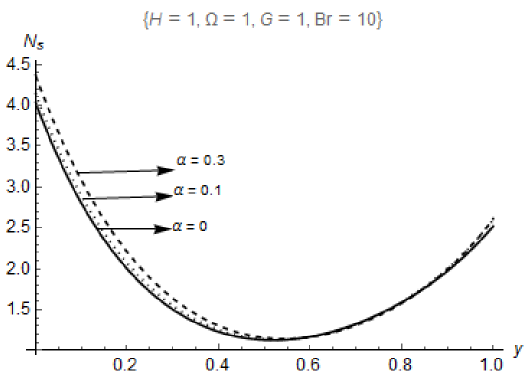

Fig. 9: Entropy generation rate with change in α

The temperature profile of the fluid flow with variations in viscosity-variation parameter (α), Brinkman number (Br) and magnetic intensity parameter (H) are displayed in Figs. 6 – 8. Generally, the minimum temperature is observed at the lower wall of the channel and a transverse increase to the maximum at the centerline and a gradual decrease to the prescribed value of $y = 1$ from equations (15) and (16) at the upper wall of the channel. The fluid temperature rises with rising values of (α) in Fig. 6 and Brinkman number (Br) in Fig. 7 while the fluid temperature decreases as the magnetic intensity parameter (H) increases in Fig. 8.

Figs. 9 – 12 displayed the entropy generation rate for various parametric values. Generally, the rate is minimum and active at the centerline of the channel and increases at the walls speedily. An increase is recorded at both ends of the wall as the values of viscosity-variation parameter (α) in Fig. 9 and Brinkman number (Br) increase in Fig. 10. While increasing values of temperature-difference parameter (Ω) and magnetic intensity parameter (H) bring down the entropy generation rate in the fluid flow in Figs. 11 and 12 respectively.

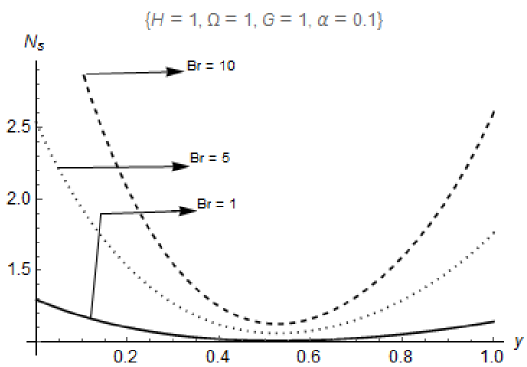

Fig. 10: Entropy generation rate with change in *Br*

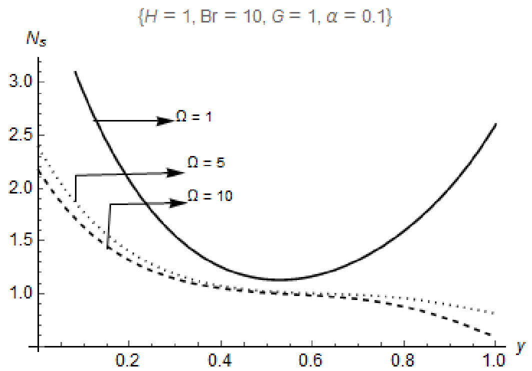

Fig. 11: Entropy generation rate with change in Ω

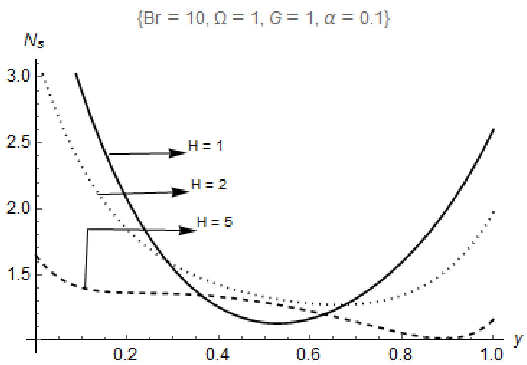

Fig. 12: Entropy generation rate with change in H

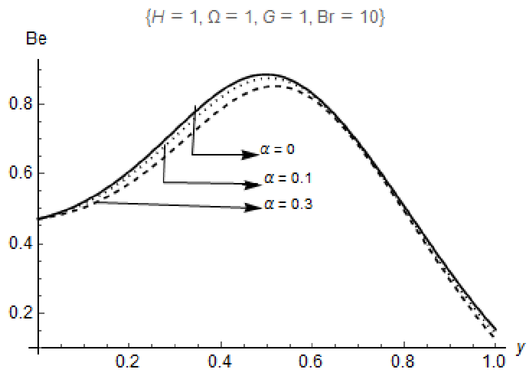

Fig. 13: Bejan number with change in α

Bejan number rates are displayed in figures 13 to 16 for variations in the respective values of viscosity-variation parameter (α), Brinkman number (Br), temperature-difference parameter (Ω) and magnetic intensity parameter (H). It is discovered that the dominant effects of heat transfer irreversibility occur around the centerline region of the channel while the dominant effects of fluid friction irreversibility occur at both ends of the wall. The increasing values of viscosity-variation parameter (α) in figure 13 and Brinkman number (Br) in figure 14 lead to increase in the dominance effect of fluid friction irreversibility near the walls while increasing values of temperature-difference parameter (Ω) in figure 15 and magnetic intensity parameter (H) in figure 16 lead to reduction in the dominant effects of fluid friction irreversibility.

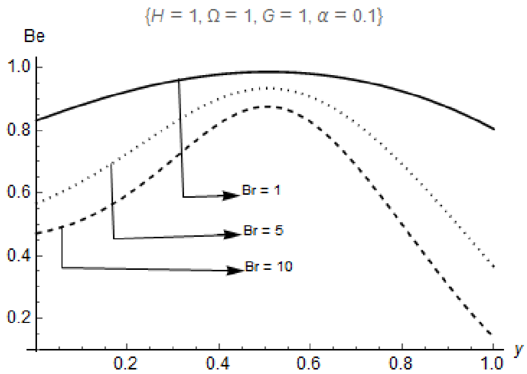

Fig. 14: Bejan number with change in *Br*

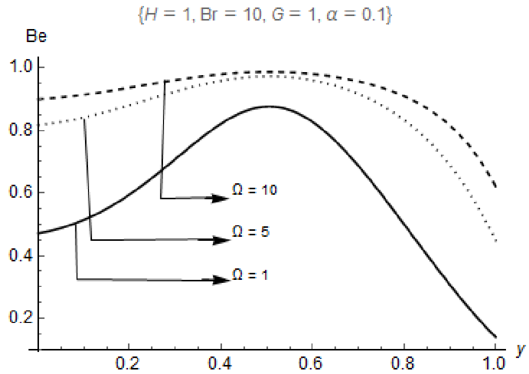

Fig. 15: Bejan number with change in Ω

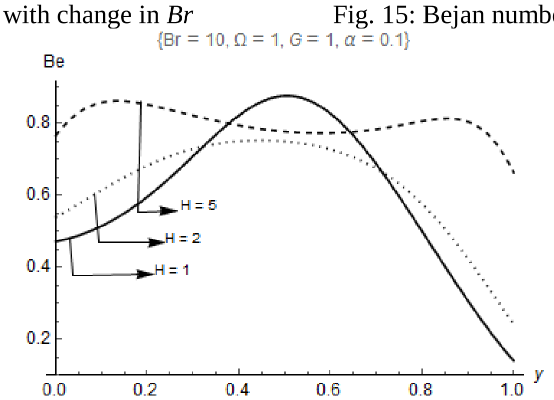

Fig. 16: Bejan number with change in *H*

6. Conclusion

In this study, we have critically examined the impact of magnetic strength for a steady, incompressible, variable-viscosity fluid flow with non-uniform wall temperature that varies linearly which has significant effect on the fluid motion, heat transfer and entropy generation rate. The analytical expressions are obtained seeking asymptotic solutions for the fluid velocity and temperature. The result showed that the fluid motion increases with rising values of viscosity-variation parameter while the reversed is observed with the increasing values of magnetic intensity parameter (H) bring a reduction in the fluid motion which is due to the presence of magnetic strength which has retarding ability on the speed of the fluid. The fluid temperature rises with rising values of (α) while the fluid temperature decreases as the magnetic intensity parameter (H). The results significantly showed that the impact of magnetic strength cannot be neglected as it can be used to control and achieve a desirable output in engineering and medical applications to reduce the fluid velocity and temperature of fluid flow because of its retarding ability to flow resistance.

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