U.P.B. Sci. Bull. Series A, Vol. 79, Iss.2, 2017 ISSN 1223-7027

GENERALIZED Ω-DISTANCE MAPPINGS AND SOME FIXED POINT THEOREMS

K.ABODAYEH[1], A.BATAIHAH[2] and W. SHATANAWI[1,3]

In this article, we introduce the concept of generalized Ω-distance mappings by using the concepts of G_b-metric spaces and Ω-distance mappings. Also, we introduce the notions of generalized Ω-Banach contraction and generalized Ω-Kannan contraction. We use such contractions to prove the existence and uniqueness fixed point.

Keywords: generalized Ω-distance, generalized b-metric space, Ω_b-distance, G_b-metric space,b-metric space, fixed point.

1. Introduction

In 1989, Bakhtin [1] introduced the concept of b-metric spaces as a generalization of the standard concept of metric spaces, while the result of Bakhtin became known more by Czerwik [2]. For some results in b-metric spaces, we refer the reader to [3]–[8]. The notion of generalization of metric spaces, called a G-metric space was investigated by Mustafa and Sims [9] in 2006. After that many authors studied many results on fixed and common fixed point in G-metric spaces; for example see [10]–[15]. In 2012 Jleli and Samet et al. [16] derived some known fixed point theorems in G-metric spaces by using the concept of quasi metric spaces. The clever paper of Jleli and Samet et al [16] reduces some fixed point theorems from G-metric spaces to standard metric spaces. In 2010 Saadati*et.al.* [17] introduced the concept of Ω-distance as a generalization of Ω-distance [18] and studied some fixed point results. Recently, many authors [19]–[23], [25] obtained many fixed point results by using the concept of Ω-distance mappings.

On the other hand, Aghanjani*et.al.* [24] utilized the concept of G-metric spaces and the concept of b-metric spaces to introduce the concept of G_b-metric spaces. The concept of generalized Ω-distance mappings (Ω_b-distance) will be considered in more details where some fixed point results using such concept will be obtained.

[1] Department of Mathematics and general courses, Prince Sultan University Riyadh, Saudi Arabia, E-mail: kamal@psu.edu.sa, wshatanawi@psu.edu.sa
[2] Department of Mathematics, Faculty of Science, University of Jordan, Amman, Jordan, email: anwerbataihah@gmail.com
[3] Department of Mathematics and general courses, Prince Sultan University Riyadh, Saudi Arabia, E-mail: kamal@psu.edu.sa, wshatanawi@psu.edu.sa and Department of Mathematics, Faculty of Science , Hashemite University, Zarqa, Jordan, email: swasfi@hu.edu.jo, wshatanawi@yahoo.com

2. Preliminary

The concept of G_b-metric spaces is defined as follows:

Definition 2.1. [24] Let X be a nonempty set and $s \geq 1$ be a given real number. Suppose that a mapping $G: X \times X \times X \to \mathbb{R}^+$ be a function satisfies:

(G_b1) $G(x,y,z) = 0$ if $x = y = z$;

(G_b2) $G(x,x,y) > 0$ for all $x, y \in X$, with $x \neq y$;

(G_b3) $G(x,y,y) \leq G(x,y,z)$, for all $x,y,z \in X$ with $y \neq z$;

(G_b4) $G(x,y,z) = G(p\{x,y,z\})$, where p is a permutation of x,y,z;

(G_b5) $G(x,y,z) \leq s[G(x,a,a) + G(a,y,z)]$ for all $x,y,z,a \in X$.

Then the function G is called a *generalized b-metric* and the pair (X, G) is called generalized b-metric space or G_b-metric space.

It is clear that the class of G_b-metric spaces is larger than that of G-metric spaces.

Example 2.1. [24] Let (X,G) be a G-metric space, and $G_*(x,y,z) = G(x,y,z)^p$, where $p > 1$ is a real number. Then we note that G_* is a G_b-metric with $s = 2^{p-1}$. To see this it is obvious that G_* satisfies conditions $(G_b1) \cdots (G_b4)$ of Definition 2.1, so it sufficient to verify Condition (G_b5). If $1 < p < \infty$, then for $x > 0$ the convexity of the function $f(x) = x^p$ implies that $(a+b)^p \leq 2^{p-1}(a^p + b^p)$. Thus for each $x,y,z,a \in X$ we obtain

$$\begin{aligned} G_*(x,y,z) &= G(x,y,z)^p \leq (G(x,a,a) + G(a,y,z))^p \\ &\leq 2^{p-1}(G(x,a,a)^p + G(a,y,z)^p) \\ &= 2^{p-1}(G_*(x,a,a) + G_*(a,y,z)). \end{aligned}$$

So G_* is a G_b-metric.

Aghanjani*et.al.* defined the G_b-convergence and G_b-Cauchy sequences as follows:

Definition 2.2. [24] Let X be a G_b-metric space. A sequence (x_n) in X is said to be

1) *G_b-convergent* to $x \in X$ if for any $\epsilon > 0$, there exists $k \in \mathbf{N}$ such that for all $n, m \geq k$, $G(x, x_n, x_m) < \epsilon$.
2) *G_b-Cauchy sequence* if for any $\epsilon > 0$, there exists $k \in \mathbf{N}$ such that for all $n, m, l \geq k$, $G(x_n, x_m, x_l) < \epsilon$.

Proposition 2.1. [24] Let X be a G_b-metric space. Then the following are equivalent:

1) the sequence (x_n) is G_b-convergent to x.
2) $G(x_n, x_n, x) \to 0$ as $n \to \infty$.

3) $G(x_n, x, x) \to 0$ as $n \to \infty$.

Proposition 2.2. [24] Let X be a G_b-metric space. Then the following are equivalent:

1) the sequence (x_n) is G_b-Cauchy .
2) for any $\epsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \epsilon$, for all $n, m \geq k$.

Definition 2.3. A G_b-metric space X is called G_b*-complete* or complete G_b-metric space if every G_b-Cauchy sequence is G_b-convergent in X.

3. Generalized Ω-distance mappings (Ω_b-distance)

First, we introduce the concept of generalized Ω-distance mappings.

Definition 3.4. Let X be a G_b-metric space. Then a mapping $\Omega: X \times X \times X \to [0, \infty)$ is called a *generalized Ω-distance mapping* or Ω_b-distance mapping on X if the following conditions are satisfied:

1) $\Omega(x, y, z) \leq s\ [\Omega(x, a, a) + \Omega(a, y, z)]$, for all $x, y, z, a \in X$ and $s \geq 1$,
2) for any $x, y \in X$, the functions $\Omega(x, y, .), \Omega(x, ., y): X \to X$ are lower semi continuous,
3) for every $\epsilon > 0$, there is $\delta > 0$ such that $\Omega(x, a, a) \leq \delta$ and $\Omega(a, y, z) \leq \delta$ imply $G_b(x, y, z) \leq \epsilon$.

Example 3.2. Consider the G_b-metric G defined on $X = \mathbb{R}$ by $G(x, y, z) = (|x - y| + |y - z| + |x - z|)^2\ \ \forall\ x, y, z \in \mathbb{R}$. Define $\Omega: X \times X \times X \to [0, \infty)$ by $\Omega(x, y, z) = (|x - y| + |x - z|)^2$ for all $x, y, z \in$ I. Then Ω is a generalized Ω-distance with $s = 2$. Note that it is a straightforward to verify that Conditions (1) and (2) of Definition 3.4 are satisfied. So we need to check that the last condition of the definition is valid.

Let $\epsilon > 0$ be given. Choose $\delta = \frac{\epsilon}{40}$ such that $\Omega(x, a, a) < \delta$ and $\Omega(a, y, z) < \delta$. Then

$$\Omega(x, a, a) = (|x - a| + |x - a|)^2 < \delta \Rightarrow |x - a| < \sqrt{\delta}.$$

Also,

$$\Omega(a, y, z) = (|a - y| + |a - z|)^2 < \delta \Rightarrow |y - a| < \sqrt{\delta} \text{ and } |z - a| < \sqrt{\delta}.$$

Now

$$
\begin{aligned}
G(x,y,z) &= (|x-y|+|y-z|+|x-z|)^2 \\
&\leq (|x-a|+|y-a|+|y-a|+|z-a|+|x-a|+|z-a|)^2 \\
&= 4|x-a|^2+4|y-a|^2+4|z-a|^2+8|x-a||y-a|+8|x-a||z-a| \\
&\quad +8|y-a||z-a| \\
&< 4\delta+4\delta+4\delta+8\delta+8\delta+8\delta \\
&= 40\delta=\varepsilon
\end{aligned}
$$

Definition 3.5. Let (X,G) be a G_b-metric space and Ω be an Ω_b-distance on X. Then we say that X is Ω-*bounded* if there exists $M>0$ such that $\Omega(x,y,z)\leq M$ for all $x,y,x\in X$.

The following result is a generalization of Lemma 1.7 in [17] for Ω_b-distance.

Lemma 3.3. Let X be a G_b-metric space and Ω_b be a generalized Ω-distance on X. Let $(x_n),(y_n)$ be sequences in X, $(\alpha_n),(\beta_n)$ be sequences in $[0,\infty)$ converging to zero and let $x,y,z,a\in X$. Then we have the following:

1) If $\Omega_b(y_n,x_n,x_n)\leq\alpha_n$ and $\Omega_b(x_n,y_m,z)\leq\beta_n$ for any $m>n\in\mathbb{N}$, then $G(y_n,y_m,z)$ converges to 0 and hence y_n converges to z;
2) If $\Omega_b(y,x_n,x_n)\leq\alpha_n$ and $\Omega_b(x_n,y,z)\leq\beta_n$ for $n\in\mathbb{N}$, then $G(y,y,z)<\epsilon$ and hence $y=z$;
3) If $\Omega_b(x_n,x_m,x_l)\leq\alpha_n$ for any $m,n,l\in\mathbb{N}$ with $n\leq m\leq l$, then (x_n) is a G_b-Cauchy sequence;
4) If $\Omega_b(x_n,a,a)\leq\alpha_n$ for any $n\in\mathbb{N}$, then (x_n) is a G_b-Cauchy sequence.

Proof.

1) Let $\epsilon>0$ be given. From the definition of a generalized Ω-distance, there exists $\delta>0$ such that $\Omega_b(s,a,a)\leq\delta$ and $\Omega_b(a,t,z)\leq\delta$ imply $G_b(s,t,z)\leq\epsilon$. Choose $n_0\in\mathbb{N}$ such that $\alpha_n\leq\delta$ and $\beta_n\leq\delta$ for every $n\geq n_0$. Then, for any $m>n\geq n_0$, we have,
$$\Omega_b(y_n,x_n,x_n)\leq\alpha_n\leq\delta,\Omega_b(x_n,y_m,z)\leq\beta_n\leq\delta,$$
and hence $G_b(y_n,y_m,z)\leq\epsilon$, so that (y_n) converges to z.
2) Part (2) follows directly from Part (1).
3) Let $\epsilon>0$ be given. As in the proof of (1), choose $\delta>0$ and $n_0\in$ I . Then, for any $\geq m>n\geq n_0,\Omega_b(x_n,x_{n+1},x_{n+1})\leq\alpha_n\leq\delta$, $\Omega_b(x_{n+1},x_m,x_l)\leq\alpha_{n+1}\leq\delta,$ and hence $G_b(x_n,x_m,x_l)\leq\epsilon$. Therefore, (x_n) is a G_b-Cauchy sequence.
4) Part (4) is a special case of Part (3). □

4. Some fixed point results through Ω_b-distance mappings

We introduce in this section the notion of Ω_b-Banach contraction.

Definition 4.6. Let (X,G) be a G_b-metric space and Ω be an Ω_b-distance on X. A mapping $T: X \to X$ is called a Ω_b-*Banach contraction* if there is $k \in [0,1)$ such that

$$\Omega(Tx, T^2x, Ty) \leq k\Omega(x, Tx, y) \quad \text{for all} \quad x, y \in X, \text{with} \quad s \geq 1 \quad \text{and} \quad ks < 1.$$

Theorem 4.4. Let (X, G) be a complete G_b-metric space and Ω be a generalized Ω-distance on X with a constant $s > 1$ such that X is Ω-bounded. Let T be a self-mapping on X that satisfies the following conditions:

1) The function T is an Ω_b-Banach contraction,
2) for all $u \in X$ assume that if $Tu \neq u$, then $\inf\{\Omega_b(x, Tx, u): x \in X\} > 0$.

Then T has a unique fixed point in X.

Proof. Let $x_0 \in X$. We construct a sequence (x_n) in X such that $x_n = Tx_{n-1}$ $n \in \mathbb{N}$. Consider $t \geq 1$. Then by using the contraction condition, we get

$$\begin{aligned} \Omega(x_n, x_{n+1}, x_{n+t}) &= \Omega(Tx_{n-1}, Tx_n, Tx_{n+t-1}) \\ &\leq k \ \Omega(x_{n-1}, x_n, x_{n+t-1}) \\ &\leq k^2\Omega(x_{n-2}, x_{n-1}, x_{n+t-2}) \\ &\ \ \vdots \\ &\leq k^n\Omega(x_0, x_1, x_t). \end{aligned}$$

Since X is Ω-bounded, there exists $M > 0$ such that $\Omega(x, y, z) \leq M$ for all $x, y, z \in X$. Therefore,

$$\Omega(x_n, x_{n+1}, x_{n+t}) \leq k^n M. \tag{4.1}$$

Now, by using part (1) of the definition of Ω_b-distance and (4.1), we have for all $l \geq m > n$

$$\Omega(x_n, x_m, x_l) \leq s\Omega(x_n, x_{n+1}, x_{n+1}) + s\Omega(x_{n+1}, x_m, x_l)$$

$$\leq s\Omega(x_n, x_{n+1}, x_{n+1}) + s^2\Omega(x_{n+1}, x_{n+2}, x_{n+2}) + s^2\Omega(x_{n+2}, x_m, x_l)$$

$$\vdots$$

$$\leq s\Omega(x_n, x_{n+1}, x_{n+1}) + s^2\Omega(x_{n+1}, x_{n+2}, x_{n+2}) + \cdots$$

$$+s^{m-n-1}\Omega(x_{m-2}, x_{m-1}, x_{m-1}) + s^{m-n-1}\Omega(x_{m-1}, x_m, x_l)$$

$$\leq sk^nM + s^2k^{n+1}M + \cdots + s^{m-n-1}k^{m-2}M + s^{m-n-1}k^{m-1}M$$

$$\leq sk^nM + s^2k^{n+1}M + s^3k^{n+2}M + \cdots$$

$$= sk^nM(1 + sk + (sk)^2 + \cdots)$$

$$= sk^nM\frac{1}{1 - sk}.$$

Taking the limit as $n \to \infty$, we have

$$\lim_{n,m,l\to\infty} \Omega\,(x_n, x_m, x_l) = 0.$$

Hence, by Lemma 3.3, (x_n) is a G_b-Cauchy sequence. Therefore there is $u \in X$ such that x_n is G_b-convergent to u. For $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that $\Omega(x_n, x_m, x_l) \leq \epsilon,$ for all $n, m, l \geq N$. Therefore $\lim_{l\to\infty} \Omega\,(x_n, x_m, x_l) \leq \epsilon.$
Now, by the lower semi continuity of Ω in its third variable, we have

$$\Omega(x_n, x_m, u) \leq \lim_{p\to\infty} \Omega\,(x_n, x_m, x_p) \leq \epsilon, \text{ for all } m, n \geq N.$$

Let $m = n + 1$. Then $\Omega(x_n, x_{n+1}, u) \leq \lim_{p\to\infty} \Omega\,(x_n, x_{n+1}, x_p) \leq \epsilon,$ for all $n \geq N$. If $Tu \neq u,$ then for each $\epsilon > 0$, the second condition of Theorem 4.4 implies that

$$0 < \inf\{\,\Omega(x, Tx, u) \colon x \in X\} \leq \inf\{\,\Omega(x_n, x_{n+1}, u) \colon n \geq N\} \leq \epsilon,$$

which is a contradiction. Therefore $Tu = u$.

To prove the uniqueness, suppose that there exists $v \in X$ such that $Tv = v$. Then by the contraction condition, we have

$$\Omega(u, u, v) = \Omega(Tu, T^2u, Tv) \leq k\Omega(u, u, v).$$

Since $k < 1$, we have $\Omega(u, u, v) = 0$. Also, by the contraction condition, we have

$$\Omega(u, u, u) = \Omega(Tu, T^2u, Tu) \leq k\Omega(u, u, u).$$

Therefore $\Omega(u,u,u)=0$. Part (3) of the Definition 3.4 implies that $G(u,u,v)=0$ and hence $u=v$. □

Now, we introduce the notion of Ω_b-Kannan contraction.

Definition 4.7. Let (X,G) be a G_b-metric space and Ω be an Ω_b-distance on X. A mapping $T: X\to X$ is called an Ω_b-*Kannan contraction* if there is $k\in[0,\frac{1}{2})$ such that

$$\Omega(Tx,T^2x,Ty)\le k\ \ [\Omega(x,Tx,Tx)+\Omega(y,Ty,Ty)]$$

for all $x,y\in X$, with $s\ge 1$ and $ks<\frac{1}{2}$.

Theorem 4.5. Let (X,G) be a complete G_b-metric space and Ω be a generalized Ω-distance on X with a constant $s>1$ such that X is Ω-bounded. Let T be a self-mapping on X that satisfies the following conditions:

1) T is an Ω_b-Kannan contraction.
2) for all $u\in X$ assume that if $Tu\ne u$, then $\inf\{\Omega_b(x,Tx,u): x\in X\}>0$.

Then T has a fixed point in X.

Proof. For $x_0\in X$, we construct a sequence (x_n) in X such that $x_n=Tx_{n-1}, n\in\mathbb{N}$.
Consider $t\ge 1$. Then by using the contraction condition, we get

$$\begin{aligned}\Omega(x_n,x_{n+1},x_{n+t}) &= \Omega(Tx_{n-1},Tx_n,Tx_{n+t-1})\\ &\le k\ \ [\Omega(x_{n-1},x_n,x_n)+\Omega(x_{n+t-1},x_{n+t},x_{n+t}).\end{aligned}$$

Now,

$$\begin{aligned}\Omega(x_{n-1},x_n,x_n) &= \Omega(Tx_{n-2},T^2x_{n-2},Tx_{n-1})\\ &\le k\ \ [\Omega(x_{n-2},x_{n-1},x_{n-1})+\Omega(x_{n-1},x_n,x_n)].\end{aligned}$$

Therefore, $\Omega(x_{n-1},x_n,x_n)\le q\ \ \Omega(x_{n-2},x_{n-1},x_{n-1})$, where $q=\frac{k}{1-k}<1$.
By applying the previous steps repeatedly, we get $\Omega(x_{n-1},x_n,x_n)\le q^{n-1}\Omega(x_0,x_1,x_1)$.
Since X is Ω-bounded, there exists $M>0$ such that $\Omega(x,y,z)\le M$, for all $x,y,z\in X$. Hence

$$\Omega(x_{n-1},x_n,x_n)\le q^{n-1}M.$$

Also,

$$\begin{aligned}\Omega(x_{n+t-1},x_{n+t},x_{n+t}) &= \Omega(Tx_{n+t-2},T^2x_{n+t-2},Tx_{n+t-1})\\ &\le k\ \ [\Omega(x_{n+t-2},x_{n+t-1},x_{n+t-1})\\ &\qquad + \Omega(x_{n+t-1},x_{n+t},x_{n+t})].\end{aligned}$$

Therefore $\Omega(x_{n+t-1},x_{n+t},x_{n+t}) \le q\ \ \Omega(x_{n+t-2},x_{n+t-1},x_{n+t-1})$.

Applying the previous steps repeatedly, gives us

$$\Omega(x_{n+t-1},x_{n+t},x_{n+t}) \le q^{n-1}\Omega(x_t,x_{t+1},x_{t+1}).$$

Hence

$$\Omega(x_{n+t-1},x_{n+t},x_{n+t}) \le q^{n-1}M.$$

Thus $\Omega(x_n,x_{n+1},x_{n+t}) \le k[q^{n-1}M + q^{n-1}M] = 2kq^{n-1}M$.
Since $k < \frac{1}{2}$, we have

$$\Omega(x_n,x_{n+1},x_{n+t}) \le q^{n-1}M. \tag{4.2}$$

For all $l \ge m > n$, we have

$$\begin{aligned}\Omega(x_n,x_m,x_l) &\le s\Omega(x_n,x_{n+1},x_{n+1}) + s\Omega(x_{n+1},x_m,x_l)\\ &\le s\Omega(x_n,x_{n+1},x_{n+1}) + s^2\Omega(x_{n+1},x_{n+2},x_{n+2}) + s^2\Omega(x_{n+2},x_m,x_l)\\ &\ \ \vdots\\ &\le s\Omega(x_n,x_{n+1},x_{n+1}) + s^2\Omega(x_{n+1},x_{n+2},x_{n+2}) + \cdots\\ &\quad + s^{m-n-1}\Omega(x_{m-2},x_{m-1},x_{m-1}) + s^{m-n-1}\Omega(x_{m-1},x_m,x_l)\\ &\le sq^{n-1}M + s^2q^nM + \cdots + s^{m-n-1}q^{m-3}M + s^{m-n-1}q^{m-2}M\\ &\le sq^{n-1}M + s^2q^nM + s^3q^{n+1}M + \cdots\\ &= sq^{n-1}M(1 + sq + (sq)^2 + \cdots)\\ &= sq^{n-1}M\frac{1}{1-sq}.\end{aligned}$$

Taking the limit as $n \to \infty$, gives us

$$\lim_{n,m,l\to\infty} \Omega\,(x_n,x_m,x_l) = 0.$$

Lemma 3.3, implies that the sequence (x_n) is a G_b-Cauchy sequence. Therefore there exits $u \in X$ such that x_n is G_b-convergent to u. Consider $\epsilon > 0$. Then there exists $N \in \mathbb{N}$ such that $\Omega(x_n, x_m, x_l) \leq \epsilon$, for all $n, m, l \geq N$ and thus $\lim_{l\to\infty} \Omega(x_n, x_m, x_l) \leq \epsilon$. Note that $\Omega(x_n, x_m, u) \leq \lim_{p\to\infty} \Omega(x_n, x_m, x_p) \leq \epsilon$, for all $m, n \geq N$. Let $m = n + 1$. Then $\Omega(x_n, x_{n+1}, u) \leq \lim_{p\to\infty} \Omega(x_n, x_{n+1}, x_p) \leq \epsilon$, for all $n \geq N$.

If $Tu \neq u$, then the second condition of the theorem implies that

$$0 < \inf\{\Omega(x, Tx, u): x \in X\} \leq \inf\{\Omega(x_n, x_{n+1}, u): n \geq N\} \leq \epsilon,$$

for each $\epsilon > 0$ which is a contradiction. Therefore $Tu = u$.

To prove the uniqueness, assume that there exist two fixed points z, v of T

The contraction condition yields

$$\begin{aligned}\Omega(z,z,z) &= \Omega(Tz, T^2z, Tz)\\ &\leq k\ [\Omega(z,z,z) + \Omega(z,z,z)]\\ &= 2k\ \Omega(z,z,z).\end{aligned}$$

Since $k < \frac{1}{2}$, we have $\Omega(z,z,z) = 0$. Similarly, we have $\Omega(v,v,v) = 0$.

Now,

$$\begin{aligned}\Omega(u,u,v) &= \Omega(Tu, T^2u, Tv)\\ &\leq k\ [\Omega(u,u,u) + \Omega(v,v,v).\\ &= k\ [0+0] = 0.\end{aligned}$$

Therefore, Definition 3.4 we have $G(u,u,v) = 0$ and hence $u = v$. □

R E F E R E N C E S

[1*] I. A. Bakhtin*, The contraction principle in quasimetric spaces, Functional Analysis, vol30, pp. 26–37 (1989)

[2]*S. Czerwik*, Nonlinear set-valued contraction mappings in b-metric spaces. AttiSemin. Mat. Fis. Univ. Modea*n* 46, 263–276 (1998).

[3] *S. Czerwik*, Contraction mappings in b-metric spaces, Acta Mathematica et InformaticaUniversitatisOstraviensis, Vol. 1 (1993), No. 1, 5–11

[4]*J. R. Roshan, N. Shobkolaei , S. Sedghi and M. Abbas*, Common fixed point of four maps in b-metric spaces, Hacettepe Journal of Mathematics and Statistics Volume 43 (4) (2014), 613–624

[5]*A. K. Dubey, R. Shukla, and R.P. Dubey*, Some Fixed Point Theorems in Complex Valued b-Metric Spaces, Journal of Complex Systems Volume 2015 (2015), Article ID 832467, 7 pages

[6]*R. George, C. Alaca, K.P. Reshma*, On Best Proximity Points In b-Metric Space, Journal of Nonlinear Analysis and Application Volume 2015, No. 1 (2015), Pages 45–56

[7]*W. Shatanawi1, A.Pitea and R.Lazovic*, Contraction conditions using comparison functions on b-metric spaces, Fixed Point Theory and Applications 2014, 2014:135
[8] *S.Sedghi, N.Shobkolaei, J. R. Roshan and W.Shatanawi*, Coupled fixed point theoremsin Gb-metric spaces, MathematiqkiVesnik, 66, 2 (2014), 190–201
[9]*Z. Mustafa and B. Sims*, A new approach to generalized metric spaces, *J. Nonlinear Convex Anal.*,Volum. 7, no. 2, (2006) 289-297.
[10*] Z. Mustafa and B. Sims*, Fixed point theorems for contractive mappings in complete G-metric spaces, *Fixed Point Theory Appl.*, (2009), ID 917175, 10 pages
[11] *H. Aydi, W. Shatanawi, and G. Vetro*, On generalized weakly G-contraction mapping in G-metric spaces, *Comput. Math. Appl.*, 62(2011).4222-4229.
[12] *W. Shatanawi, S. Chauhan, M. Postolache, M*. Abbas and S. Radenovic, common fixed points for contractive mappings of integral type in G-metric spaces, J. Adv. Math. Stud. Vol. 6, No. 1, (2013) 53–72.
[13] *W.Shatanawi and M.Postolache*, Some fixed point results for a G-weak contraction in G-metric spaces, Abstract and Applied Analysis, Vol. **2012**, (2012) Article ID 815870, 19 pages.
[14]*W. Shatanawi*, Some fixed point theorems in ordered G-metric spaces and applications, Fixed Point Theory and Appl., vol. **2011** , Article ID 126205, 11 pages, (2011).
[15]*W. Shatanawi*, Fixed point theory for contractive mappings satisfying Φ-maps in G-metric spaces, Fixed Point Theory and Appl., vol. **2010**, Article ID 181650,9 pages, (2010).
[16*] M. Jleli, B. Samet*, Remarks on G-metric spaces and fixed point theorems, Fixed Point Theory Appl., 2012 (2012), 7 pages.
[17] *R. Saadati, S.M. Vaezpour, P. Vetro and B.E. Rhoades*, Fixed point theorems in generalized partially ordered G-metric spaces. Mathematical and Computer Modeling**52** (797-801) 2010.
[18]*O. Kada, T. Suzuki and W. Takahashi*, Nonconvex minimization theorems and fixed point theorems in complete metric spaces, *Math. Japonica*,**44**, 381–391, (1996).
[19] *L. Gholizadeh, R.Saadati, W.Shatanawi, and Vaezpour*, Contractive mapping in generalized, ordered metric spaces with application in integral equations, Math. Probl. Eng. 2011, Article ID 380784 (2011)
[20]*L. Gholizadeh*, A fixed point theorem in generalized ordered metric spaces with application, J. Nonlinear Sci. Appl. 6 (2013), 244–251.
[21] *W. Shatanawi and A. Pitea*, Fixed and coupled fixed point theorems of omega-distance for nonlinear contraction, Fixed Point Theory and Appl. 2013, 2013:275.
[22] *W. Shatanawi, A. Bataihah and A. Pitea*, Fixed and common fixed point results for cyclic mappings of Ω-distance, *J*. Nonlinear Sci. Appl. 9 (2016), 727–735.
[23] *W. Shatanawi, and A. Pitea*, Ω-Distance and coupled fixed point in G-metric spaces, Fixed Point Theory and Appl. 2013, 2013:208
[24] *A. Aghajani, M. Abbas and J.R. Roshan*, Common fixed point of generalized weak contractive mappings in partially ordered G_b-metric spaces, Filomate28:6 (2014) 1087–1101.
[25] *K. Abodayeh, W. Shatanawi and A. Bataihah*, Fixed point theorem through Ω-distance of Suzuki type contraction condition, *Gazi University Journal of Science.* (2016) **29** (1), 129-133.