COMPLEX SYSTEM DYNAMICS THROUGH A FRACTAL PARADIGM

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Assimilating the complex with a fractal, non – differentiable behaviors in their dynamics are analyzed through a fractal paradigm. It results that complex system dynamics in the framework of hydrodynamic – type fractal regimes imply "holographic implementation" of the velocity fields at non – differentiable scale resolution, by means of fractal solitons, fractal solitons – fractal kinks and fractal minimal vortices. These vortices become turbulence sources in complex systems dynamics at non – differentiable scale resolutions.

Keywords: complex systems, non – differentiability, fractal hydrodynamic regimes, fractal paradigm

1. Introduction

Complex systems are large interdisciplinary research topics that have been studied by means of a combination of basic theory, derived especially from physics and computer simulation. Such kind of systems are composed of many interacting entities that were called „agents" (structural units). Examples of complex systems can be found in human societies, the brain, internet, ecosystems, biological evolution, stock markets, economies and many others [1-3]. On the same topic, probably one of the most intriguing complex systems in nature is DNA, who creates

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cells by means of a simple but very elegant language. It is responsible for the remarkable way in which individual cells organize into complex systems like organs and these organs form even more complex systems like organisms.

The way in which such a system manifests can't be predicted only by the behavior of individual elements or by adding their behavior, but is determined by the manner in which the elements relate to influence global behavior. Among the most significant properties of complex systems are emergence, self-organization, adaptability etc. [4].

Usually, models used to describe complex system dynamics are based on the uncertain hypothesis that the variables describing it are differentiable [5-7]. The success of these models must be understood gradually on domains in which differentiability is still valid. However, the differential procedures are not suitable when describing processes related to complex system dynamics, which imply nonlinearity and chaos (it is reminded that this is the de – facto case [8-11]).

Since the non-differentiability appears as a universal property of the complex systems, it is necessary to construct a non-differentiable physics. In such conjecture, by considering that the complexity of the interactions processes is replaced by non-differentiability, it is no longer necessary to use the whole classical "arsenal" of quantities from the standard physics (differentiable physics).

Therefore, in order to describe complex system dynamics by remaining faithful to the differentiable mathematical procedures, it is necessary to employ a fractal paradigm, which explicitly introduces scale resolutions, both in the expression of the physical variables and in the fundamental equations which govern complex system dynamics. This means that, instead of "working" with a single physical variable described by a strict non-differentiable function, it is possible to "work" only with approximations of these mathematical functions obtained by averaging them on different scale resolutions. As a consequence, any physical variable purposed to describe complex system dynamics will perform as the limit of a family of mathematical functions, this being non – differentiable for null scale resolutions and differentiable otherwise [8, 11].

In the present paper, considering the fractal paradigm as being functional, a non – differentiable model describing the complex system dynamics is proposed.

2. Mathematical Model

2.1. Scale covariant derivative and geodesics equation

Our fundamental hypothesis is the one that the structural units' dynamics of any complex system are described by continuous but non-differentiable curves (fractal curves). Indeed, such an assumption is sustained by the following example, related to the collision processes in a complex fluid: between two successive collisions, the trajectory of the complex fluid structural unit is a straight line that becomes non – differentiable in the impact point. Considering that all the collision impact points form an uncountable set of points, it results that the trajectories of the complex fluid structural unit become continuous and non – differentiable curves, i.e. fractal curves. Obviously, the reality is much more complicated, taking into account both the diversity of particles which compose a complex fluid, and the various interactions between them, in the form of double / triple collisions etc. Then, the complex system's structural units trajectories become multifractal.

In such a context, the dynamics of the complex system structural units become operational through the scale covariant derivative [9, 10]:

$$\frac{\hat{d}}{dt} = \partial_t + \hat{V}^l \partial_l + \frac{1}{4}(dt)^{\left(\frac{2}{D_f}\right)-1} D^{lp} \partial_l \partial_p, \tag{1}$$

where

$$\begin{gathered} \hat{V}^l = V_D^l - V_F^l \\ D^{lp} = d^{lp} - i\hat{d}^{lp} \\ d^{lp} = \lambda_+^l \lambda_+^p - \lambda_-^l \lambda_-^p \\ \hat{d}^{lp} = \lambda_+^l \lambda_+^p + \lambda_-^l \lambda_-^p \\ \partial_t = \frac{\partial}{\partial t}, \partial_l = \frac{\partial}{\partial x^l}, \partial_l \partial_p = \frac{\partial}{\partial x^l}\frac{\partial}{\partial x^p}, i = \sqrt{-1}, l, p = 1,2,3 \end{gathered} \tag{2}$$

In the above – written relations, x^l is the fractal spatial coordinate, t is the non – fractal time having the role of an affine parameter of the motion curves, $\hat{V}^l$ is the complex velocity, V_D^l is the differential velocity independent on the scale resolution dt, V_F^l is the non – differentiable velocity dependent on the scale resolution, D_F is the fractal dimension of the movement curve, D^{lp} is the constant tensor associated with the differentiable – non – differentiable transition, $\lambda_+^l(\lambda_+^p)$ is the constant vector associated with the backward differentiable – non – differentiable physical processes and $\lambda_-^l(\lambda_-^p)$ is the constant vector associated with the forward differentiable – non – differentiable physical processes. There are many modes, and thus a varied selection of definitions of fractal dimensions: more precisely, the fractal dimension in the sense of Kolmogorov, the fractal dimension in the sense of Hausdorff – Besikovitch etc. [11-13]. Selecting one of these definitions and operating it in the complex system dynamics, the value of the fractal dimension must be constant and arbitrary for the entirety of the dynamical analysis: for example, it is regularly found $D_F < 2$ for correlative processes, $D_F > 2$ for non – correlative processes, etc. [8, 12, 13].

Now, accepting the functionality of the scale covariance principle i.e. applying the operator (1) to the complex velocity field from (2), in the absence of any external constraint, the motion equations (i.e. the geodesics equation on a fractal space) takes the following form [8, 9]:

$$\frac{\hat{d}\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \partial_l \hat{V}^i + \frac{1}{4}(dt)^{\left(\frac{2}{D_f}\right)-1} D^{lk} \partial_l \partial_k \hat{V}^i = 0, \tag{3}$$

This means that the fractal acceleration $\partial_t \hat{V}^i$, the fractal convection $\hat{V}^l \partial_l \hat{V}^i$ and the fractal dissipation $D^{lk}\partial_l \partial_k \hat{V}^i$, make their balance in any point of the fractal curve.

If the fractalisation is achieved by Markov – type stochastic processes [11 – 13], then:

$$\lambda_+^i \lambda_+^l = \lambda_-^i \lambda_-^l = 2\lambda \delta^{il}, \tag{4}$$

where λ is a coefficient associated to the differentiable – non – differentiable transition and δ^{il} is Kronecker's pseudo – tensor.

Under these conditions, the geodesics equation (3) takes the simple form:

$$\frac{\hat{d}\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \partial_l \hat{V}^i - i\lambda (dt)^{\left(\frac{2}{D_f}\right)-1} \partial^l \partial_l \hat{V}^i = 0 \tag{5}$$

2.2. Dynamics of complex systems in the form of hydrodynamic – type fractal "regimes".

The separation of the complex system's dynamics on scale resolutions implies, through (5), both the conservation law of the specific fractal momentum at differentiable scale resolution:

$$\frac{\partial V_D^i}{dt} = \partial_t V_D^i + V_D^l \partial_l V_D^i - \left[V_F^l + \lambda (dt)^{\left(\frac{2}{D_f}\right)-1} \partial_l\right] \partial^l V_F^i = 0, \tag{6}$$

and also the conservation laws of the specific momentum at non – differentiable scale resolutions:

$$\frac{\partial V_F^i}{dt} = \partial_t V_F^i + V_D^l \partial_l V_F^i + \left[V_F^l + \lambda (dt)^{\left(\frac{2}{D_f}\right)-1} \partial_l\right] \partial^l V_D^i = 0, \tag{7}$$

From (6) it results the specific fractal force:

$$f_F^i = \left[V_F^l + \lambda (dt)^{\left(\frac{2}{D_f}\right)-1} \partial_l\right] \partial^l V_F^i, \tag{8}$$

induced by the velocity fields V_F^i which is a "measure" of non – differentiability of motion curves of complex system entities.

In the case of stationary complex system dynamics $\left(\partial_t V_D^i = 0, \partial_t V_F^i = 0\right)$, the conservation laws (6), (7) become:

$$V_D^l \partial_l V_D^i - \left[V_F^l + \lambda (dt)^{\left(\frac{2}{D_f}\right)-1} \partial_l\right] \partial^l V_F^i = 0, \tag{9}$$

$$V_D^l \partial_l V_F^i + \left[V_F^l + \lambda (dt)^{\left(\frac{2}{D_f}\right)-1} \partial_l \right] \partial^l V_D^i = 0, \tag{10}$$

while in the static case ($\partial_t V_D^i = 0, V_D^i = 0, \partial_t V_F^i = 0$) these take the form:

$$\left[V_F^l + \lambda (dt)^{\left(\frac{2}{D_f}\right)-1} \partial_l \right] \partial^l V_F^i = 0, \tag{11}$$

The result (11) specifies that, although at differentiable scale resolution, the complex system dynamics are absent while, at the non – differentiable scale resolution, the complex system dynamics can be "dictated" by the hydrodynamic fractal – type equations:

$$V_F^l \partial_l V_F^i + \lambda (dt)^{\left(\frac{2}{D_f}\right)-1} \partial_l \partial^l V_F^i = 0 \tag{12}$$

$$\partial_l V_F^l = 0. \tag{13}$$

Equation (13) corresponds to the fractal fluid incompressibility at the non – differentiable scale resolution (i.e. the states' density ρ at the non – differentiable scale resolution is constant).

Generally, it is difficult to obtain an analytical solution for the previous equation system, taking into account its non – linear nature. However, it is still possible to obtain an analytic solution in the case of plane symmetry (for example, in (x, y) coordinates) of the complex system dynamics. In such a context, let it be considered the equations system (12) and (13) in the form:

$$U_0 \partial_x U_0 + V_0 \partial_y U_0 = \sigma_0 \partial_{yy}^2 U_0, \tag{14}$$

$$\partial_x U_0 + \partial_y V_0 = 0, \tag{15}$$

where:

$$V_{Fx} = U_0(x,y), \qquad V_{Fy} = V_0(x,y), \qquad \sigma_0 = \lambda (dt)^{\left(\frac{2}{D_f}\right)-1} \tag{16}$$

Imposing the following restrictions:

$$\lim_{y\to 0} V_0(x,y) = 0, \lim_{y\to 0} \frac{\partial U_0}{\partial y} = 0, \lim_{y\to\infty} U_0(x,y) = 0, \tag{17}$$

and considering the constant flux moment per unit of depth:

$$Q = \rho \int_{-\infty}^{+\infty} U_0^2 dy = const., \tag{18}$$

the velocity fields as solution of the equations system ((14), (15)) takes the form (for details on the similarities method, see [14, 15]):

$$U_0 = \frac{1.5 \left(\frac{Q}{6\rho}\right)^{\frac{2}{3}}}{(\sigma_0 x)^{\frac{1}{3}}} \operatorname{sech}^2 \left[\frac{0.5 y \left(\frac{Q}{6\rho}\right)^{\frac{1}{3}}}{(\sigma_0 x)^{\frac{2}{3}}} \right], \tag{19}$$

$$V_0 = \frac{1.9\left(\frac{Q}{6\rho}\right)^{\frac{2}{3}}}{(\sigma_0 x)^{\frac{1}{3}}}\left\{\frac{y\left(\frac{Q}{6\rho}\right)^{\frac{1}{3}}}{(\sigma_0 x)^{\frac{2}{3}}}\operatorname{sech}^2\left[\frac{0.5y\left(\frac{Q}{6\rho}\right)^{\frac{1}{3}}}{(\sigma_0 x)^{\frac{2}{3}}}\right] - \tanh\left[\frac{0.5y\left(\frac{Q}{6\rho}\right)^{\frac{1}{3}}}{(\sigma_0 x)^{\frac{2}{3}}}\right]\right\}, \qquad (20)$$

The previous equations are simplified through non – dimensional variables and non – dimensional parameters:

$$X = \frac{x}{x_0}, Y = \frac{y}{y_0}, U = \frac{U_0}{w_0}, V = \frac{V_0}{w_0}, \qquad (21)$$

$$\mu = \frac{\sigma_0}{\upsilon_0}, \upsilon_0 = \frac{y_0^{\frac{3}{2}}}{x_0}\left(\frac{Q}{6\rho}\right)^{\frac{1}{2}}, w_0 = \frac{1}{(y_0)^{\frac{1}{2}}}\left(\frac{Q}{6\rho}\right)^{\frac{1}{2}}, \qquad (22)$$

where x_0, y_0, w_0 and ν_0 represent specific lengths, specific velocity and "fractal degree" of the complex system dynamics. In these conditions, the normalized velocity fields become:

$$U = \frac{1.5}{(\mu X)^{\frac{1}{3}}}\operatorname{sech}^2\left[\frac{0.5Y}{(\mu X)^{\frac{2}{3}}}\right], \qquad (23)$$

$$V = \frac{1.9}{(\mu X)^{\frac{1}{3}}}\left\{\frac{Y}{(\mu X)^{\frac{2}{3}}}\operatorname{sech}^2\left[\frac{0.5Y}{(\mu X)^{\frac{2}{3}}}\right] - \tanh\left[\frac{0.5Y}{(\mu X)^{\frac{2}{3}}}\right]\right\}, \qquad (24)$$

Any of the above relations specify the nonlinearity of the velocity fields: fractal soliton (i.e. soliton depending on non – differentiable scale resolution) for the velocity field across the Ox axis, respectively "mixtures" of fractal soliton – fractal kink (i.e. kink dependent on non – differentiable scale resolution), for the velocity fields across the Ox axis. The specificities in the complex system dynamics are "explained" in Figures 1a – d, and 2a – d. Details on the soliton, kink and other classical nonlinear solutions are given in [12, 13].

The velocity fields (23) and (24) induce the fractal minimal vortex Ω as in the following expression (Figures 3a-d).

$$\Omega = \left(\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial Y}\right) = \frac{0.57Y}{(\mu X)^2} + \frac{0.63\mu}{(\mu X)^{\frac{4}{3}}} \tanh\left[\frac{0.5Y}{(\mu X)^{\frac{2}{3}}}\right] + \frac{1.9Y}{(\mu X)^2} \mathrm{sech}^2\left[\frac{0.5Y}{(\mu X)^{\frac{2}{3}}}\right] - \frac{0.57Y}{(\mu X)^2} \tanh^2\left[\frac{0.5Y}{(\mu X)^{\frac{2}{3}}}\right] - \left[\frac{1.5}{\mu X} + \frac{1.4Y^2}{X(\mu X)^{\frac{5}{3}}}\right] \mathrm{sech}^2\left[\frac{0.5Y}{(\mu X)^{\frac{2}{3}}}\right] \tanh\left[\frac{0.5Y}{(\mu X)^{\frac{2}{3}}}\right], \qquad (25)$$

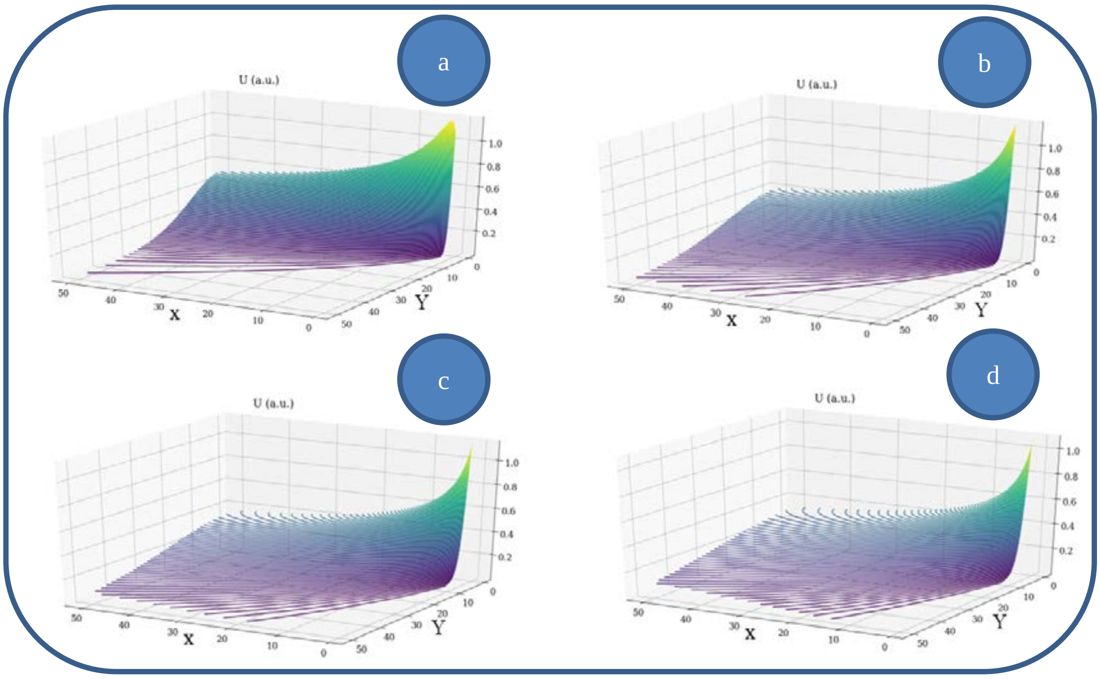

Fig. 1. Normalized velocity field U for various fractal degrees: a) $\mu = 0.45$; b) $\mu = 1$; c) $\mu = 1.55$; d) $\mu = 2.70$

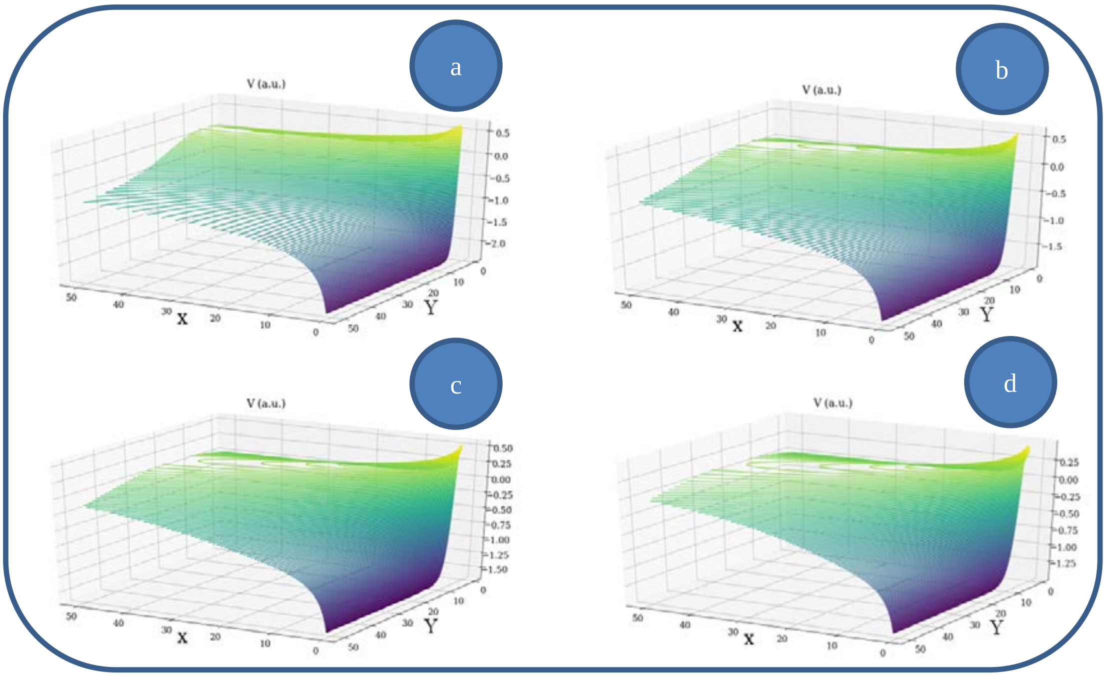

Fig. 2. Normalized velocity field V for various fractal degrees: a) $\mu = 0.45$; b) $\mu = 1$; c) $\mu = 1.55$; d) $\mu = 2.70$

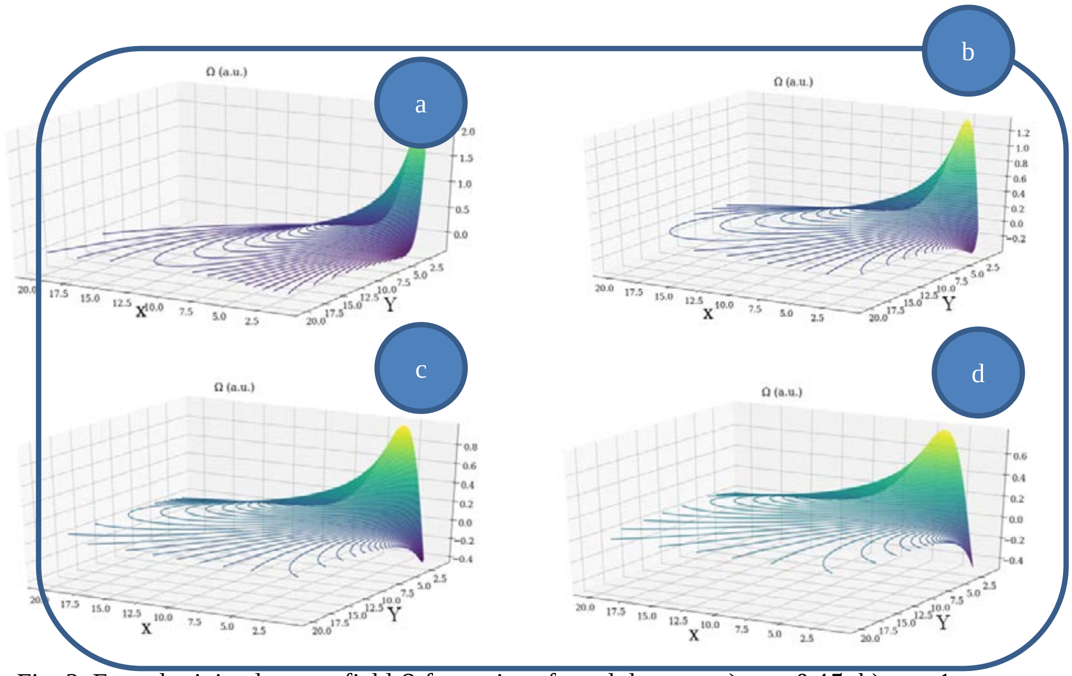

Fig. 3. Fractal minimal vortex field Ω for various fractal degrees: a) $\mu = 0.45$; b) $\mu = 1$; c) $\mu = 1.55$; d) $\mu = 2.70$

This previous result was used to specify the fact that the turbulence sources [16-18] may be induced by fractal vortices. Such turbulences can appear in many complex systems and phenomena, from microscales to macroscales (for details see [19-22]).

3. Conclusions

The main conclusions of the present paper are the following:

Assimilating the complex sytem with a fractal, dynamics at non-differentiable scale resolution are analyzed. The following results have been obtained:

i) The complex system dynamics in the framework of hydrodynamic – type fractal regimes, specify velocity fields at non – differentiable scale resolution, in the form of fractal solitons, fractal solitons – fractal kinks and fractal minimal vortices.

ii) The fractal vortices can be linked to turbulence sources in complex systems dynamics at non – differentiable scale resolutions. As long as the complex system is not constrained externally, fractal vortices do not manifest themselves. In other words, they are “virtual” fractal vortices and manifest as “virtual” turbulence sources. In the presence of an external constraint, they become “real” and the turbulence mechanism is triggered. Essentially, the discussion revolves around “holographic implementation” of turbulences in the complex system dynamics at non-differentiable scale resolution. Since the dynamics of complex system entities are described by continuous but non – differentiable curves, curves which exhibit the property of self – similarity in every one of its points, these can be viewed as a holographic mechanism (every part reflects the whole) of the dynamics description.

Authors’ contributions:

All authors have equally contributed to this paper.

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