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ON THE ELECTRODYNAMICAL RELATIVITY

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Pentru două sisteme neinerţiale aflate în mişcare relativă unul faţă de celălalt (unul inductor şi altul indus), matricile Lorentz rezultă prin transformarea mărimilor specifice câmpului electromagnetic şi a operatorilor de derivare de la un sistem la celălalt. Folosind expresia maticială pentru valorile componentelor paralelă şi normală a vitezei relative, se pot obţine în formă explicită, ecuaţiile covariant tensoriale. Pentru mişcare uniformă a unui sistem inerţial, folosind bune aproximaţii ale mărimilor câmpului electromagnetic este stabilită o formă distinctă a transformării cinematice Galilei respectiv "anti-Galilei". Relaţiile cinematice dintre operatorii spaţio-temporari de derivare rezultă ca subordonaţi proprietăţilor câmpului electromagnetic.

For a pair of non-inertial systems (inductor and induced) in relative non-uniform movement or relative rest of each system, Lorentz matrices result by transition from own electromagnetic values and space–time derivative operators of one system to another. Using the matrix expressions for the values of the parallel and normal components to the relative velocity intrinsecal, manifest covariant tensorial equations are obtained. For uniform movement of the inertial systems, using some good approximations of the electromagnetic quantities is established a peculiar form of the kinematic transformation Galilei or "anti-Galilei". The kinematic relations between the time-space derivative operators result as subordinated to the electromagnetic field properties.

Keywords: bitensors, bivectors, space-time operators, covariant matrix equation .

Introduction

The relativity theory of Einstein and Minkowski, based on the principle of inertial systems equivalence and Lorentz's kinematic transformation, was born from the necessity of agreement between the theory of moving mediums and experimental observations.

Although Lorentz's transformation was found in electrodynamics [1], later the electrodynamic phenomena were explained by kinematic relations [2, 3]. But relativistic aspects of pre-relativistic electrodynamics were rendered manifest without kinematical relations [4, 5], Lorentz's transformation resulting from the

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covariance of a differential equation obtained through time-space derivative operators.

In the following we use the principle of electromagnetic state dependence of reference systems attached to the bodies on their relative motion, each system being able to be considered either as fixed or in relative movement.

Faraday's experiences rendered manifest the importance of the relative movement between the inductor and the induced system, this phenomenon is well known today from the theory and practice of electrical machines.

Using the components of vector state physical values parallel and normal to the speed vector for passing from the electromagnetic field values and also from the suitable space-time derivative operators associated to a system to the quantities of the other system, we have obtained Lorentz matrices without any restrictive hypotheses regarding the space-time dependence of the relative speed.

The tensorial state and evolution equations of Maxwell-Minkowski are presented as matrices in intrinsic, intuitive and manifest covariant form; in this way we have avoided the abstract symbolism with indices, elegant but unfriendly [6], considered also a "mathematical trick" [7].

For the inertial systems in the relativistic electrodynamics, taking the magnetic flux density and electric displacement as absolute magnitudes the Galilei transformation results (absolut time) if the constitutive relation are not used.

If the constitutive relations are used in the hypothesis of absolute magnetic flux density and relative electric displacement, a kinematic transformation "anti-Galilei" is obtained (absolute space and relative time), so that is justified the subordination of the kinematic properties to the electrodynamic ones, and not vice-versa.

1. Relativistic transformation of electromagnetic physical values

Let be a pair of referential systems S and S^0 attached to the moving bodies in relative movement ("reciprocal systems"): the system S is moving with the speed v relative to the system S_0, and system S^0 is moving with the speed $\mathbf{v}^0 = -\mathbf{v}$ relative to the system S.

Decomposing the electric and magnetic field vectors into parallel component (index p) and normal component (index n) to the speed vector (generally non constant), for the non-inertial systems we will have the following classical equations:

$$\begin{bmatrix} \mathbf{E}_\mathrm{p} \\ \mathbf{B}_\mathrm{p} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{E}_\mathrm{p}^0 \\ \mathbf{B}_\mathrm{p}^0 \end{bmatrix}, \qquad \begin{bmatrix} \mathbf{H}_\mathrm{p} \\ \mathbf{D}_\mathrm{p} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{H}_\mathrm{p}^0 \\ \mathbf{D}_\mathrm{p}^0 \end{bmatrix}, \tag{1}$$

$$\begin{bmatrix}\mathbf{E}_n\\ \mathbf{B}_n\end{bmatrix}=\begin{bmatrix}1 & \mathbf{v}\times\\ -\dfrac{\mathbf{v}}{c^2}\times & 1\end{bmatrix}\begin{bmatrix}\mathbf{E}_n^0\\ \mathbf{B}_n^0\end{bmatrix},\quad \begin{bmatrix}\mathbf{H}_n\\ \mathbf{D}_n\end{bmatrix}=\begin{bmatrix}1 & -\mathbf{v}\times\\ \dfrac{\mathbf{v}}{c^2}\times & 1\end{bmatrix}\begin{bmatrix}\mathbf{H}_n^0\\ \mathbf{D}_n^0\end{bmatrix}, \tag{2}$$

$$\begin{bmatrix}\mathbf{E}_n^0\\ \mathbf{B}_n^0\end{bmatrix}=\begin{bmatrix}1 & \mathbf{v}^0\times\\ -\dfrac{\mathbf{v}^0}{c^2}\times & 1\end{bmatrix}\begin{bmatrix}\mathbf{E}_n\\ \mathbf{B}_n\end{bmatrix},\quad \begin{bmatrix}\mathbf{H}_n^0\\ \mathbf{D}_n^0\end{bmatrix}=\begin{bmatrix}1 & -\mathbf{v}^0\times\\ \dfrac{\mathbf{v}^0}{c^2}\times & 1\end{bmatrix}\begin{bmatrix}\mathbf{H}_n\\ \mathbf{D}_n\end{bmatrix}. \tag{3}$$

If after a motion of system S with the speed v relative to S^0 is considered also a motion of the system S^0 with the speed $\mathbf{v}^0 = -\mathbf{v}$ relative to the system S, the initial state must be re-established and so the above mentioned matrices must be in an inversion relation. This means that their multiplication must be equal to the unit matrix. Since $\left(-\mathbf{v}\times\mathbf{v}^0/c^2\right)\times\mathbf{F}_n^0 \equiv \mathbf{v}\mathbf{v}^0/c^2\,\mathbf{F}_n^0$, because $\mathbf{v}\mathbf{F}_n^0 = -\mathbf{v}^0\mathbf{F}_n^0 = 0$, a factor different of unit is obtained.

$$\begin{bmatrix}1 & \mathbf{v}\times\\ -\mathbf{v}/c^2\times & 1\end{bmatrix}\begin{bmatrix}1 & \mathbf{v}^0\times\\ -\mathbf{v}^0/c^2\times & 1\end{bmatrix}=\begin{bmatrix}1+\mathbf{v}\mathbf{v}^0/c^2\times & 0\\ 0 & 1+\mathbf{v}\mathbf{v}^0/c^2\times\end{bmatrix}= \\ =\left(1+\mathbf{v}\mathbf{v}^0/c^2\times\right)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}. \tag{4}$$

For removing this factor $1+\mathbf{v}\mathbf{v}^0/c^2 = 1-v^2/c^2 = 1-v^{0\,2}/c^2$, symmetrically is attached to each matrix a norming factor $\gamma=\left(1-v^{0\,2}/c^2\right)^{-1/2}$, specific to the Lorentz matrix.

Instead of matrices from relations (2) and (3), that contain vector products of speeds, it can be used a matrix with algebraic values of the speed in order to transform some bi-tensors of the electromagnetic field as follows:

$$\begin{bmatrix}\mathbf{E}_n\\ \mathbf{i}\times\mathbf{B}_n\end{bmatrix}=\gamma\begin{bmatrix}1 & v\\ v/c^2 & 1\end{bmatrix}\begin{bmatrix}\mathbf{E}_n^0\\ \mathbf{i}\times\mathbf{B}_n^0\end{bmatrix},\quad \begin{bmatrix}\mathbf{H}_n\\ \mathbf{i}\times\mathbf{D}_n\end{bmatrix}=\gamma\begin{bmatrix}1 & -v\\ -v/c^2 & 1\end{bmatrix}\begin{bmatrix}\mathbf{H}_n^0\\ \mathbf{i}\times\mathbf{D}_n^0\end{bmatrix}, \tag{5}$$

$$\begin{bmatrix}\mathbf{i}\times\mathbf{E}_n\\ \mathbf{B}_n\end{bmatrix}=\gamma\begin{bmatrix}1 & -v\\ -v/c^2 & 1\end{bmatrix}\begin{bmatrix}\mathbf{i}\times\mathbf{E}_n^0\\ \mathbf{B}_n^0\end{bmatrix},\quad \begin{bmatrix}\mathbf{i}\times\mathbf{H}_n\\ \mathbf{D}_n\end{bmatrix}=\gamma\begin{bmatrix}1 & v\\ v/c^2 & 1\end{bmatrix}\begin{bmatrix}\mathbf{i}\times\mathbf{H}_n^0\\ \mathbf{D}_n^0\end{bmatrix}. \tag{6}$$

Here **i** is the unit vector of the speed orientation $\mathbf{v} = \mathbf{i}v = -\mathbf{i}v^0$.

Similar equations can be obtained also for other physical values (polarization vector **P** and magnetization **M**), between which are similar relations.

The tensor relations (5) and (6) written above in an intrinsic form, are identical with the equations between the essential components of electromagnetic field tensors grouped explicitly in matrices with bi-vector properties [5].

Similar transformation matrices result also for the bi-vector components [**J**n ρ] of the current density quadri-vector **J** = **J**n + **J**p and electric charge density ρ [5].

$$\mathbf{J}_{\mathrm{n}} = \mathbf{J}_{\mathrm{n}}^{0}, \quad \begin{bmatrix} \mathbf{J}_{\mathrm{p}} \\ \rho \end{bmatrix} = \gamma \begin{bmatrix} 1 & -v \\ -v/c^{2} & 1 \end{bmatrix} \begin{bmatrix} J_{\mathrm{p}}^{0} \\ \rho^{0} \end{bmatrix}, \quad \begin{bmatrix} \mathbf{J}_{\mathrm{p}}^{0} \\ \rho \end{bmatrix} = \gamma \begin{bmatrix} 1 & v \\ v/c^{2} & 1 \end{bmatrix} \begin{bmatrix} J_{\mathrm{p}} \\ \rho \end{bmatrix}. \quad (7)$$

2. The relativistic transformation of the derivatives and space-time differential operators

This transformation is obtained by imposing the covariance property to a space-time differential equation, from electrodynamics.

The expression of electric charge conservation law

$$\nabla \mathbf{J} + \frac{\partial \rho}{\partial t} = 0, \quad (8)$$

with the operator $\nabla = \nabla_{\mathrm{n}} + \nabla_{\mathrm{p}} = \nabla_{\mathrm{n}} + \mathbf{i}\frac{\partial}{\partial x}$ is written in matrix form as

$$\begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial t} \end{bmatrix} \begin{bmatrix} \mathbf{J}_{p} \\ \rho \end{bmatrix} + \nabla_{n} \mathbf{J}_{n} = 0. \quad (9)$$

Because $\mathbf{J}_{\mathrm{n}} = \mathbf{J}_{\mathrm{n}}^{0}$ and $\nabla_{\mathrm{n}} = \nabla_{\mathrm{n}}^{0}$, after the substitution of bi-vector $[\mathbf{J}_{\mathrm{n}} \quad \rho]$ components from (7), it results the following equation:

$$\begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial t} \end{bmatrix} \gamma \begin{bmatrix} 1 & -v \\ -v/c^{2} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{J}_{p}^{0} \\ \rho^{0} \end{bmatrix} + \nabla_{n}^{0} \mathbf{J}_{n}^{0} = 0. \quad (10)$$

The equations (9) and (10) are covariant if we associate to each referential system own operators of space-time derivatives according the following relations:

$$\begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial t} \end{bmatrix} \gamma \begin{bmatrix} 1 & -v \\ -v/c^{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x^{0}} & \frac{\partial}{\partial t^{0}} \end{bmatrix}. \quad (11)$$

The differential transformation results after imposing the total diferential‘s invariance of any scalar function:

$$\begin{bmatrix} \mathrm{d}x & \mathrm{d}t \end{bmatrix} \begin{bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial t} \end{bmatrix} = \begin{bmatrix} \mathrm{d}x^0 & \mathrm{d}t^0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial}{\partial x^0} \\ \dfrac{\partial}{\partial t^0} \end{bmatrix}. \tag{12}$$

Further on, for inertial systems (v = const.) the equations become:

$$\begin{aligned} \begin{bmatrix} \mathrm{d}x & \mathrm{d}t \end{bmatrix} &= \begin{bmatrix} \mathrm{d}x^0 & \mathrm{d}t^0 \end{bmatrix} \begin{bmatrix} 1 & -v/c^2 \\ -v & 1 \end{bmatrix}, \\ \begin{bmatrix} \mathrm{d}x \\ \mathrm{d}t \end{bmatrix} &= \gamma \begin{bmatrix} 1 & -v \\ -v/c^2 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{d}x^0 \\ \mathrm{d}t^0 \end{bmatrix}. \end{aligned} \tag{13}$$

By integration in null initial conditions and for the origin of coordinates the special Lorentz transformations of the time and space coordinates result:

$$\begin{bmatrix} x \\ t \end{bmatrix} = \gamma \begin{bmatrix} 1 & -v \\ -v/c^2 & 1 \end{bmatrix} \begin{bmatrix} x^0 \\ t^0 \end{bmatrix}, \quad \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y^0 \\ z^0 \end{bmatrix}. \tag{14}$$

The same transformation matrix was obtained previously (5, 6) in more general conditions of non-uniform relative speed (non-inertial systems) for bi-tensors $\begin{bmatrix} \mathbf{i} \times \mathbf{E}_n \\ \mathbf{B}_n \end{bmatrix}$ and $\begin{bmatrix} \mathbf{H}_n \\ \mathbf{i} \times \mathbf{D}_n \end{bmatrix}$ respectively.

3. The equations of electromagnetic field and their covariance

For the analyse of the state and evolution equations covariance of the electromagnetic field, these are grouped in pairs of homogeneous and inhomogenous equations:

$$\begin{aligned} &\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \quad \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}, \\ &\nabla \mathbf{B} = 0, \qquad\qquad\quad \nabla \mathbf{D} = \rho. \end{aligned} \tag{15}$$

By decomposing the different physical values and operators in parallel and normal components towards the relative speed ($\mathrm{v} = \mathbf{i}\mathrm{v} = -\mathrm{v}^0 = -\mathbf{i}\mathrm{v}^0$), they may be obtained the following equations.

3.1. *Implicit intrinsic equations*

$$
\begin{aligned}
&[\nabla \times \mathbf{E}]_{\mathrm{n}} + \frac{\partial \mathbf{B}_{\mathrm{n}}}{\partial t} = 0, \quad [\nabla \times \mathbf{H}]_{\mathrm{n}} - \frac{\partial \mathbf{D}_{\mathrm{n}}}{\partial t} = \mathbf{J}_{n}, \\
&[\nabla \times \mathbf{E}]_{\mathrm{p}} + \frac{\partial \mathbf{B}_{\mathrm{p}}}{\partial t} = 0, \quad [\nabla \times \mathbf{H}]_{\mathrm{p}} - \frac{\partial \mathbf{D}_{\mathrm{p}}}{\partial t} = \mathbf{J}_{p}, \\
&\nabla_{\mathrm{n}} \mathbf{B}_{\mathrm{n}} + \nabla_{\mathrm{p}} \mathbf{B}_{\mathrm{p}} = 0, \quad \nabla_{\mathrm{n}} \mathbf{D}_{\mathrm{n}} + \nabla_{\mathrm{p}} \mathbf{D}_{\mathrm{p}} = \rho.
\end{aligned} \tag{16}
$$

3.2. *Intrinsic explicit tensor equations*

Let F be an arbitrary vector with normal component Fn and parallel component **F**p = **i**Fx to the speed vector v = **i**v_x, with spatial derivative operator $\nabla = \nabla_{\mathrm{n}} + \nabla_{\mathrm{p}} = \nabla_{\mathrm{n}} + \mathbf{i}\frac{\partial}{\partial x}$, we have:

$$
\begin{aligned}
&(\nabla \times \mathbf{F})_{\mathrm{n}} = \nabla_{\mathrm{n}} \times \mathbf{F}_{\mathrm{p}} + \nabla_{\mathrm{p}} \times \mathbf{F}_{\mathrm{n}} = \nabla_{\mathrm{n}} \times \mathbf{i} F_x + \frac{\partial}{\partial x}(\mathbf{i} \times \mathbf{F}_{\mathrm{n}}); \\
&(\nabla \times \mathbf{F})_{\mathrm{p}} = \nabla_{\mathrm{n}} \times \mathbf{F}_{\mathrm{n}}; \qquad \mathbf{i}(\nabla_{\mathrm{n}} \times \mathbf{F}_{\mathrm{n}}) = \nabla_{\mathrm{n}}(\mathbf{F}_{\mathrm{n}} \times \mathbf{i}); \\
&\nabla \mathbf{F} = \nabla_{\mathrm{n}} \mathbf{F}_{\mathrm{n}} + \frac{\partial}{\partial x} F_x .
\end{aligned} \tag{16'}
$$

With these relations, the implicit intrinsic equations (16) can be explicitly expressed using matrices as follow:

$$
\begin{bmatrix} \frac{\partial}{\partial t} & \frac{\partial}{\partial x} \end{bmatrix} \begin{bmatrix} \mathbf{B}_{\mathrm{n}} \\ \mathbf{i} \times \mathbf{E}_{\mathrm{n}} \end{bmatrix} + \nabla_{\mathrm{n}} \times \mathbf{i} E_x = 0; \quad \begin{bmatrix} -\frac{\partial}{\partial t} & \frac{\partial}{\partial x} \end{bmatrix} \begin{bmatrix} \mathbf{D}_{\mathrm{n}} \\ \mathbf{i} \times \mathbf{H}_{\mathrm{n}} \end{bmatrix} + \nabla_{\mathrm{n}} \times \mathbf{i} H_x = \mathbf{J}_{\mathrm{n}}, \tag{17}
$$

$$
\nabla_{\mathrm{n}} \begin{bmatrix} \mathbf{E}_{\mathrm{n}} \times \mathbf{i} \\ \mathbf{B}_{\mathrm{n}} \end{bmatrix} + \begin{bmatrix} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \end{bmatrix} B_x = 0; \quad \nabla_{\mathrm{n}} \begin{bmatrix} \mathbf{H}_{\mathrm{n}} \times \mathbf{i} \\ \mathbf{D}_{\mathrm{n}} \end{bmatrix} + \begin{bmatrix} -\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \end{bmatrix} D_x = \begin{bmatrix} J_x \\ \rho \end{bmatrix}. \tag{17'}
$$

These equations are manifest covariant because in relations (17) we have produced terms with inverse matrix transformations:

$$\begin{bmatrix} \frac{\partial}{\partial t} & \frac{\partial}{\partial x} \end{bmatrix} = \gamma \begin{bmatrix} \frac{\partial}{\partial t^0} & \frac{\partial}{\partial x^0} \end{bmatrix} \begin{bmatrix} 1 & v/c^2 \\ v & 1 \end{bmatrix},$$

$$\begin{bmatrix} -\frac{\partial}{\partial t} & \frac{\partial}{\partial x} \end{bmatrix} = \gamma \begin{bmatrix} -\frac{\partial}{\partial t^0} & \frac{\partial}{\partial x^0} \end{bmatrix} \begin{bmatrix} 1 & -v/c^2 \\ -v & 1 \end{bmatrix}, \tag{18}$$

$$\begin{bmatrix} \mathbf{B}_\text{n} \\ \mathbf{i}\times\mathbf{E}_\text{n} \end{bmatrix} = \gamma \begin{bmatrix} 1 & -v/c^2 \\ -v & 1 \end{bmatrix} \begin{bmatrix} \mathbf{B}^0_\text{n} \\ \mathbf{i}\times\mathbf{E}^0_\text{n} \end{bmatrix}, \quad \begin{bmatrix} \mathbf{D}_\text{n} \\ \mathbf{i}\times\mathbf{H}_\text{n} \end{bmatrix} = \gamma \begin{bmatrix} 1 & v/c^2 \\ v & 1 \end{bmatrix} \begin{bmatrix} \mathbf{D}^0_\text{n} \\ \mathbf{i}\times\mathbf{H}^0_\text{n} \end{bmatrix},$$

and added terms are manifest covariant

$$\nabla_\text{n} \times \mathbf{i}E_x = \nabla^0_\text{n} \times \mathbf{i}E^0_x; \quad -\nabla_\text{n} \times \mathbf{i}H_x = -\nabla^0_\text{n} \times \mathbf{i}H^0_x.$$

In equations (17’) the added terms have the same tranformation matrix

$$\begin{bmatrix} \mathbf{E}_\text{n}\times\mathbf{i} \\ \mathbf{B}_\text{n} \end{bmatrix} = \gamma \begin{bmatrix} 1 & v \\ v/c^2 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{E}^0_\text{n}\times\mathbf{i} \\ \mathbf{B}^0_\text{n} \end{bmatrix}, \quad \begin{bmatrix} \mathbf{H}_\text{n}\times\mathbf{i} \\ \mathbf{D}_\text{n} \end{bmatrix} = \gamma \begin{bmatrix} 1 & -v \\ -v/c^2 & 1 \end{bmatrix}, \tag{19}$$

and for inertial systems (v = const).

$$\begin{bmatrix} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \end{bmatrix} = \gamma \begin{bmatrix} 1 & v \\ v/c^2 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial t^0} \\ \frac{\partial}{\partial x^0} \end{bmatrix}, \quad \begin{bmatrix} -\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \end{bmatrix} = \gamma \begin{bmatrix} 1 & -v \\ -v/c^2 & 1 \end{bmatrix} \begin{bmatrix} -\frac{\partial}{\partial t^0} \\ \frac{\partial}{\partial x^0} \end{bmatrix}. \tag{20}$$

$$\begin{bmatrix} J_x \\ \rho \end{bmatrix} = \gamma \begin{bmatrix} 1 & -v \\ -v/c^2 & 1 \end{bmatrix} \begin{bmatrix} J^0_x \\ \rho^0 \end{bmatrix}.$$

4. Relativistic aspects of pre-relativistic electrodynamics

In pre-relativistic electrodynamics theory of moving bodies [8] four vector physical values of electromagnetic field were used **E**, **B**, **H**, **D**, without constitutive relationship between them. Hertz condidered magnetic flux density B, and electric displacement **D** as absolute physical values ($\mathbf{B} = \mathbf{B}^0$ for homogenous equations and $\mathbf{D} = \mathbf{D}^0$ for non homogenous equations). In these hypotheses for inertial systems a kinematic transformation with absolute time and relative space is obtained (Galilei).

If the constitutive relationship is considered $\mathbf{B} = \mu\mathbf{H}$, absolute **B** implies absolute **H** ($\mathbf{H} = \mathbf{H}^0$) and from non homogenous equations for inertial systems it results a kinematic transformation with absolute space ($x = x0$) and relative time $\left(t = t^0 - x^0\, v/\mathrm{c}^2\right)$, complementary to Galilei transformation.

5.1. Galilei kinematic transformation

Considering as absolute induction and displacement ($\mathbf{B}_n = \mathbf{B}_n^0$ and $\mathbf{D}_n = \mathbf{D}_n^0$) in tensor equations (5) and (6) we have $v/c^2 = 0$ and $\gamma = 1$

$$\begin{bmatrix} \mathbf{E}_\mathrm{n} \\ \mathbf{i}\times\mathbf{B}_\mathrm{n} \end{bmatrix} = \begin{bmatrix} 1 & v \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{E}_\mathrm{n}^0 \\ \mathbf{i}\times\mathbf{B}_\mathrm{n}^0 \end{bmatrix}, \quad \begin{bmatrix} \mathbf{i}\times\mathbf{H}_\mathrm{n} \\ \mathbf{D}_\mathrm{n} \end{bmatrix} = \begin{bmatrix} 1 & v \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{i}\times\mathbf{H}_\mathrm{n}^0 \\ \mathbf{D}_\mathrm{n}^0 \end{bmatrix}. \tag{21}$$

The same operation $v/c^2 = 0$ and $\gamma = 1$ must be performed to the transformation matrix of space-time derivative operators for preserving the covariance of equations.

But the transformation of operators is also valid for non-inertial systems

$$\begin{bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial t} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ v & 1 \end{bmatrix} \begin{bmatrix} \dfrac{\partial}{\partial x^0} \\ \dfrac{\partial}{\partial t^0} \end{bmatrix} \tag{22}$$

and for inertial systems and null initial condition, is added a space–time equation (kinematic) with absolute time and relative space (Galilei).

$$\begin{bmatrix} x \\ t \end{bmatrix} = \begin{bmatrix} 1 & -v \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x^0 \\ t^0 \end{bmatrix}. \tag{23}$$

5.2. Complementary kinematic transformation

For common values of magnetic flux density and magnetic strength in air $\left(\mathbf{H} = \mathbf{B}/\mu_0\right)$, and for electric displacement and electric strength $\left(\mathbf{E} = \mathbf{D}/\varepsilon_0\right)$, the movement in magnetic field has major electric field effects, and the movement in electric field has minor magnetic field effects [9].

Therefore in applications (in technique) is very important the value of absolute magnetic flux density $\left(\mathbf{B} \cong \mathbf{B}^0\right)$ and as a consequence the absolute value of magnetic field strength $\left(\mathbf{H} \cong \mathbf{H}^0\right)$ for non-homogeneous equations.

In this case in tensor equations (6) the substitutions $v = 0$ and $\gamma = 1$ must be made. So:

$$\begin{bmatrix} \mathbf{i}\times\mathbf{H}_\mathrm{n} \\ \mathbf{D}_\mathrm{n} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ v/\mathrm{c}^2 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{i}\times\mathbf{H}_\mathrm{n}^0 \\ \mathbf{D}_\mathrm{n}^0 \end{bmatrix}, \quad \begin{bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial t} \end{bmatrix} = \begin{bmatrix} 1 & v/\mathrm{c}^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dfrac{\partial}{\partial x^0} \\ \dfrac{\partial}{\partial t^0} \end{bmatrix}. \tag{24}$$

For inertial systems and null space-time initial conditions a kinematic equation with absolute space and relative time is obtained, in fact a complementary transformation matrix to Galilei transformation

$$\begin{bmatrix} x \\ t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -v/\mathrm{c}^2 & 1 \end{bmatrix} \begin{bmatrix} x^0 \\ t^0 \end{bmatrix}. \tag{25}$$

As a consequence, the properties of electromagnetic field are prior to the kinematic properties. That is because the kinematic properties result as a consequence (not as a premise), what justifies the name of electodynamic relativity given to the theory of relativity based on the objective properties of the electromagnetic field.

Conclusions

1. The physical values attached to the electromagnetic state expressed in the intrinsic form, with normal and parallel components to the relative speed vector, of non-inertial reference systems attached to the bodies and the field were grouped as bi-tensors and bi-vectors, with matrix special Lorentz transformation relations.

2. Using covariance of a differential space-time equation from electrodynamics similar transformations were resulted for space-time derivatives and differential operators.

3. At the integration of kinematic relations of differential equations attached to the inertial systems with null space-time initial conditions, the special Lorentz kinematic relations were obtained.

4. Using bi-tensors and bi-vectors attached to the electromagnetic field quantities and space-time derivative operators, intuitive matrix equations manifest covariant for state and evolution equations of electromagnetic field are obtained.

5. In the practical hypothesis (advantageous for technical applications) considering the magnetic flux density as an absolute quantity (independent of the reference system in the relative inertial movement), a kinematic Galilei transformation results.

Using the same practical hypothesis to the magnetic strength, according to the constitutive relation, a complementary kinematic transformation results (named anti-Galilei). This additional fact justifies the name of electrodynamics relativity given to the relativity theory based on the objective properties of the electromagnetic state.

REFERENCES

1. *H. Poincaré*, Électricité et optique. Paris 1901.
2. *A. Einstein*, Zur Electrodinamik der bewegter Körper. Ann Phys. 17,1905.
3. *E. Purcell*, Cursul de fizică Berkley – vol.2, 1988.
4. *F. Manea*, Sur l' électrodynamique des corps en mouvement. Rev. Roum. Sci. Techn.– Électrotechn. et Énerg. , 32, 4, pp. 435–441 (1987).
5. *F. Manea*, Sur les propriétés relativistes des états stationnaires et variables de l'électrodynamique des corps en mouvement. Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., 41, 3, pp. 275–282 (1996).
6. *J. Jackson,* Classical Electrodynamics, John Wiley, 1974.
7. *R. Feynman*, Lectures on Physics, Addison Wesley, Massachusetts, 1963.
8. *H. Hertz*, Sur les équations fondamentales de l'électrodynamique pour les corps en mouvement, La lumière électrique XXXVIII, 1, 1890, Paris.
9. *F. Manea*, Approximations et ambiguités dans l'électrodynamique des corps en mouvement. Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., 42, 4, pp. 419–424 (1997).