MULTI-OBJECTIVE OPTIMIZATION OF AUTOMOBILE GEARBOX DESIGN BASED ON MATLAB

JIN Xiangjie[1]*, ZHANG Cai[1], YANG Fei[1], ZHANG Xiaoying[1], WANG Yahui[1]

Aiming at the characteristic that the parameter design of the important parts in the gearbox has an effect on the vehicle smoothness, the multi-objective genetic algorithm theory is used to optimize the structure of the automobile gearbox. Based on the MATLAB multi-objective genetic algorithm toolbox, the data comparison analysis before and after multi-objective optimization of objective function shows that the multi-objective optimization method realizes the lightweight of the box while meeting the higher performance requirements of the gearbox, and finally obtains the ideal optimization results, at the same time provides design methods and theoretical support for other relevant parts of the gearbox.

Keywords: Automobile gearbox; Multi-objective optimization; MATLAB; Genetic algorithm

1. Introduction

With the progress of the times and the development of society, people put forward higher requirements for the performance of the transmission. Ideal automotive gearbox not only has good dynamic performance, but also has small size, less material, light weight, reliable working conditions. At the same time, it can take into account the light shift and effectively reduce working noise [1]. This is also the necessary development trend of gearbox manufacturing industry. Therefore, the optimal design of gearbox has become an important issue for many automobile enterprises.

2. Establishment of Three-dimensional Solid Model

2.1 Virtual Assembly of Gearbox

Virtual assembly of gearbox is helpful for function analysis and performance optimization. The three-dimensional model of each part is inserted into the assembly environment one by one in top-down order, and then the fit relationship is selected to assemble. The internal structure of the gearbox hides the gearbox case and upper cover, as shown in Figure 1.

[1] School of Mechanical Engineering, North China University of Water Resources and Electrical Power Zhengzhou, Henan 450045, China.
*Email: jing_hc123@126.com

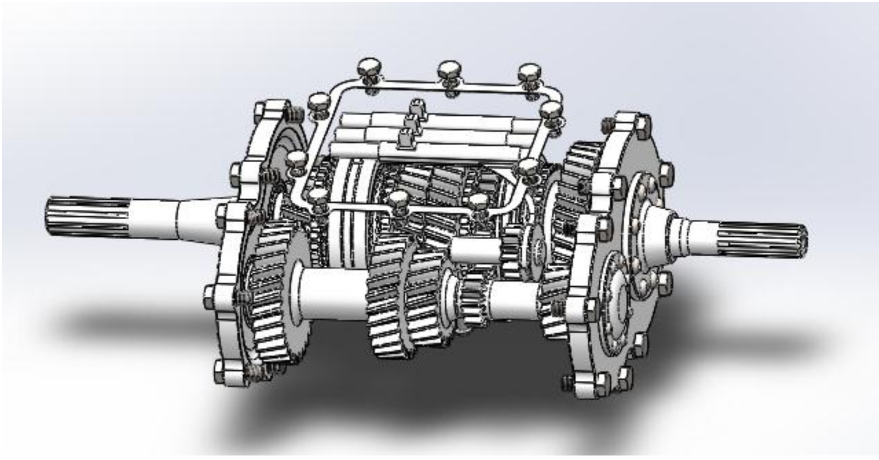

Fig.1 Internal structure of assembly body

2.2 Simplified Gear Model of Gearbox

The gearbox studied in this subject is mainly aimed at light vehicles. The internal structure of the gearbox is compact and complex, and there are many factors affecting the bearing capacity and overall volume. If all the parameters affecting the design structure in the design process are taken as the target parameters, the optimization problem will become complex and difficult to calculate. Therefore, the main parameters that affecting the mechanical properties of gearbox, such as tooth width b, gear modulus m and tooth number z, are selected as the research parameters.

The reverse gear of the middle shaft, the output shaft and the reverse gear of the reverse shaft are spur gears with the same shape. The first, second and third gears on the output shaft are helical gears with the same shape and different sizes except reverse gears on the middle gear shaft.

2.3 Gear Parameters

According to the specific conditions of gearbox design, refer to the gear design in the mechanical design manual and refer to the reference [2], the obtained parameters are shown in Table 1 and Table 2.

Table 1

Computational Gear Parameters Table

Gears	Spur Gears	Helical Gears
Modulus	2.75	3.25
Pressure Angle	20°	20°
Helix Angle	0°	23.802°
Tooth Width (mm)	15	25

Table 2

Distribution Table of Gears

Gear Position	Number of Driving Gear Teeth	Number of Driven Gear Teeth
First gear	15($Z1$)	39($Z2$)
Second gear	20($Z3$)	34($Z4$)
Third gear	26 ($Z5$)	28($Z6$)
Forth gear	30($Z7$)	24($Z8$)
Reverse gear	14($Z9$)	49($Z10$)

3. Multi-objective Genetic Algorithm Based on MATLAB

3.1 Introduction and Application of Genetic Algorithms

Genetic algorithm is derived from the biological laws of heredity and the theory of evolution. It was first proposed by Professor Holland and applied the idea of genetic crossover and mutation. It is an efficient global optimization algorithm which combines the survival rule of the fittest in the evolution of biology with the random exchange system in the group [3].

Since the 1970s, genetic algorithm has been widely applied to various disciplines as an efficient, practical and robust optimization technology, and has achieved remarkable theoretical and scientific research results. It provides a wide range of problem-solving frameworks with wide applicability and does not rely on a specific condition. It has been applied successfully in many fields, such as function optimization, image learning, genetic coding, and so on.

However, as a separate optimization method, genetic algorithm has its own limitations. For example, when the internal operation data is large or random search is performed, there may be a repeated computation of the encoding, which takes up a certain amount of time and resources.

Therefore, if the initial coding is incorrect or inaccurate, the optimal solution can be easily deviated from the actual optimal solution at runtime. If the coding grouping is not standardized or the operational mechanism is deviated, premature convergence may occur in the iterative solution. At present, there is no suitable method to solve and improve the search accuracy and computational complexity of genetic algorithm [4].

3.2 Improved Genetic Algorithms

Although genetic algorithm has many advantages, some problems such as slow convergence near the optimal solution and premature phenomena still need to be solved. Therefore, many scholars have tried to study coding methods, control parameters and crossover mechanisms to promote the development of genetic algorithm. Therefore, various improved genetic algorithms, such as hierarchical genetic algorithm, adaptive genetic algorithm, relaxed genetic algorithm, parallel genetic algorithm and niche based genetic algorithm, have been proposed. The improved genetic algorithm can significantly reduce the blindness of search and increase the probability of finding the optimal solution. It can largely avoid the deception problem of genetic algorithm, avoid falling into local optimum and improve the ability of global optimization.

In this paper, NSGA-II algorithm is applied, which is improved by NSGA. By using the mechanism of elite strategy, the excellent population individuals are guaranteed not to be discarded in the evolution process, and the accuracy of the optimization results is improved. The algorithm introduces crowding degree and crowding degree comparison operator, which overcomes the shortcomings of NSGA that need to specify the shared parameters artificially, and makes Pareto frontier more uniform and distributed. In addition, the algorithm greatly reduces the computational complexity by using fast non dominated sorting criteria. Therefore, NSGA-II algorithm has faster operation speed, better convergence solution set and higher robustness.

3.3 Establishment of mathematical model for multi-objective optimization problems

Multi-objective optimization problem refers to an optimization problem in which the objective function is not unique. Its main process is to find a set of design variables $[x_1, x_2, \ldots x_n]$, which not only satisfy the constraints and vector objective functions, but also enable decision makers to accept all the sub-objective values. The objective function is the specific description of the system performance of the design variables, and its mathematical model is as follows.

$$\min F(\vec{x}) = [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_m(\vec{x})]^T \quad (\vec{x} = [x_1, x_2, \ldots, x_n]^T)$$

$$s.t. \begin{cases} g_i(\vec{x}) \leq 0 & (i = 1, 2, \ldots, p) \\ h_j(\vec{x}) = 0 & (j = 1, 2, \ldots, q) \end{cases} \tag{1}$$

Among them, the decision variables $[x_1, x_2, \ldots x_n]$ constitute the decision vector; the numerical objective function $f_i(x) = f_i(x_1, x_2, \ldots, x_n)$ constitute the vector objective function; and the $g_i(x)$ and $h_j(x)$ represent the optimization constraint function [5]. Supposing that the constraint set of the multi-objective optimization model is $x \subseteq R^m$ and the vector objective function is $f_i(x) \in R^n$.

If $x_1 \in X$ is solved and x_1 is superior to all other solutions in X, x_1 is the optimal solution of the multi-objective optimization model. If $x_2 \in X$ is found and there is no better solution than x_2, then x_2 is the optimal (Pareto) solution of the multi-objective optimization model. The concept of Pareto was proposed by Vilfredo Pareto in 1896, so it is called the Pareto optimal solution [6].

4. Mathematical Model of Automotive Gearbox

4.1 Design Variables

The research object of the subject is four-speed gearbox. Therefore, the main parameters affecting the mechanical properties of gearbox, such as tooth width b, gear modulus m and tooth number z, are selected as design variables [7]. Forward gear z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8 are helical gears, whose modulus is m_1, tooth width is b_1 and helical angle is β; reverse gear z_9, z_{10} and z_{11} are spur gears, whose modulus is m_2, tooth width is b_2, and $z_9 = z_{11}$. The modulus and width of the forward gear are equal. Because the axes of the input and output axes of the gearbox are in the same straight line, the center spacing of each gear must be equal. So

$$\begin{cases} z_4 = z_1 + z_2 - z_3 \\ z_6 = z_1 + z_2 - z_5 \\ z_8 = z_1 + z_2 - z_7 \end{cases} \tag{2}$$

So the design variable is

$$\begin{aligned} X &= [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}]^T \\ &= [z_1, z_2, z_3, z_5, z_7, z_9, z_{10}, m_1, m_2, b_1, b_2]^T \end{aligned} \tag{3}$$

4.2 Objective Function

The minimum volume of the gearbox should be selected as the first optimization objective. Gear center distance is a key parameter to measure the compactness of automobile gearbox structure. Small center distance should be selected under the condition of guaranteeing the contact strength of gears in design and calculation. Therefore, the minimum center distance is chosen as the second optimization objective. The coincidence degree has an important influence on the improvement of vehicle transmission smoothness, that is, ride comfort. Therefore, the maximum coincidence degree is chosen as the third goal [8].

(1) Volume objective function

The overall size of the gearbox is close to the size of the box, and the size of the box depends on the size of the internal gear train. When the sum of all the gears in the gearbox is the smallest, the size of the box is the smallest, and the overall volume of the gearbox is the smallest. Therefore, the total volume of the gearbox can be expressed by the sum of all the gears in the gearbox, that is, the volume optimization goal can be transformed into the minimum of the total volume of the gearbox. In fact, the calculation process of gear volume is rather complicated, especially the helical gear volume. In order to simplify the calculation process and not change the essence of optimization, the cylinder volume with the base circle diameter of gear indexing circle diameter is selected to replace the gear volume [9].

$$f_1(x) = Min(\frac{\pi}{4}\frac{m_1^{\ 2}}{\cos^2\beta}b_1\sum_{i=1}^{8} z_i^{\ 2} + \frac{\pi}{4}m_2^{\ 2}b_2\sum_{i=9}^{11} z_i^{\ 2}) \tag{4}$$

(2) Center distance objective function

The axes of the input and output axes of the gearbox are in the same straight line. The center distance between the driving wheel and the driven wheel of the first, second, third and fourth gears are equal. The reverse gear on the reverse shaft is used as the transmission gear between the reverse gear on the output shaft and the reverse gear on the intermediate shaft. The center distance between the reverse gear on the output shaft and the intermediate gear is larger, and different from that of the forward gear. Therefore, the minimum centre distance refers to the maximum centre distance of forward gear and reverse gear, which is the smallest under the condition of satisfying the constraints. Because the center distance of forward gear is the same, four center distance is chosen here to represent [10].

$$f_2(x) = Min(max(\frac{m_1(z_7+z_8)}{2\cos\beta}, \frac{m_2(z_{10}+z_{11})}{2})) \tag{5}$$

(3) Coincidence objective function

The first, second, third and fourth gears of gearbox are in constant meshing state, which requires high load-carrying capacity and transmission stability. Therefore, the optimization objective is to find the maximum total coincidence of four pairs of meshing gears in forward gear. Since the genetic algorithm is usually calculated by minimizing the value of the objective function, the reciprocal of the total coincidence of the four pairs of meshing gears is transformed into the objective function [11].

$$A_{i-1} = \sqrt{z_{2i-1}^{\ 2}\sin^2\alpha + 4h\alpha^* z_{2i-1} + 4h\alpha^{*2}} - z_{2i-1}\sin\alpha' \tag{6}$$

$$A_i = \sqrt{z_{2i}{}^2 \sin^2 \alpha + 4h\alpha^* z_{2i} + 4h\alpha^{*2}} - z_{2i} \sin \alpha' \tag{7}$$

$$f_3(x) = Min(\sum_{i=1}^{4} (\frac{A_{i-1} + A_i}{2\pi \cos \alpha} + \frac{b_1 \tan \beta}{\pi m_1})^{-1}) \tag{8}$$

In the formula, α is the pressure angle, taking α =20°; $h\alpha*$ is the coefficient of tooth top height, $h\alpha*$=1; α' is the meshing angle of gear, taking α' =20°.

4.3 Constraints

(1) Modular constraints

The modulus of gears with power transmission function is generally larger than 2 mm[12]. Selection is made according to the design performance requirement and the modulus of common gearbox.

$$\begin{cases} 3 \le m_1 \le 4 \\ 2 \le m_2 \le 3 \end{cases} \tag{9}$$

(2) Tooth width constraint

Large tooth width can improve the bearing capacity of gears, but large tooth width will lead to tooth error and deflection deformation, resulting in unbalanced force in the direction of tooth width, and increase the overall volume and axial length of gearbox [13]. Therefore, the tooth width constraint is:

$$\begin{cases} b_1 = (4.5 \sim 8.0) m_1 \\ b_2 = (6.0 \sim 8.5) m_2 \end{cases} \tag{10}$$

(3) Number of teeth and constraints

In general, the number of teeth of the driving and driven wheels of each pair of meshing gears in the gearbox is not greater than 100. And because the center distance of each helical gear in forward gear is the same, the modulus and helical angle of helical gear are the same, so the number of teeth of the first, second, third and fourth gears and the same are required [14].

(4) Drive Ratio Error Constraints

In the design process, the selection of transmission ratio is determined by the best transmission system scheme, and the number of gear teeth will be rounded when calculating, resulting in the emergence of transmission ratio error. Therefore, in order not to affect the performance of the whole vehicle, the change rate of the limit transmission ratio is less than or equal to 3%.

(5) Coincidence constraints

When solving the problem, not only the total coincidence degree should be increased, but also the coincidence degree of each pair of meshing gears should be improved.

$$\varepsilon_r' - \varepsilon_r \geq 0 \quad (11)$$

In the formula, ε_r' is the optimal meshing gear coincidence degree, and ε_r is the pre-optimization meshing gear coincidence degree [15].

5. Analysis of Optimization Results

5.1 Optimization Results

"X" is the optimal design point after optimization, and 70 groups of optimized data, 11 data in each group; 70 groups of optimized objective function values, three data in each group. That is to say, the result of this optimization is a set of 70 Pareto optimal solutions. Because of the large amount of data in Pareto optimal solution set, the first 16 groups are selected to form a table. As shown in Table 3 and Table 4.

Table 3

Pareto Optimal Set of the First 16 Groups

Serial Number	z_1	z_2	z_3	z_5	z_7	z_9	z_{10}	m_1	m_2	b_1	b_2
1	15	34	19	27	27	19	49	3.37	2.12	26.97	22.61
2	16	34	19	27	27	19	49	3.37	2.14	26.97	22.62
3	15	35	19	27	27	19	49	3.37	2.18	26.99	22.65
4	17	34	19	27	27	19	49	3.37	2.17	26.98	22.64
5	15	34	19	27	27	19	49	3.37	2.12	26.97	22.61
6	15	34	19	27	27	19	49	3.37	2.13	26.97	22.64
7	17	34	19	27	27	19	49	3.37	2.16	26.98	22.64
8	17	34	19	27	27	19	49	3.37	2.15	26.98	22.64
9	17	34	19	27	27	19	49	3.37	2.17	26.98	22.66
10	17	34	19	27	27	19	49	3.37	2.14	26.97	22.65
11	16	34	19	27	27	19	49	3.37	2.12	26.97	22.61
12	15	34	19	27	27	19	49	3.37	2.14	26.97	22.63
13	16	34	19	27	27	19	49	3.37	2.13	26.97	22.62
14	16	34	19	27	27	19	49	3.37	2.13	26.97	22.63
15	16	34	19	27	27	19	49	3.37	2.14	26.97	22.61
16	16	34	19	27	27	19	49	3.37	2.14	26.97	22.67

Note: In Table 3, the number of teeth Z is the data after rounding. Both modulus m and tooth width b retain two decimal digits.

Table 4

The corresponding objective function values of the first 16 Pareto optimal solution sets

Serial Number	Total Volume (V)	Center Distance (A)	Reciprocal of Total Coincidence ($1/Q$)
1	1703011.39	89.9734	0.09194
2	1789440.67	92.8324	0.09164
3	1797773.61	92.4868	0.09170

4	1852842.13	94.6013	0.09150
5	1703090.07	89.9729	0.09194
6	1739522.78	91.1325	0.09183
7	1842625.97	94.3221	0.09152
8	1830191.18	94.0171	0.09153
9	1863029.91	94.9002	0.09145
10	1822173.30	93.9012	0.09155
11	1776981.29	92.5637	0.09167
12	1725846.77	90.6200	0.09188
13	1763421.35	92.0225	0.09174
14	1770607.25	92.1977	0.09173
15	1752642.18	91.5903	0.09177
16	1807003.00	93.2936	0.09162

Note: In Table 4, the volume unit is mm^3 and the center distance unit is mm.

Table 3 is the optimal design point of the first 16 Pareto optimal solution sets, and Table 4 is the objective function value corresponding to the first 16 Pareto optimal solution sets. The results are shown in Figures 2-5.

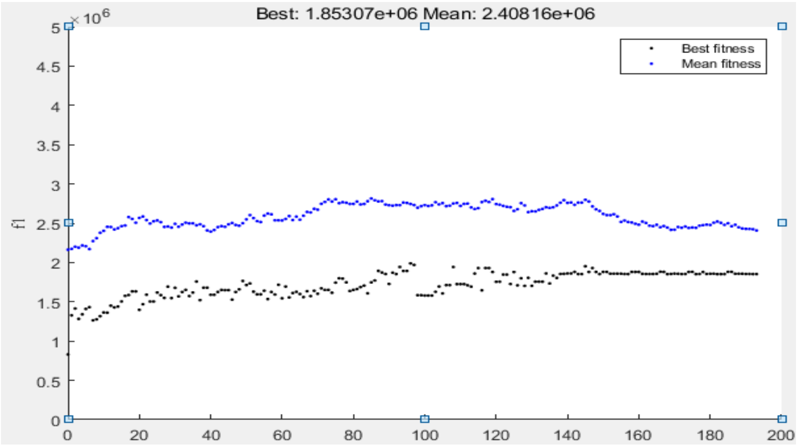

Fig. 2 Iterative diagram of volume objective function

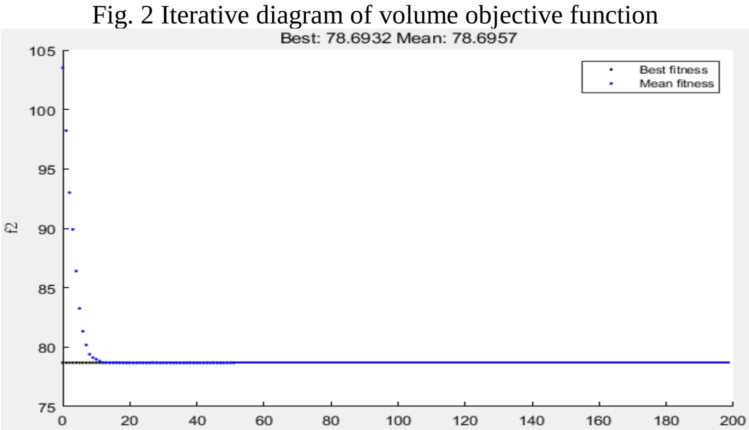

Fig.3 Iterative diagram of the central distance objective function

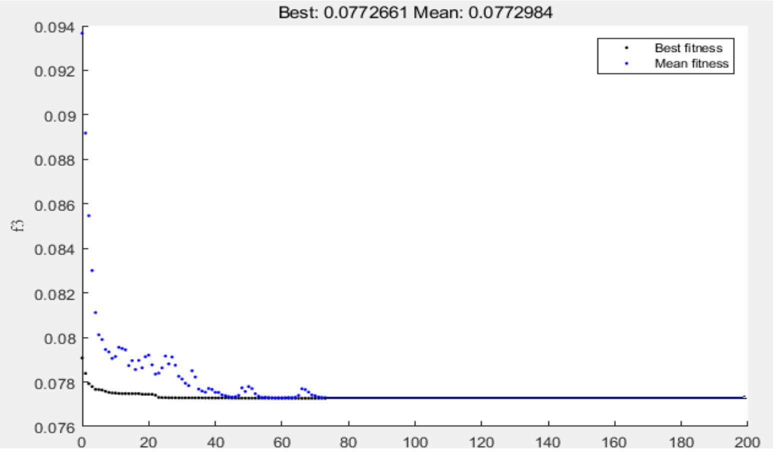

Fig.4 Iterative diagram of objective function of coincidence degree

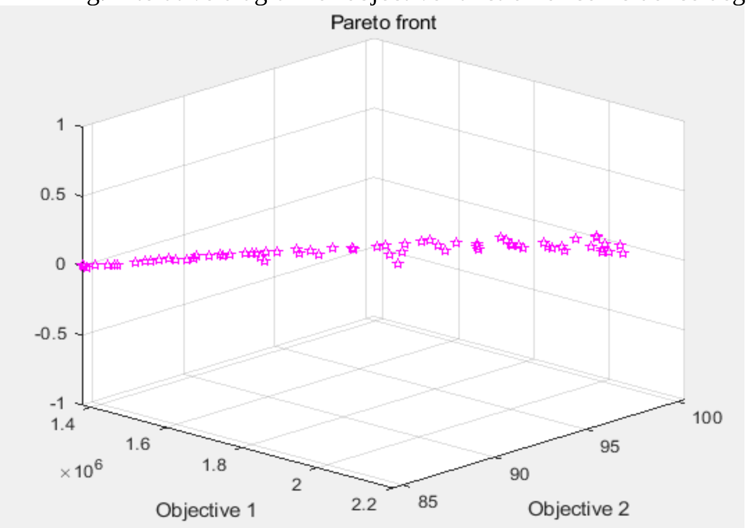

Fig.5 Front edge of multi-objective optimization for gearbox

5.2 Iteration Process and Frontier Analysis

From the iteration graphs of three objective functions, we can see that:

In Figure 2, the volume objective function $f1$ does not decrease significantly during the iteration process, but only fluctuates slightly up and down a certain value and tends to stabilize at last, which shows that the value of the total volume of the gearbox in each generation does not change much compared with that of the initial population.

In Figure 3, the center distance objective function $f2$ decreases gradually in the iteration process, showing a convergence shape, and tends to be stable at the earlier iteration times (less than 20 generations). It shows that the minimum value of the center distance of the forward gear and the center distance of the reverse gear reduces fastest when the constraints are satisfied.

In Figure 4, the objective function $f3$ of coincidence degree shows a downward trend in the initial iteration and a slightly scattered jump. When the population evolves to about 80 generations, it gradually converges to a stable value. This shows that the reciprocal of total coincidence degree of gears decreases slowly in the iteration process, that is, the speed of increasing total coincidence degree of gears is slower.

In Figure 5, 70 sets of Pareto optimal solution sets are evenly distributed in the three-dimensional space composed of three objective functions, and approximately form a straight line. The results are satisfactory.

5.3 Contrastive Analysis before and after Optimization

From the first 16 sets of Pareto optimal solution set after optimization, a group of solutions are randomly selected as representatives and compared with those before optimization. The change rates of values before and after optimization of each parameter and objective function are calculated and analyzed.

Because the objective function of the total gear coincidence in the optimized mathematical model has reciprocal transformation between the minimum and the maximum, the value of the optimized third objective function needs to be corrected.

In order to make the comparison results clearer and easier to see, a histogram is made, as shown in Figure 6 and Figure 7.

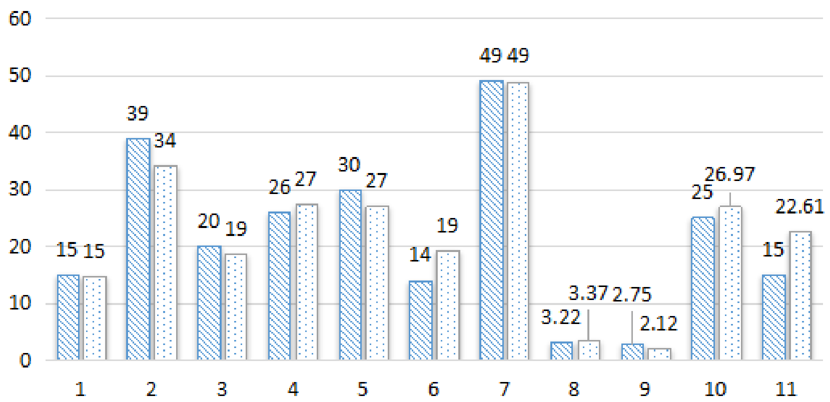

Fig.6 Variation of design variables before and after optimization

From Figure 6, we can see that the changes in the values of each parameter are, x_4 (Intermediate shaft third gear number z_5), x_6 (Intermediate shaft reverse gear teeth z_9), x_8 (Forward gear helical gear modulus m_1), x_{10} (Forward gear helical gear width b_1) and x_{11} (Reverse spur gear width b_2) have increased; x_1 (Intermediate shaft once gear number z_1) and x_7 (Output shaft reverse gear teeth z_{10}) have not changed; The remaining parameters are reduced. Among the increased parameters, x_4 (Intermediate shaft third gear number z_5) changes the least and x_{11} (Reverse spur gear width b_2) changes the most.; Among the reduced parameters, z_6 changes the most, z_3 the changes the least.

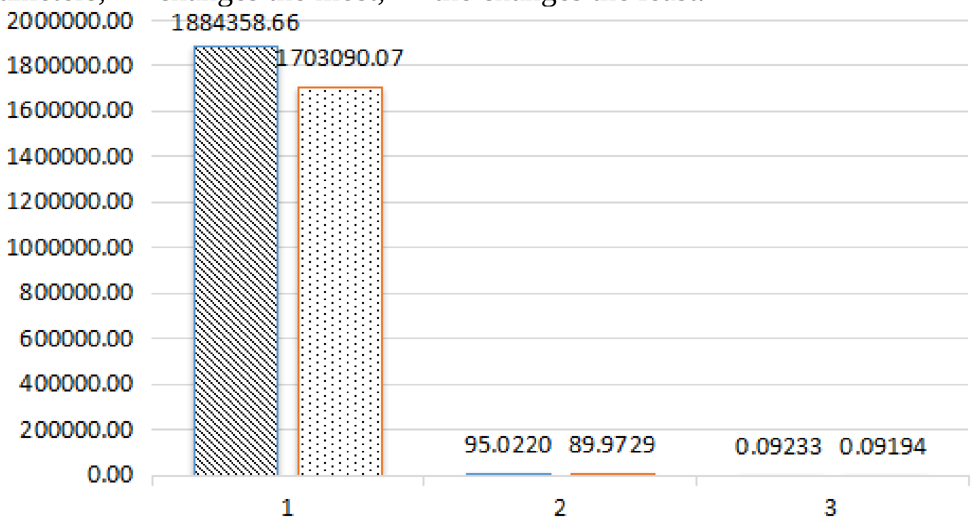

Fig.7 Variation of objective function before and after optimization

From Figure 7, it can be seen that the optimized three objective function values have decreased, which are as follows: the total volume of gearbox has decreased by 9.6%, the minimum value of center distance of forward gear and reverse gear has decreased by 5.3%, the reciprocal of total gear coincidence has decreased by 0.4%, that is, the total gear coincidence has increased by 0.4%.Changing the above conclusion into another angle, that is, under the condition of the same material used, the reduction of volume reduces the mass and cost of the gearbox and realizes lightweight; the reduction of center distance makes the structure of the gearbox more compact and indirectly reduces the radial size of the gearbox; the increase of the total coincidence degree improves the transmission stability, thereby improving the ride comfort of the car.

6. Conclusion

By analyzing the data of the objective function iteration curve, the optimization front and the optimization result, it is found that the total volume, center distance and coincidence degree of the gearbox are improved after optimization. Specifically, the total volume of all gears in the gearbox decreases, the minimum value of the maximum center distance of the forward gear and the reverse gear decreases under the constraint conditions, and the reciprocal of the gear coincidence decreases, that is, the gear coincidence degree increases. It not only reduces the cost of transmission manufacturing and realizes lightweight, but also improves the stability and bearing capacity of transmission.

Acknowledgment

The project was supported by a key research project of Higher Education of Henan Province (20A460019), North China University of Water Resources and Electric Power in Demonstration virtual simulation experiment teaching project.

R E F E R E N C E S

[1]. *He Jiaying, Xiakui, Meng Qinglong et al.* Application of optimization design in automobile gearbox. Automotive Practical Technology, (16):140-142. 2018.
[2]. *Hou jun.* Thoughts on gear design of automobile transmission. Akako (upper middle), (04): 222.2015.
[3]. *Zhou Linjin.* An Immune Genetic Algorithms for Solving Multiobjective Functions [D]. Xi`an University of Science and Technology, 2-3. 2010
[4]. *Garrett W. Mann, Steven Eckels.* Multi-objective heat transfer optimization of 2D helical micro-fins using NSGA-II. International Journal of Heat and Mass Transfer. 132: 1250-1261. 2019.
[5]. *Zhou Shenghai, Yin Hang, Gu Ying et al.* Fault recovery and reconstruction of distribution network based on POP NSGA-II. Journal of Electrical Engineering, 13(06): 28-35. 2018.
[6]. *Wen Shaoxiong, Zhou Wei, Li Shaolin and others.* Multi-objective parameter inversion method of rockfill dam based on NSGA-II algorithm. Journal of Hydropower, 36(01):86-95. 2017.
[7]. *Lingyu Li, Xiangming Huang.* A Compound Optimization Algorithm and its Application to Optimization of Automobile Gearbox. Revista de la Facultad de Ingeniería U.C.V. (32): 269-275.2017.
[8]. *Yu Bin, Pan Jiangru.* Reliability optimization analysis of wheel-side deceleration system based on fuzzy optimization. Mechanical design and manufacture. (10): 83-90. 2016.
[9]. *Ke Fengqin, Tang Shixing.* Reliability optimization design of secondary planetary gear reducer based on MATLAB. Mechanical Drive. (07): 54-61. 2017.
[10]. *Engin Afacan,Gunhan Dundar.* A comprehensive analysis on differential cross-coupled CMOS LC oscillators via multi-objective optimization. Integration. 67: 162-169. 2019.
[11]. *Zeng Chen, Li Bing, Wang Xiaoyong, etc*. based on NSGA- II algorithm multi-objective optimization design of micro tiller gearbox. Mechanical Drive, 40(07): 87-91. 2016.

[12]. *Kristian Ejlebjerg Jensen.* A MATLAB Script for Solving 2D/3D Minimum Compliance Problems using Anisotropic MeshAdaptation. Procedia Engineering. (07): 102-114. 2017.

[13]. *Yue Jinghui, Li Hua.* Structural parameter optimization of 7-axle transmission system based on MATLAB. Mechanical Drive, 39(05): 80-84. 2015.

[14]. *Tamer M. Ismail, Khaled Ramzy, Basem E. Elnaghi, M. N. Abelwhab, M.Abd El-Salam.* Using MATLAB to model and simulate a photovoltaic system to produce hydrogen. Energy Conversion and Management. (04):101-129. 2019.

[15]. *Shi Yunfang, Hu Yadong, Wu Haixia.* Optimum design of gearbox interlocking mechanism. Agricultural equipment technology. (10): 42-43. 2018.