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REGRESSION MODEL APPROACH THROUGH PROPER ROOTS

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Se prezinta MLG (model linear general) în comparație cu modelul ANOVA, precum şi aplicațiile practice ce folosesc aceste modele. Se calculeaza catul Rayleigh folosit in modelul Rayleigh Rayleigh şi se studiaza implicațiile sale in alte modele matematice. Este data o abordare alternativă pentru modelul liniar general față de analiza a modelului ANOVA. Este calculata metoda celor mai mici patrate modificata, obtinuta prin valori proprii şi vectorii proprii corespunzători matricii de corelare a modelului.

The General Linear Model MLG is presented in comparison to ANOVA model as well as the practical applications that are making use of these models. Rayleigh ratio is calculated by use of Rayleigh model and its implication is studied in other mathematical models. An alternative approach for general linear model versus the analysis of ANOVA model is given. The modified least squares procedure obtained using the eigen values and the corresponding eigen vectors of the model's correlation matrix are calculated.

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1. MLG versus ANOVA model

The General Linear Model (MLG) is considered ([4], [7], [9]):

$$\underline{y} = \beta_0 \underline{1} + \underline{x}\underline{\beta} + \underline{\varepsilon} \tag{1}$$

where $\underline{y}$ is the vector of observed random variables (r.v.) - (response), of dimension (n,1), β_0 and the components $\beta_1, \beta_2, \dots, \beta_k$ of the vector (1,k) $\underline{\beta}' = (\beta_1, \dots, \beta_k)$ are unknown parameters which are going to be estimated in certain conditions imposed to model (1);

$\underline{1}$ is the (n,1) – unit vector;

$\underline{X} = (X_{ij})$=$(\underline{X}_1, \underline{X}_2, \dots, \underline{X}_k)$ is the (n,k) matrix of rank k with elements (X_{ij}), $i = \overline{1,n}, j = \overline{1,k}$ independent known r.v., standardized so that

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$$\sum_{i=1}^{n} X_{ij} = \underline{X}_j^{'}\underline{1} = 0, \quad (3)$$

$$\sum_{i=1}^{n} \underline{X}_j^{'}\underline{X}_j = 1, j = \overline{1,k} \quad (3)$$

$\underline{\varepsilon}$ is (n,1) – r.v. vector uncorrelated ~N(0,σ^2) which represent errors for model (1). Conditions (1)-(3) are synthesized into the matrix form of MLG:

$$\Omega \begin{cases} \underline{y} = \beta_0\underline{1} + \underline{x\beta} + \underline{\varepsilon} \\ v.a.necorelate\ \underline{\varepsilon} \sim N(0,\sigma^2 \end{cases}$$

Within Ω-MLG, the following theorem holds:

Theorem 1: In the fundamental hypothesis Ω, if MLG is correct, then l.s.m. (least square method) estimators of parameters β_0 and $\underline{\beta}$ are unmoved and of minimal dispersion, i.e., the solutions of the normal equations system:

$$\underline{X'}\,\underline{X}\ \underline{\beta} = \underline{X'}\,\underline{Y} \quad (4)$$

are of the following form:

$$b_0 = \hat{\beta}_0 = \underline{\overline{Y}} \quad (5)$$

$$\underline{\hat{\beta}} = (X'X)^{-1}\underline{X'}\,\underline{Y}$$

Note 1: MLG in Ω form is also called MLG with intercept β_0.

2. MLG through proper roots

In ([17]) is presented a modified method of obtaining l.s.m.- parameter estimators β_0 and $\underline{\beta}$; A'A is the correlation matrix of MLG Ω of standardized dependable variables and of independent variables, with proper roots $\lambda_j, j = \overline{0,k}$ and eigen vectors ζ_j defined by the following equations:

$$\det|\underline{A}'\underline{A} - \lambda_j \underline{I}| = 0 \quad (6)$$

$$(|\underline{A}'\underline{A} - \lambda_j \underline{I}\ \underline{\zeta_j}) = 0, j = \overline{0,k} \quad (7)$$

where $y_i^* = (y_i - \overline{y})/\eta$, with $\eta^2 = \sum_{i=1}^{n}(y_i - \overline{y})^2$, A=$\left(\underline{y^*}\middle|\underline{X}\right)$, $(\mathrm{n}, \mathrm{k}+1)$ matrix of standardized dependable variables and of independent variables; the eigen vector $\zeta_j^{'} = (\zeta_{0j}\colon \zeta_j^{0'})$ is used in the following relation:

$$\mathrm{A}\zeta_j = \left(\underline{y^*}\middle|\underline{X}\right)\begin{pmatrix}\zeta_{0j}\\ \underline{\zeta_j^0}\end{pmatrix} = y^*\zeta_{0j} + X\zeta_j^0 \quad (8)$$

Theorem 2: If $\zeta_{0j} \neq 0$ for $\forall\, k$, then l.s.m.- $\underline{\beta}$ estimator in MLG (1) with the assumptions (6)-(8) ,(the prediction solution of equations) is:

$$\underline{\hat{y}_\iota} = \underline{\overline{y}} \cdot \underline{1} - \eta\zeta_{0j}^{-1}\underline{X}\underline{\zeta_j^0} \quad (9)$$

$$\underline{\hat{y}} = \sum_{j=0}^{k} a_j\zeta_j^0\,\hat{y}_j \mathrm{y} \quad (10)$$

Proof: The relation (10) is a linear combination of predictors of type (9). The following condition is imposed:

$$\sum_{j=0}^{k} a_j \zeta_j^0 = 1$$

and using l.s.m. the estimator for (1) is obtained with the same expression:

$$\underline{\hat{y}} = \underline{\overline{y}} \cdot \underline{1} - \eta \underline{X}(\sum_{j=0}^{k} a_j \zeta_j^0)$$

i.e. in form (5) in the model Ω with $\hat{b}_0 = \hat{\beta}_0$ and $\underline{b} = \underline{\hat{\beta}} = -\eta \frac{\lambda_j}{\zeta_{0j}^2}$

Consequence: By using these predictors, SS is obtained i.e. the sum of squares corresponding residue

$$SS_e = \sum_{i=1}^{n} (\underline{y}_i - \underline{\hat{y}}_{ij})^2 = \left\| \underline{y} - \underline{\hat{y}}_j \right\|^2 = \eta^2 \frac{\zeta_j^0}{\zeta_{0j}^2} \tag{11}$$

With the general form of:

$$SS_e = \left\| \underline{y} - \underline{\hat{y}} \right\|^2. \tag{11'}$$

The use of l.s.m.-modified estimators leads to obtaining other expression for the sum of squares residue. Thus, the following theorem is established:

Theorem 3: If l.s.m.- modified estimators are used, then $SS_e; mod \geq SS_e$ is obtained

Proof:

From previous results using l.s.m., the parameter estimators β_0 and $\underline{\beta}$ from solutions (5) of the system (4) are obtained. SS_e is calculated from relations (11)-(11)'.

From the equations:

$$(X'X + kI)\hat{\beta}^* = X'Y,\ k \geq 0 \tag{12}$$

the $\hat{\beta}^*$solution is obtained, i.e. using l.s.m., the modified estimators of the parameters are obtained in the form of:

$$\widehat{\beta}^* = (X'X + kI)^{-1} X'Y\ \ (= \hat{\beta}_{mod}) \tag{13}$$

The relation between (13) and (5) is:

$$\hat{\beta}^* = \left(I_p + k(X'X)^{-1}\right)^{-1} \hat{\beta}. \tag{14}$$

Results:

$$(\hat{\beta}^*)'\hat{\beta}^* < \hat{\beta}'\hat{\beta}. \tag{15}$$

The sum of square residues is: ([6])

$$SS_e = \eta^2 (\sum_{j=0}^{k} \lambda_j^{*-1})^{-1} \tag{16}$$

where $\eta^2 = \sum_{i=1}^{n} (y_i - \bar{y})^2$

The sum of modified square residues is:

$$SS_{e;mod} = \left(y - X\hat{\beta}^*\right)' (y - X\hat{\beta}^*)$$

A similar form with relation (16) is immediately obtained:

$$SS_{e;mod} = \eta^2 (\sum_{l=p}^{k} \lambda_l^{*-1})^{-1} \quad (17)$$

using (13) expressed by:

$$\hat{\beta}_{mod} = -\eta (\sum_{l=p}^{k} \lambda_l^{*-1})^{-1} \sum_{j=p}^{k} \zeta_{oj} \lambda_j^{-1} \zeta_j \quad (13)'$$

3. ANOVA in proper squares

The general linear model (MLG)

$$\Omega \begin{cases} y = X\beta + e \\ E(e) = 0_n \\ E(ee') = \sigma^2 G \ (in\ particular\ \sigma^2 I) \end{cases}$$

$y_{(n,1)}$ is the observed response vector r.v.,

$\underline{\beta}_{(k+1,1)} = (\beta_0, \beta_1, \dots, \beta_k)' = (\beta_0 \vdots \underline{\beta'})$ is a vector of unknown parameters. $\underline{1}$ is the (n,1) unit vector.

$\underline{X}_{(n,k)} = (X_0, X_1, \dots, X_k)$ r.v. matrix independent known a.i. $\sum_{i=1}^{n} X_{ij} = \underline{X}_j' \underline{1} = 0; \sum_{i=1}^{n} X_{ij}^2 = \underline{X}_j' X_j = 1, j = \overline{1,k}$

ε – vector (n,1) of r.v. uncorrelated$\sim N(0, \sigma^2)$

Estimators of l.s.m.:

$$\begin{cases} \underline{\hat{\beta}} = \left(\underline{X}'\,\underline{X}\right)^{-1} X' y \\ \widehat{\beta_0} = b_0 = \overline{y} \end{cases} \quad \begin{cases} E(\hat{\beta}) = \underline{\beta} \\ var\ \hat{\beta} = c_{jj}\sigma^2 \end{cases}$$

where:

- $(X'X)^{-1} = c$ with the diagonal element c_{jj},
- $\hat{\beta}$ is l.s.m : unbiased, minimum dispersion
- Solutions of normal equations: $\frac{\partial}{\partial \beta_c} \sum_{i=1}^{I} \sum_{j=1}^{J} (y_{ij} - \beta_i)^2$, $\underline{\hat{\beta}} = \sum_i J_i y_{i*/n}, n = \sum_i J_i$
- In the case of the balanced model n=IJ.

General: normal equations for β estimation are:

$$X'(X'X)^{-1} X\beta = X'(X'X)^{-1} y \rightarrow \hat{\beta}$$

If

var$\hat{\beta}$= $\sigma^2 (X'(X'X)^{-1} X)^{-1} X)^{-1} \rightarrow min \rightarrow var\hat{\beta}_j \geq \sigma^2 / tr(G^{-1})), j = \overline{1,J} =$ min $Var\hat{\beta}_j G^{-1} = ((X'X)^{-1} X)^{-1}$

MLG suggests the particular model when G=I, obtaining the general model of variable analysis (MLG ANOVA) ([5],[13]):

$$\text{MLG ANOVA} \begin{cases} y_{(n,1)} = X_{(n,p)} \beta_{(p,1)} + e_{(n,1)} \\ Ee = 0 \\ Eee' = \sigma^2 I \end{cases}$$

-> ANOVA: rankX=rankX'X (incomplete rank case)<p

The normal equation system: X'Xβ=X'y

with the following solution: $\hat{\beta} = \left(\underline{X}'\,\underline{X}\right)^{-1} X'y$

Cov$\hat{\beta} = \sigma^2 (X'X)^{-1}$> in the case of complete rank: rankX=rankX'X=p (the number of the parameters β_i)

EX1 single factor model ANOVA

$$\Omega \begin{cases} y_{ij} = \beta_i + e_{ij}, i = \overline{1,p} \\ e_{ij}\ v.a.: E e_{ij} = 0_n, j = \overline{1, n_i} \end{cases}$$

The MLG **Ω** with X'X=$\begin{pmatrix} n_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & n_p \end{pmatrix}$

X'y=$(y_{1*}, \dots, y_{p*})$'

NOTE Choosing X a.i. $\hat{\beta}$ elements to be estimated, with minimum variable using the following steps:

- choosing G=$g[(1-\rho)I_n + \rho 1_n 1_n']$
- g, ρ whatever scalings, with $\frac{-1}{(n-1)} < \rho < 1$
- $\underline{1}_n$ vector (n,1) with elements 1
- I_n identity matrix, n order

Comparison of parameters β:

1) null hypothesis: $H_0: \psi = \sum_i c_i \beta_i$, with $\sum \ \ c_i = 0$, contrast in parameters β
2) (1-α)% simultaneous reliable intervals through S method, respectively, T of multiple comparison ([4],[9])
3) The preference of choosing S or T on the basis of R ratio of square intervals of adequate reliability S and T

$$R = \frac{L_S^2}{L_T^2} = H(I-1)\frac{F_{1-\alpha}\,, (I-1), I(J-1)}{q_{1-\alpha}^2, I, I(J-1)} f$$

where

$$f = \frac{\sum c_i^2}{(\sum |c_i|)^2}, with \sum c_i = 0$$

The particular form of Rayleigh floor of vector $\underline{c}$ in relation with G matrix:

$$R(\underline{c}) = \frac{c'c}{\underline{c}'\,\underline{G}\,\underline{c}}, \underline{c}'\,\underline{1} = 0$$

Algebraic and probable inequalities establish the extremes of the functions f and R, respectively:

$f_{max} = 1/2$ involves T is preferable to S
g.l. υ=+∞
$f_{min} = \frac{1}{4}\left(\frac{1}{n^+} + \frac{1}{n^-}\right)$ involves S is preferable to T
g.l. υ=+∞
<u>Theorem 4:</u> In balanced ANOVA model:

$$f_{max} = \begin{cases} f(1, I-1), I \geq 13 \\ f(2, I-2), I \geq 60 \end{cases}$$

$$\begin{cases} f\left(\frac{I}{2}, \frac{I}{2}\right), I\ even \\ f\left(\frac{I-1}{2}, \frac{I+1}{2}\right), I\ odd \end{cases}$$

Note. Here was noted: $[n^+, n^-]$ a contrast of a superior order. Numerical comparisons for I (or k=no. parameters.); α=0,05; α=0,01.
The numerical demonstration and comparisons are given in ([5] and [4]).
<u>Theorem 5:</u> If the correlation matrix G is almost singular, then eigen values and eigen vectors are used for obtaining $\hat{\beta}_{mod}$ l.s.m.-modified estimators (instead of $\hat{\beta}$ l.s.m.-estimators).
$\hat{\beta}$ and $\hat{\beta}_{mod}$ are calculated from (5) respectively (13) for ANOVA model.
The following inequation is obtained, which is used for Rayleigh ratio expression:

$$\frac{1}{\sqrt{\max \lambda_i}} \leq \frac{c^{'}c}{c^{'}Gc} \leq \frac{1}{\sqrt{\min \lambda_i}}$$

4. The calculation of eigen values and eigen vectors

The calculation technique of values and proper vectors of the matrices with triangular structure –the simple case, but also in the general case was largely treated in ([1]). Here a few aspects are mentioned on the problems that occur MLG and ANOVA discussed in chapter 3.
The eigen values are the roots of the equation:

$$\det(\lambda I_n - A) = 0$$

where A is a square matrix with real elements in the above mentioned context from chapter 2.
The nonzero vector with real elements a.i. Ax=λx is the proper vector of A matrix associated to the proper value λ.
<u>Consequence:</u> If A is diagonal or triangular then the proper values are the diagonal elements.
<u>Definition</u> The set of eigen values is called the matrix spectrum $A(\in R^{nxn}\ or\ C^{nxn})$:

$$\lambda(A) = \{\lambda_1, \lambda_2, \dots, \lambda_n\} = \{\lambda \in R^{nxn} / \det(\lambda I - A) = 0\}$$

The non-negative real number:

$$\rho(A) = \max\{|\lambda_1|, |\lambda_2|, \dots, |\lambda_n|\}$$

is named the spectral ray of the matrix A.

The following affirmations are true for$A \in R^{nxn}$:

1) The matrices: $A^k, k\epsilon N^*; A-\mu I_n; A^{-1}$ (if A is not singular) have the same eigen vectors with the matrix A with the spectrums

$$\lambda(A^k) = \{\lambda_1^k, \lambda_2^k, \dots, \lambda_n^k\}$$
$$\lambda(A-\mu I_n) = \{\lambda_1 - \mu, \lambda_2 - \mu, \dots, \lambda_n - \mu\}$$
$$\lambda(A^{-1}) = \{1/\lambda_1, 1/\lambda_2, \dots, 1/\lambda_n\}$$

2) The eigen values of A matrix satisfy the relations:

$$\sum_{i=1}^{n} \lambda_i = \sum_{i=1}^{n} a_{ii} =^{def} tr(A)$$

where tr(A) is the trace of the matrix A;

$$\prod_{i=1}^{n} \lambda_i = detA$$

3) Spectrum λ(A) of A matrix (hermitian) occurs between the extremes of the function:

$$\mu = \frac{x^H A x}{x^H x} : S \to R$$

where $S = \{x\epsilon R^n / \|x\|^2 = x^H x = 1\}$

is the unit sphere of radius=1 . H represents the cumulated operator of transposition and conjugation

4) A spectrum is considered downward:

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$$

The extremes of μ function are:

$$M = \max_{x \in S} x^H A x = \lambda_1$$
$$m = \min_{x \in S} x^H A x = \lambda_n$$

That goes back to the relation: $\lambda_n \leq \mu \leq \lambda_1$

5) It is important to know the eigen values and respectively the eigen vectors associated to A matrix. The problem becomes difficult when iterative calculation method is applied to a eigen vector with (and especially, without) the knowledge of proper associated values.

   In this last point, iterative calculation methods of a proper vector were developed among which the power method and inverse power method and we recommend the excellent cited monograph: ([1]).

Applications

Different principles of some stationary functions used in the simple approximation of functions that depend on two unknown constants of a multiplicative type or of an exponent form met in engineering applications is solved through the shown models. ([10])

The characteristic taken into study represents the operating time without faults, T, i.e. per total meaning the sum of the operating time until its total breakage: $T = \sum t_i$, where t_i is the operating time between two consecutive overhauls.

The following elements are calculated:

-the distribution of the hazard variable (r.v.) T;

- Statistical inference over T for reliability assessment (good functioning) and sustainability (lifetime, life testing) of T.

1. The use of Rayleigh method is proposed for calculating the superior limits of the first proper values. Rayleigh-Ritz method is preferred to some cases by justifying the simplicity and its reliability ([10]).

The examples are taken from the potential power field, π triggered at the primary elastic displacement, vertical from x=0 until la x=L (arbitrary) by the stationary condition:

$$\frac{\partial \pi}{\partial c} = 0 \; si \; \frac{\partial \pi}{\partial n} = 0$$

where they are obtained from n≈1,725; k≈2,4747.

The exact values are k=2,4674 for the particular case n=2.

Response function f($x_1, \dots x_N$) is to be determined on the basis of independent observations: $x_1, \dots x_N$ of r.v. x.

Different forms of the response function were proposed within the organized experiments and the ones that are best representative can be chosen i.e. with an accepted fault of the studied phenomenon.

2. Rayleigh ratio of (A,$y^{(k-1)}$) pair is the pseudo-solution meaning c.l.s.m. of the system:

$$y^{(k-1)}\mu_k = Ay^{(k-1)}$$

where

$$\mu_k = \frac{(y^{(k-1)})^H A y^{(k-1)}}{\|y^{(k-1)}\|^2} = (y^{(k-1)})^H A y^{(k-1)}$$

Example: Rayleigh method uses for determining the absolute frequency of a vibration: $f = \omega/2\pi$

5. Rayleigh general distribution

It starts with taking into consideration the distribution of generalized gamma $G(\theta, \alpha, \lambda, \nu), \theta > 0$ that has d.p. ([14],[15]):

$$f\big(x;(\theta,\alpha,\lambda,\nu\,)\big)=\begin{cases}\dfrac{\theta\lambda^{\nu}}{\Gamma(\nu)}(x-\alpha)^{\nu-1}e^{-|\lambda(x-\alpha)|^{\theta}}, x>\alpha\\ 0, in\ rest\end{cases}$$

Having the particular cases:

(i) $G(\theta=1,\alpha,\lambda,\nu)$= $G(\alpha,\lambda,\nu)$~ gamma distribution with d.p.:

$$f\big(x;(\alpha,\lambda,\nu\,)\big)=\begin{cases}\dfrac{\lambda^{\nu}}{\Gamma(\nu)}(x-\alpha)^{\nu-1}e^{-\lambda(x-\alpha)}, x>\alpha\\ 0, in\ rest\end{cases}$$

with $\alpha\in\mathbb{R};\lambda,\nu>0$ location and scale parameter, respectively the gamma function $\Gamma(\nu)$:

$$\Gamma(\nu)=\int_0^{\infty}x^{\nu-1}e^{-x}dx$$

ii) r.v. $X{\sim}G(0,1,\nu)$ from i) with α=0;λ=1

iii) If r.v. $X{\sim}G(0,1,\nu)$, then r.v. $y=\frac{1}{\lambda}X^{1/\theta}\sim G(0,1,\frac{\nu}{\theta})$

6. Consequences

1. Generation of r.v. Y is reducing to generation of r.v. $X\sim G\ (0,1,\nu)$
2. From the distribution of generalized gamma $G\ (\theta,\alpha,\lambda,\nu)\ \ \theta>0$ the above mentioned particular cases are obtained: $G\ (\alpha,\lambda,\nu)$ and $G(0,1,\nu)$.
3. From the distribution $G(0,1,\nu)$, is obtained the particular case of Weibull distribution:

   3.i) From the distribution $G(0,1,\nu)$ with $\nu=1$ r.v. is obtained $X\sim Expo(1)$.

   3.ii) If r.v. $X\sim Expo(1)$, then r.v.

   $W=\ \alpha+\left(\frac{X}{\lambda}\right)^{1/\nu}\sim W(\alpha,\lambda,\nu)$ with d.p.

   $$f_W(x)=\begin{cases}\nu\lambda(x-\alpha)^{\nu-1}e^{-\lambda(x-\alpha)^{\nu}},\ x\geq\alpha\\ 0,\quad in\ rest\end{cases}$$
4. For $\nu=2$ from the $W(\alpha,\lambda,2)$ distribution, the Rayleigh distribution is obtained ([2]).
5. Generally, r.v. $T{\sim}GRV$ (generalized Rayleigh model with d.p. : ([16])

   $$f(t,\theta,k)=\begin{cases}\dfrac{2\theta^{k+1}}{\Gamma(k+1)}t^{2k+1}e^{-\theta t^2},\quad t\geq 0,\theta>0,k>-1\\ 0,\quad in\ rest\end{cases}$$

   And for $k=0,\theta=\frac{1}{\eta^2}$ the Rayleigh model is obtained with d.p. $f_R(x,\theta)=2\theta xe^{-\theta x^2}$

6. Exposed program packages were designed that will be subsequently applied to concrete data. The finite approach assumes that the sample is a mixture of ingredients in an unknown number but limited. Each component has a specified distribution.

Relative to the Ω model, two types of finite SEM mixtures are distinguished (Structural Equation Modelling). Unconditioned models (Yung, 1997; Jedidi, Jag pal, de Sarbo 1997; Dolan, van der Maas, 1998) use the hypothesis as the endogenous and exogenous variables have the normal law multivariable with different components.

The conditioned models have the hypothesis more weakened but more realistic that assumes that the dependable variables have the normal law conditioned by some certain exogenous variables regression (i.e. demographically).

One of the programs for estimating the situation of finite mixture of the conditioned models is MECOSA (Arminger, Wittenberg, Schepersm, 1996;Mplus, Muthen, ~, 1998).

In the last 30 years, the LISREL model has become a synonym to SEM.

SEM allows the researchers from the social sciences, science management, biological science and educational science and others to empiric establish their theory. These theories are formulated as theoretical models for observed and latent (unobserved) variables. If the data is taken for the observable variables of the theoretical model, the LISREL program may be used to fit the model to the data.

LISCOMP (the analysis of the equations with the Linear Structure with a comprehensive measuring model) is a modelling program with the structure equation adequate to the qualitative data (dichotomic polytomous arranged) and continous multidimensional non-normal data. The standard aspects are included for continous multidimensioned normal data and mixtures of variable types. LISCOMP may be used for Monte-Carlo studies with the automatic generation of data.

MATLAB-is simpler to use in different applications instead of using the above softwares for particular problems.

MATLAB is used for SEM in the field of fMRI problem (functional MRI or functional Magnetic Resonance Imaging) a special MRI scanning type. The hemodynamic response is measured (change in the blood flow) relatively to the neural brain activity or the spinal cord in humans or animals.

Since 1990 fMRI is to dominate the image due to the relative low aggressiveness, the absence of radiation exposure and the relative large approach.

The standard approach for free SEM is usually taken with the help of Mx software.

Unfortunately, it is difficult to use Mx for high data sets when we are to use fMRI-the source of the code for fMRI is not available.

It is easier to program the calculations by using MATLAB which is the platform for SPM, the most popular software package for data analyze with fMRI.

As a result, the SEM-MATLAB code for the fMRI analysis is necessary.

7. Conclusion

The GRV model, recently presented, is brought in the attention of researchers and engineers , being a subtle "under cover" of the other models that were presented.

Regarding the work of authors, we are thinking of the theory: estimation of the GRV parameters and the statistic inference for this model, but also in new fields of application and concrete examples.

R E F E R E N C E S

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