ADAPTIVE CUBATURE KALMAN FILTERING WITH NOISE STATISTIC ESTIMATION BASED ON EXPECTATION-MAXIMUM ALGORITHM WITH OPTIMIZED MOVING HORIZON STEPS

Xiaoyi ZHENG[1*], Xiaomin FANG[2], Peijiang LI[2]

For noise statistic estimation problem in nonlinear Gaussian systems, an adaptive Cubature Kalman Filtering (CKF) with noise statistic estimation is presented in this paper, while the noise statistic estimation is derived by an expectation-maximum (EM) algorithm with optimized moving horizon steps. Firstly, we propose a common form of nonlinear Gaussian system that satisfies some assumptions in order to facilitate further research, and some reasonable assumptions are presented. Secondly, the maximum likelihood principle (MLP) based noise statistic estimation model is proposed, then the moving horizon estimation (MHE) is used to optimize this model. One step further, EM Algorithm is introduced to iteratively estimate the noise statistics, and the moving horizon steps is optimized by the given index to reduce the computational cost while maintaining the accuracy of calculations. Finally, an experiment is implemented and three common situations are conducted to verify the proposed algorithm from different perspectives, and the experimental results show the effectiveness of our work product.

Keywords: Noise Statistic Estimation, CKF, MLP, EM Algorithm, MHE

1. Introduction

In recent years, various control scenarios and control systems have increasingly high requirements for filtering accuracy [1]. Due to its ability to achieve high filtering accuracy, nonlinear filtering algorithms have received widespread attention from experts and scholars in related fields [2,3]. The extended Kalman filter (EKF) is a popular nonlinear filtering algorithm, and it is based on the idea of performing first-order Taylor expansion on the nonlinear system model and then using the Kalman filter algorithm for calculation [4]. However, for strongly nonlinear systems, significant estimation errors or even divergence occurred when EKF is adopted. Moreover, the complex calculation of Jacobian matrix also limits the application of EKF in practical problems [5,6].

Based on the above problems, the researchers from McMaster University, Ienkaran Arasaratnam and Simon Haykin [7] proposed a new filtering algorithm

[1] Associate Prof., Dept. of Information Engineering, Quzhou College of Technology, Quzhou, China, e-mail: zxy_xmu@163.com.
[2] Prof., Dept. of Information Engineering, Quzhou College of Technology, Quzhou, China.

named CKF. The detailed description of CKF can be found in literature [7], and it is omitted here. The same with EKF algorithm, when we use the CKF to solve nonlinear problems, the prior statistical characteristics of system noise are also assuming already known. However, the assumption is difficult to satisfy in most practical situations, leading to a decrease in filtering accuracy or even divergence [8-10].

Regarding the existing issues, it is necessary to conduct research on estimating the statistical characteristics of noise, and some related works have been done in last few years [11-13]. In literature [14], an adaptive SRCKF algorithm with noise estimator is designed by introducing the principles of strong tracking filter and maximum a posterior (MAP). In literature [15], a new adaptive UKF based on MAP and random weighting is proposed. In literature [16], a singular value decomposition and maximum likelihood criterion combined adaptive CKF algorithm is presented to apply in integrated navigation systems. In literature [17], a noise estimation and filtering method is proposed by combining EM algorithm and suboptimal unbiased MAP to form an adaptive UKF. In literature [18], the authors present a new adaptive Kalman filter based on moving weighted average and MLP, and the estimated result is optimized with computationally efficient. There are also some other results and developments in this domain, see in literatures [19-21] and the references therein.

From the above literature, we can see the methods of MAP, MLP, EM and so on are usually used in the problem of noise statistic estimation, but the calculation complexity are always not considered. In our work, we deal with the problem of process noises and measurement noises statistic estimation based on MLP and EM algorithm, and the calculation complexity is considered simultaneously. The main contributions of this article are as follows. (1) The MHE concept is introduced to optimize the noise statistic estimation model in order to reduce the calculation complexity, and the moving horizon steps is calculated by the index function defined by us. (2) By introducing the EM algorithm, the iterative calculation process is accomplished to estimate the noise statistics. Therefore, the computed result can be given step by step with efficiency. In addition, an experiment with three situations is conducted, and shows the effectiveness of the scheme we propose clearly.

We make the arrangements for the rest of our work as follows. In part 2, a common form of nonlinear Gaussian system is presented, and some reasonable assumptions are stated. In Part 3, noise statistic estimation model based on MLP is designed, then the model is optimized by MHE, based on which EM algorithm is introduced to achieve system noise statistic estimation. In part 4, an illustrated example is presented to prove the availability of our proposed scheme, and conclusion is given in part 5.

2. Problem formulation

Firstly, we consider the following nonlinear Gaussian system:

$$\begin{cases} x_k = f\left(x_{k-1}\right) + v_{k-1} \\ z_k = h\left(x_k\right) + w_k \end{cases} \tag{1}$$

where $x_k \in R^{n_x}$ denotes the state vector of the system; $z_k \in R^{n_z}$ denotes the measurement vector of the system; $f\left(\cdot\right): R^{n_x} \times R^{n_u} \to R^{n_x}$ and $h\left(\cdot\right): R^{n_x} \times R^{n_u} \to R^{n_z}$ are the nonlinear state function and measurement function, respectively; $v_k \in R^{n_x}$ and $w_k \in R^{n_z}$ are process noises and measurement noises satisfy constraints as below:

$$\begin{cases} E\left[v_k\right] = 0 \\ E\left[w_k\right] = 0 \\ E\left[v_k w_j^T\right] = 0 \end{cases} \tag{2}$$

and

$$\begin{cases} E\left[v_k v_j^T\right] = Q\delta_{kj} \\ E\left[w_k w_j^T\right] = R\delta_{kj} \end{cases} \tag{3}$$

which means that Q is a non-negative definite symmetric matrix about v_k with $n_x \times n_x$ dimensions, meanwhile R is a positive definite symmetric matrix about w_k with $n_z \times n_z$ dimensions, δ_{kj} is the $Kronecker-\delta$ function.

For simplicity, the following notations are used:

$$[1] = \left\{ \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \cdots, \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ \vdots \\ 0 \end{pmatrix}, \cdots, \begin{pmatrix} 0 \\ 0 \\ \vdots \\ -1 \end{pmatrix} \right\},$$

$$\xi_i = \sqrt{\frac{m}{2}}[1]_i,$$

$$\omega_i = \frac{1}{m}$$

where $i = 1, 2, 3, \cdots\cdots, m$, $m = 2n_x$, m denotes the total number of cubature points, $[1]_i$ is the ith column of $[1]$.

Remark 1:

The nonlinear Gaussian system (1) describes a common class of system model with additive noises from both internal and external sources that often be seen in various domains, such as aerospace, automotive engineering, power system, manufacturing industry and so on. Our goal is to estimate the additive noises with three general situations, one of Q and R is unknown and Q and R are both unknown, respectively.

Remark 2:

The CKF algorithm provides a novel scheme for nonlinear systems to handle the filtering problems, it's main core is a spherical-radial cubature rule used to calculate cubature points. There are two basic steps about the CKF, for the first step, the history posterior density at previous step is used to calculate the predicted error covariance, while for the second step, the calculated error covariance at first step is used to compute the error covariance, and it is obviously an iterative process.

3. Noise Statistic Estimation based on EM Algorithm

From the above, we can see that the effectiveness of CKF is influenced by noise statistics Q and R, and we introduce a novel adaptive CKF that combines EM algorithm and MLP approach to estimate Q and R effectively.

3.1 Noise Statistic Estimation Model based on MLP

Let $\theta = \{Q, R\}$, based on the MLP, we can estimate θ as follows:

$$\theta = \arg\max_{\theta} \left\{ \ln \left[L\left(\theta \middle| z_{1:k}, x_{0:k}\right) \right] \right\} \tag{4}$$

where $L\left(\theta \middle| z_{1:k}, x_{0:k}\right)$ is the likelihood function about θ. Then we have:

$$L\left(\theta \middle| z_{1:k}, x_{0:k}\right) = p\left(x_{0:k} \middle| \theta\right) p\left(z_{1:k} \middle| x_{0:k}, \theta\right) \tag{5}$$

On account of the system (1) is a first-order Markov process [10], then (5) is factorized as:

$$p\left(z_{1:k}, x_{0:k} \middle| \theta\right) = p\left(x_0 \middle| \theta\right) \times \prod_{j=1}^{k} p\left(x_j \middle| x_{j-1}, \theta\right) \times \prod_{j=1}^{k} p\left(z_j \middle| x_j, \theta\right) \tag{6}$$

Provided that the initial state and the noises obey normal distribution, then (6) can be rewritten as:

$$p\left(z_{1:k},x_{0:k}\left|\theta\right.\right)=\frac{1}{\left(2\pi\right)\left|P_0\right|^{1/2}}\exp\left[-\frac{\left(x_0-x_0\right)^T P_0^{-1}\left(x_0-x_0\right)}{2}\right]$$
$$\times\prod_{j=1}^{k}\frac{1}{\left(2\pi\right)^{n_x/2}\left|Q\right|^{1/2}}\exp\left[-\frac{v_j^T Q^{-1}v_j}{2}\right] \tag{7}$$
$$\times\prod_{j=1}^{k}\frac{1}{\left(2\pi\right)^{n_z/2}\left|R\right|^{1/2}}\exp\left[-\frac{w_j^T R^{-1}w_j}{2}\right]$$

Take the logarithm of (7) both side, we obtain:

$$\ln\left[L\left(\theta\left|z_{1:k},x_{0:k}\right.\right)\right]=-\ln\left(2\pi\right)-\frac{1}{2}\ln\left|P_0\right|-\frac{\left(x_0-\hat{x}_0\right)^T P_0^{-1}\left(x_0-\hat{x}_0\right)}{2}$$
$$-\frac{kn_x}{2}\ln\left(2\pi\right)-\frac{k}{2}\ln\left|Q\right|-\frac{1}{2}\sum_{j=1}^{k}v_j^T Q^{-1}v_j \tag{8}$$
$$-\frac{kn_z}{2}\ln\left(2\pi\right)-\frac{k}{2}\ln\left|R\right|-\frac{1}{2}\sum_{j=1}^{k}w_j^T R^{-1}w_j$$

According to the above, θ can be obtained as follows:

$$\hat{\theta}=\arg\max_{\theta}\left\{\ln\left[L\left(\theta\left|z_{1:k},x_{0:k}\right.\right)\right]\right\}$$
$$=\arg\max_{\theta}\left\{\begin{array}{l}-\ln\left(2\pi\right)-\frac{1}{2}\ln\left|P_0\right|-\frac{\left(x_0-\hat{x}_0\right)^T P_0^{-1}\left(x_0-\hat{x}_0\right)}{2}\\ -\frac{kn_x}{2}\ln\left(2\pi\right)-\frac{k}{2}\ln\left|Q\right|-\frac{1}{2}\sum_{j=1}^{k}v_j^T Q^{-1}v_j\\ -\frac{kn_z}{2}\ln\left(2\pi\right)-\frac{k}{2}\ln\left|R\right|-\frac{1}{2}\sum_{j=1}^{k}w_j^T R^{-1}w_j\end{array}\right\} \tag{9}$$
$$=\arg\min_{\theta}\left\{\begin{array}{l}\left(1+\frac{kn_x}{2}+\frac{kn_z}{2}\right)\ln\left(2\pi\right)+\frac{1}{2}\ln\left|P_0\right|+\frac{k}{2}\ln\left|Q\right|+\frac{k}{2}\ln\left|R\right|+\\ \frac{\left(x_0-\hat{x}_0\right)^T P_0^{-1}\left(x_0-\hat{x}_0\right)}{2}+\frac{1}{2}\sum_{j=1}^{k}v_j^T Q^{-1}v_j+\frac{1}{2}\sum_{j=1}^{k}w_j^T R^{-1}w_j\end{array}\right\}$$

Remark 3:

According to the above deduction, the summation range is from $j=1$ to k, and with the calculation process continues, the computational range will continue to increase as well, this is unrealistic when dealing with practical problems. To solve this problem, we introduce the MHE as below.

3.2 Noise Statistic Estimation Model Optimized by MHE

Denote the moving horizon steps as N, let $X_N=\left\{x_j : j=k-N+1,\cdots\cdots,k\right\}$, $Z_N=\left\{z_j : j=k-N+1,\cdots\cdots,k\right\}$, then θ can be rewritten as:

$$\theta=\arg\max\left\{\ln\left\{L\left(\theta\middle|Z_N,X_N\right)\right\}\right\} \tag{10}$$

where $L\left(\theta\middle|Z_N,X_N\right)$ can be calculated as follows:

$$\begin{aligned} L\left(\theta\middle|Z_N,X_N\right)&=p\left(Z_N,X_N\middle|\theta\right)\\ &=p\left(x_{k-N}\middle|\theta\right)\times\prod_{j=k-N+1}^{k}p\left(x_j\middle|x_{j-1},\theta\right)\times\prod_{j=k-N+1}^{k}p\left(z_j\middle|x_j,\theta\right) \end{aligned} \tag{11}$$

To simplify the analysis, assume that $p\left(x_{k-N}\middle|\theta\right)\sim N\left(\hat{x}_{k-N},P_{k-N}\right)$ is the common practice, then (11) can be further rewritten as:

$$\begin{aligned} L\left(\theta\middle|Z_N,X_N\right)=&\frac{1}{(2\pi)\left|P_{k-N}\right|^{1/2}}\exp\left[-\frac{\left(x_{k-N}-\hat{x}_{k-N}\right)^T P_{k-N}^{-1}\left(x_{k-N}-\hat{x}_{k-N}\right)}{2}\right]\\ &\times\prod_{j=k-N+1}^{k}\frac{1}{(2\pi)^{n_x/2}|Q|^{1/2}}\exp\left[-\frac{v_j^TQ^{-1}v_j}{2}\right]\\ &\times\prod_{j=k-N+1}^{k}\frac{1}{(2\pi)^{n_z/2}|R|^{1/2}}\exp\left[-\frac{w_j^TR^{-1}w_j}{2}\right] \end{aligned} \tag{12}$$

Take the logarithm of (12) both sides, we obtain:

$$\begin{aligned} \ln\left[L\left(\theta\middle|Z_N,X_N\right)\right]=&-\ln(2\pi)-\frac{1}{2}\ln\left|P_{k-N}\right|-\frac{\left(x_{k-N}-\hat{x}_{k-N}\right)^T P_{k-N}^{-1}\left(x_{k-N}-\hat{x}_{k-N}\right)}{2}\\ &-\frac{Nn_x}{2}\ln(2\pi)-\frac{N}{2}\ln|Q|-\frac{1}{2}\sum_{j=k-N+1}^{k}v_j^TQ^{-1}v_j\\ &-\frac{Nn_z}{2}\ln(2\pi)-\frac{N}{2}\ln|R|-\frac{1}{2}\sum_{j=k-N+1}^{k}w_j^TR^{-1}w_j \end{aligned} \tag{13}$$

According to the above, θ can be obtained as follows:

$$\hat{\theta}=\arg\min_{\theta}\left\{\begin{array}{l}\left(1+\frac{Nn_x}{2}+\frac{Nn_z}{2}\right)\ln(2\pi)+\frac{1}{2}\ln\left|P_{k-N}\right|+\frac{N}{2}\ln|Q|+\frac{N}{2}\ln|R|+\\+\frac{\left(x_{k-N}-\hat{x}_{k-N}\right)^T P_{k-N}^{-1}\left(x_{k-N}-\hat{x}_{k-N}\right)}{2}+\frac{1}{2}\sum_{j=k-N+1}^{k}v_j^T Q^{-1}v_j+\frac{1}{2}\sum_{j=k-N+1}^{k}w_j^T R^{-1}w_j\end{array}\right\} \tag{14}$$

3.3 Noise Statistic Estimation with EM Algorithm

Based on the above-mentioned analyses, then the EM algorithm is introduced to iteratively estimate the noise statistics.

(1) E step:

Firstly, we calculate the expectation of log-likelihood function:

$$\begin{aligned}E\ln\left[-L\left(\theta|Z_N,X_N\right)\right]=&\left(1+\frac{Nn_x}{2}+\frac{Nn_z}{2}\right)\ln(2\pi)+\frac{1}{2}\ln\left|P_{k-N}\right|+\frac{N}{2}\ln|Q|+\frac{N}{2}\ln|R|+\\&\frac{1}{2}E\left\{tr\left[P_{k-N}^T\left(x_{k-N}-\hat{x}_{k-N}\right)\left(x_{k-N}-\hat{x}_{k-N}\right)^T\right]\right\}+\\&\frac{1}{2}E\left\{\sum_{j=k-N+1}^{k}tr\left(Q^{-1}v_j^T v_j\right)\right\}+\frac{1}{2}E\left\{\sum_{j=k-N+1}^{k}tr\left(R^{-1}w_j^T w_j\right)\right\}\end{aligned} \tag{15}$$

Then ignoring the constant term of (15), let:

$$\begin{aligned}J=&\ln\left|P_{k-N}\right|+N\ln|Q|+N\ln|R|+E\left\{tr\left[P_{k-N}^T\left(x_{k-N}-\hat{x}_{k-N}\right)\left(x_{k-N}-\hat{x}_{k-N}\right)^T\right]\right\}+\\&E\left\{\sum_{j=k-N+1}^{k}tr\left(Q^{-1}v_j v_j^T\right)\right\}+E\left\{\sum_{j=k-N+1}^{k}tr\left(R^{-1}w_j w_j^T\right)\right\}\end{aligned} \tag{16}$$

(2) M step:

Let $\frac{\partial J}{\partial Q}=0$ and $\frac{\partial J}{\partial R}=0$ respectively, system noise estimator can be obtained as follows:

$$\hat{Q}_k=\frac{1}{N}\sum_{j=k-N+1}^{k}\left(x_j x_j^T-x_j f^T\left(x_{j-1}\right)-f\left(x_{j-1}\right)x_j^T+f\left(x_{j-1}\right)f^T\left(x_{j-1}\right)\right) \tag{17}$$

and

$$\hat{R}_k=\frac{1}{N}\sum_{j=k-N+1}^{k}\left(z_j z_j^T-z_j h^T\left(x_j\right)-h\left(x_j\right)z_j^T+h\left(x_j\right)h^T\left(x_j\right)\right) \tag{18}$$

Substituting the filter estimate into the above equations (17) and (18), sub-optimal noise estimator is concluded:

$$\hat{Q}_k \approx \frac{1}{N}\sum_{j=k-N+1}^{k}\begin{bmatrix}\hat{x}_{j|j}\hat{x}_{j|j}^T + P_{j|j} - \frac{1}{m}\sum_{i=1}^{m} X_{i,j|j-1} f^T\left(X_{i,j-1|j-1}\right) - \\ \frac{1}{m}\sum_{i=1}^{m} f\left(X_{i,j-1|j-1}\right) X_{i,j|j-1}^T + \frac{1}{m}\sum_{i=1}^{m} f\left(X_{i,j-1|j-1}\right) f^T\left(X_{i,j-1|j-1}\right)\end{bmatrix} \tag{19}$$

$$\hat{R}_k \approx \frac{1}{N}\sum_{j=k-N+1}^{k}\begin{bmatrix} z_j z_j^T - \frac{1}{m}\sum_{i=1}^{m} z_j h^T\left(X_{i,j|j-1}\right) - \frac{1}{m}\sum_{i=1}^{m} h\left(X_{i,j|j-1}\right) z_j^T + \\ \frac{1}{m}\sum_{i=1}^{m} h\left(X_{i,j|j-1}\right) h^T\left(X_{i,j|j-1}\right)\end{bmatrix} \tag{20}$$

Remark 4:

From (19) and (20), we can see that the calculate of the obtained sub-optimal noise estimator is a process of iterative summation, and the summation range is from $k-N+1$ to k. In order to reduce computational complexity, when we design program code to calculate the noise at time k, the previous calculation result at time $k-1$ can be used, so the summation from $k-N+1$ to $k-1$ is omitted, this operation can significantly reduce the computational burden.

3.4 Calculation of Optimized Moving Horizon Steps

Both considering the accuracy and speed of noise estimation, we need to choose the optimized steps of moving horizon.

Provided that the minimum steps of moving horizon is p, and the maximum steps is q, then we should initialize some related parameters in time domain from first step to q^{th} step. The initializing method is presented as follows.

We define the index function $f(\cdot) = \Gamma_1 t_k + \Gamma_2 l_k$, where t_k is the elapsed time of the estimation algorithm at sampling time k, l_k is the accuracy index of the algorithm defined as (37). Γ_1 and Γ_2 are two known constant represent the weight value of accuracy and speed and satisfy that $\Gamma_1 + \Gamma_2 = 1$.

$$l_k = \left[z_k - \frac{1}{m}\sum_{i=1}^{m} h\left(X_{i,k|k-1}\right)\right]^T \left[z_k - \frac{1}{m}\sum_{i=1}^{m} h\left(X_{i,k|k-1}\right)\right] \tag{21}$$

Then the calculation of optimized moving horizon steps can be described as follows:

$$\begin{aligned}&\min\ \Gamma_1 t_k + \Gamma_2 l_k \\ &\text{subject to } p \le k \le q \text{ and } k \in N^+\end{aligned} \tag{22}$$

Remark 5:

In order to keep the validity of the optimal steps in the moving horizon estimation, while reduce the computational cost, it is suggested to update the optimal steps every certain number of steps.

4. Performance Evaluation and Discussions

At last, an experiment is implemented to prove the availability of the scheme we propose. The following nonlinear Gaussian system is given, and satisfies constraints (2) and (3):

$$x_k = 0.5x_{k-1} + \frac{0.2x_{k-1}}{1+x_{k-1}^2} + v_{k-1} \quad (23)$$

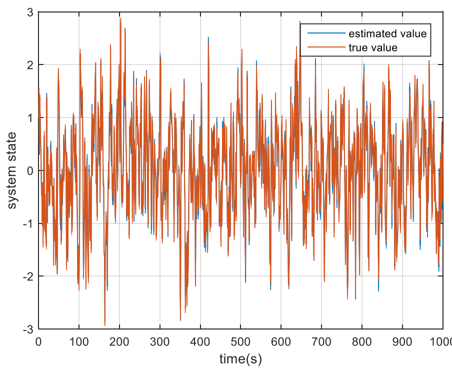
$$z_k = 10x_k + \frac{x_k^2}{20} + v_k \quad (24)$$

Here, the theoretical initial state of the systems are set as $x_0 = 1$, while the Kalman filter initial parameters are set as $\hat{x}_0 = 1.1$ and $P_0 = 0.01$. The variances of v_k and w_k are $Q = 0.6$ and $R = 0.8$, respectively.

On the basis of above given parameters, we present three simulation results corresponding to three situations, separately.

4.1 Situation A: Q known, R unknown

In this situation, we assume that the variance of v_k $Q = 0.6$ is known, while the variance of w_k $R = 0.8$ is unknown, simulation results are displayed as follows. Fig. 1 shows the system state x_k, Fig. 2 shows the measurement noise variance R, Fig. 3 shows the estimated error of system state.



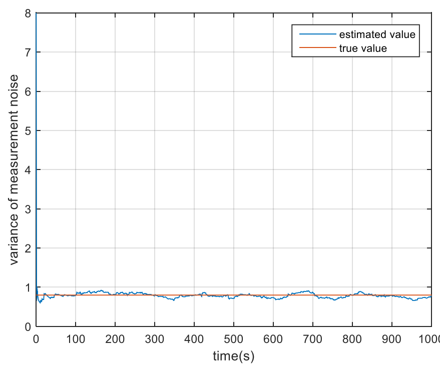


Fig. 1 system state and its estimated value(situation A) Fig. 2 variance of measurement noise(situation A)

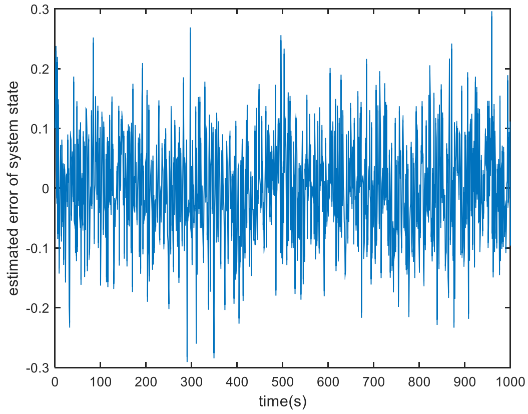
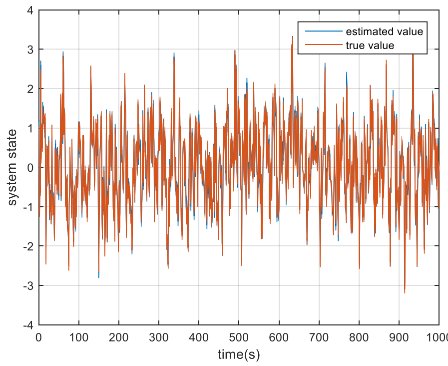


Fig. 3 estimated error of system state(situation A)

4.2 Situation B: Q unknown, R known

In this situation, we assume that the variance of v_k $Q = 0.6$ is unknown, while the variance of w_k $R = 0.8$ is known, simulation results are displayed as follows. Fig. 4 shows the system state x_k, Fig. 5 shows the process noise variance Q, Fig. 6 shows the estimated error of system state.



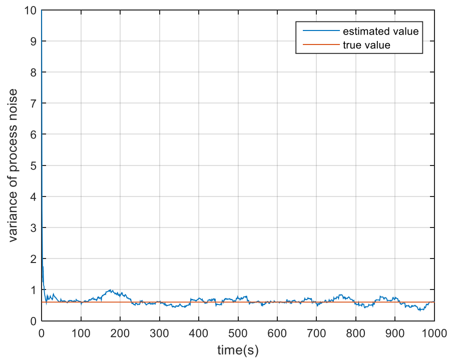


Fig. 4. system state and its estimated value (situation B) Fig. 5. variance of process noise (situation B)

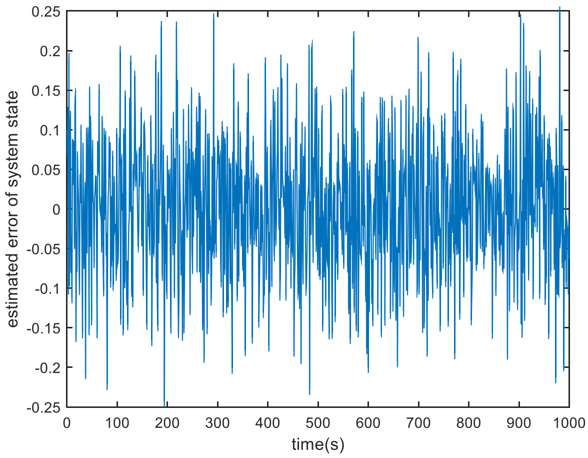
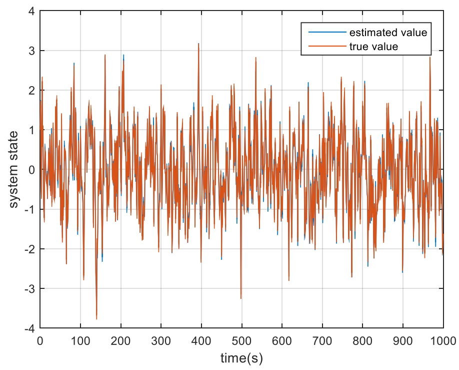


Fig. 6 estimated error of system state(situation B)

4.3 Situation C: Q unknown, R unknown

In this situation, we assume that the variance of v_k $Q=0.6$ and the variance of w_k $R=0.8$ are all unknown, simulation results are displayed as follows. Fig. 7 shows the system state x_k, Fig. 8 shows process noise variance Q, Fig. 9 shows the measurement noise variance R, Fig. 10 shows the estimated error of system state.



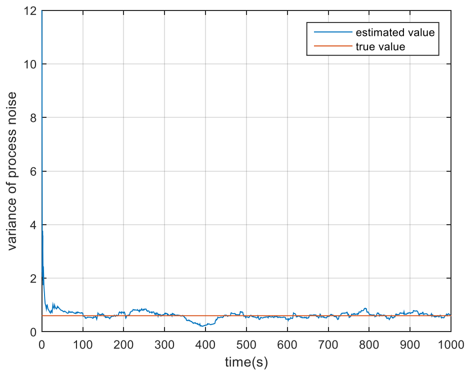


Fig. 7 system state and its estimated value (situation C) Fig. 8 variance of process noise(situation C)

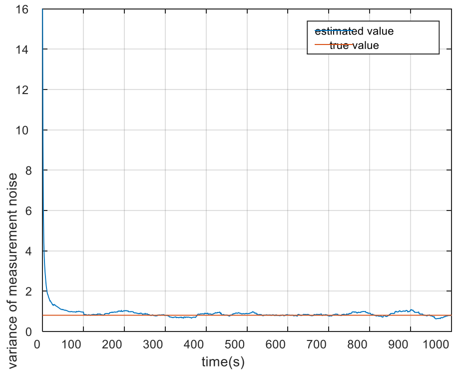


Fig. 9 variance of measurement noise (situation C)

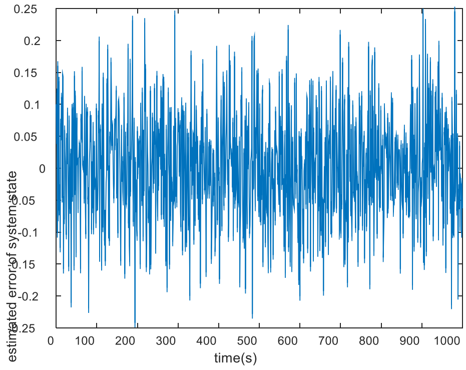


Fig. 10 estimated error of system state(situation C)

Remark 6:

On the above, we design three situations to test the validity of the algorithm we propose. The test results indicate that in all three situations, the noises can be quickly estimated in a relative small error, and the calculated state value can track the actual state well.

Remark 7:

At the same time, we should also recognize the limitations of the scheme we propose. Firstly, Γ_1 and Γ_2 in the index function $f(\cdot) = \Gamma_1 t_k + \Gamma_2 l_k$ are assumed to be known, therefore the selection of Γ_1 and Γ_2 values will affect the experimental results. Secondly, the experiment we designed only consider the first-order scenario, while higher-order scenarios are more common in reality.

5. Conclusions

The problem of adaptive CKF with noise statistic estimator based on MLP and EM algorithm is investigated in this paper. In order to reduce the calculation complexity, the moving horizon steps is introduced and optimized by the given index function. The scheme is verified by an illustrative example with three situations, and the noise statistic estimator can estimate the noises well, meanwhile the adaptive CKF with the noise statistic estimator give a good performance to estimate the system state. Our next step of work is how to calculate the moving horizon steps by using neural network algorithms, such as Genetic Algorithm to improve the steps calculation.

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REFERENCES

[1] Urrea, Claudio & Agramonte, Rayko. (2021). Kalman Filter: Historical Overview and Review of Its Use in Robotics 60 Years after Its Creation. Journal of Sensors. 2021. 1-21. 10.1155/2021/9674015.

[2] Mao, Jingyang & Sun, Ying & Yi, Xiaojian & Liu, Hongjian & Ding, Derui. (2021). Recursive filtering of networked nonlinear systems: a survey. International Journal of Systems Science. 52. 1-19. 10.1080/00207721.2020.1868615.

[3] Jiang S, Shi J, Moura S. A New Framework for Nonlinear Kalman Filters[J]. arxiv preprint arxiv:2407.05717, 2024.

[4] Setoodeh, P.; Habibi, S.; Haykin, S. Nonlinear Filters: Theory and Applications; John Wiley & Sons: Hoboken, NJ, USA, 2022.

[5] Wang, Lu & Li, Guang-Chun & Qiao, Xiang-Wei & Wang, Zhao-Long & Ma, Tao. (2012). An Adaptive UKF Algorithm Based on Maximum Likelihood Principle and Expectation Maximization Algorithm. Acta Automatica Sinica. 38. 1200. 10.3724/SP.J.1004.2012.01200.

[6] Özkurt, Gizem & Zerdali, Emrah. (2022). Design and Implementation of Hybrid Adaptive Extended Kalman Filter for State Estimation of Induction Motor. IEEE Transactions on Instrumentation and Measurement. 71. 7500212. 10.1109/TIM.2022.3144729.

[7] Arasaratnam, Haran & Haykin, Simon. (2009). Cubature Kalman Filters. Automatic Control, IEEE Transactions on. 54. 1254 - 1269. 10.1109/TAC.2009.2019800.

[8] Lee A S, Hilal W, Gadsden S A, et al. Combined Kalman and sliding innovation filtering: An adaptive estimation strategy[J]. Measurement, 2023, 218: 113228.

[9] Rout S, Das S. A robust modified adaptive extended Kalman filter for state-of-charge estimation of rechargeable battery under dynamic operating condition[J]. Electrical Engineering, 2024: 1-20.

[10] Uribe-Murcia K, Ibarra-Manzano O G, Andrade-Lucio J A, et al. Moving Vehicle Tracking Under Measurement Uncertainties, Multi-Step Random Delays, and Packet Dropouts[J]. IEEE Access, 2023, 11: 52381-52391.

[11] Gao, Bingbing & Gao, Shesheng & Hu, Gaoge & Zhong, Yongmin & Gu, Chengfan. (2017). Maximum likelihood principle and moving horizon estimation based adaptive unscented Kalman filter. Aerospace Science and Technology. 73. 10.1016/j.ast.2017.12.007.

[12] Li, Qiangqiang & Chen, Zhiyong & Shi, Wenku. (2022). A novel state estimation approach for suspension system with time-varying and unknown noise covariance. 10.21203/rs.3.rs-2147338/v1.

[13] Prakash, Rahul & Dheer, Dharmendra. (2023). Vehicle state estimation using a maximum likelihood based robust adaptive extended kalman filter considering unknown white Gaussian process and measurement noise signal. Engineering Research Express. 5. 10.1088/2631-8695/acd73e.

[14] Liqiang, Zhao & Jianlin, Wang & Tao, Yu & Huan, Jian & Tangjiang, Liu. (2015). Design of adaptive robust square-root cubature Kalman filter with noise statistic estimator. Applied Mathematics and Computation. 256. 10.1016/j.amc.2014.12.036.

[15] Gao, Zhaohui & Mu, Dejun & Gao, Shesheng & Zhong, Yongmin & Gu, Chengfan. (2017). Adaptive Unscented Kalman Filter Based on Maximum Posterior and Random Weighting. Aerospace Science and Technology. 71. 10.1016/j.ast.2017.08.020.
[16] Li S, Ma M, Tang X, et al. Maximum Likelihood Criterion based Adaptive CKF Algorithm for Integrated Navigation[C]//Proceedings of the 11th China Satellite Navigation Conference, Chengdu, China. 2020: 23-25.
[17] Guangwu C, Yue Y U, Wenyuan L I, et al. Noise estimation and filtering method of MEMS gyroscope based on EMMAP[J]. Journal of Measurement Science & Instrumentation, 2021 (2).
[18] Fu, Hongpo & Cheng, Yongmei. (2022). A novel maximum likelihood and moving weighted average based adaptive Kalman filter. Journal of Instrumentation. 17. P08036. 10.1088/1748-0221/17/08/P08036.
[19] Jiang, Chao & Wang, Zhiling & Liang, Huawei & Wang, Yajun. (2023). A Novel Adaptive Noise Covariance Matrix Estimation and Filtering Method: Application to Multiobject Tracking. IEEE Transactions on Intelligent Vehicles. PP. 1-15. 10.1109/TIV.2023.3286979.
[20] Gu, Peng & Jing, Zhongliang & Wu, Liangbin. (2024). A novel adaptive maximum correntropy cubature Kalman filter based on multiple fading factors. Transactions of the Institute of Measurement and Control. 10.1177/01423312241229039.
[21] Wang, Zili & Cheng, Jianhua & Qi, Bing & Cheng, Sixiang & Chen, Sicheng. (2023). An adaptive Kalman filtering algorithm based on maximum likelihood estimation. Measurement Science and Technology. 34. 10.1088/1361-6501/ace9ef.