PLANE HARMONIC WAVES IN ROTATING MEDIUM UNDER THE EFFECT OF MICRO-TEMPERATURE AND DUAL-PHASE-LAG THERMOELASTICITY

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The present work is supposed to analyze the propagation of plane harmonic waves in a homogeneous isotropic medium in the context of generalized dual phase lag model of thermoelasticity. The concept micro-temperature where the microelements have different temperatures is considered. Further, the medium is set on rotation with some specific rotating frequency to examine the wave propagation in rotating media like Earth. The governing equations for present model are solved by using Normal mode analysis method. The theoretical results are obtained for displacement, stress, temperature and micro-temperature distributions, which are also plotted against vertical and horizontal distances. It is found that rotation increases the absolute value of amplitude for plane wave propagating through the medium. By increasing rotation harmonic behavior of wave also increases which in result reduces the attenuation factor.

Keywords: Time Harmonic waves; Thermo-elasticity; Rotation; Micro-temperature; Dual phase Lag; Normal Mode Analysis.

1. Introduction

Studies of propagation of elastic waves in a medium have long been interest to researchers in the field of gophysics, acoustics and nondestructive evaluation. Eringen [1] introduced a general theory of non-linear micro elastic continuum in which the balance laws of continuum mechanics are supplemented and the intrinsic motions of the microelements contained in a macro-volume are taken into account. Grot [2] extended the thermodynamics of a continuum with microstructure by assuming that the microelements have different temperatures. To describe this phenomenon the concept of micro-temperatures is introduced. Riha [3] presented a study of heat conduction in materials with micro-temperatures. Iesan and Quintanilla [4] developed a linear theory of thermoelastic

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material with micro-temperatures, in which the particles of the material are subjected to classical displacement and temperature field, and may possess micro-temperatures. Iesan [5] showed that there exists coupling of micro-rotation vectors fields with the micro-temperatures even for isotropic bodies. Later, Iesan and Quintanilla [6] modified their theory to include macro-temperature and developed the theory of thermoelastic bodies with inner structure and micro-temperatures, which permits the transmission of heat as thermal waves at finite speed. Several experts worked on the extensions of the theory, some notable authors are Iesan and Scalia [7], Svanadze [8], Steeb et al. [9], Singh [10] and Kumar et al. [11].

Fourier law after changing becomes $q\left(P,t+\tau_q\right)= -\left[k\nabla T\left(P,t+\tau_T\right)\right]$. The τ_q gives relaxation time of thermal inertia and τ_T is the time lag due to micro-structural interactions. Stability of DPL model was encountered by Quintanilla and Racke [13]. Some researchers have investigated the problems on elastic deformation by using dual-phase-lag model as, Othman et al. [14], Mondal et al. [15] and Singh et al. [16].

Schoenberg and Censor, [17], Clarke and Burdness, [18], Destrade, [19] studied the effect of rotation on elastic waves. Ting [20] investigated the interfacial waves in a rotating anisotropic elastic half-space. Singh and Othman [21], Sharma and Walia [22], Othman et al. [23], presented the effect of rotation in magneto thermoelastic medium.

Theme of present work is to consider the concept of microstructure which develops new type of waves that are not in classical theory of elasticity. Metals, polymers, composites, solids, rocks, and concrete are typical media with microstructures. More generally, most natural and manmade materials, including engineering, geological, and biological media, possess a microstructure. In this paper we encounter the propagation of plane harmonic waves in a homogeneous and isotropic unbounded medium in the context of generalized DPL thermoelastic model. We have also considered the effect of micro-temperature and rotation on waves propagating through the medium. Firstly, we obtained the theoretical solution of the problem by using Normal mode analysis method. Secondly results obtained theoretically are computed by using the packages of Matlab and shown graphically in figures. In graphical structure, we have also compared the results obtained by dual phase lag model (DPL) with coupled theory of linear thermo-elasticity (CL).

2. Formulation of the problem:

We use a rectangular coordinate system (x, y, z) having origin on the surface $y = 0$ and z-axis pointing vertically into the medium. The equations of motion for linear generalized thermo-elasticity in a rotating frame of reference and without body force and heat source is given by [21]

$$\mu\nabla^2 u+(\lambda+\mu)\nabla(\nabla.u)-\beta\nabla \mathrm{T}=\rho\left[\ddot{u}+\vec{\Omega}\times\left(\vec{\Omega}\times\vec{u}\right)+2\vec{\Omega}\times\dot{u}\right]_i \tag{1}$$

The dual phase heat conduction equation along with influence of micro-temperature is

$$\left(1+\tau_{\mathrm{T}}\frac{\partial}{\partial t}\right)KT_{,jj}+k_1\left(\nabla\cdot\vec{w}\right)=\left(1+\tau_q\frac{\partial}{\partial t}\right)\left(\rho c_e\dot{\mathrm{T}}+\beta \mathrm{T}_{\circ}\dot{e}_{kk}\right) \tag{2}$$

The equation of micro-temperature is

$$k_6\nabla^2 w+(k_4+k_5)\nabla(\nabla\cdot w)-k_3\nabla\mathrm{T}-k_2 w-b\dot{w}=0 \tag{3}$$

Where $\lambda,\mu,\alpha,\beta,k,k_i\ (i=1,2...,6)$ are constitutive coefficients, ρ is the density of the medium, $T=T_t-T_0$, $u(x,z,t)$ is the displacement vector and $\vec{w}$ is the micro-temperature vector. The constitutive relations for thermoelastic medium with micro-temperatures are given by [5].

$$\begin{cases}\sigma_{ij}=\lambda\delta_{ij}e_{\kappa\kappa}+2\mu e_{ij}-\beta\mathrm{T}\delta_{ij}\ ; \quad q_{ij=-\kappa_4}w_{r,r\delta_{ij}}-\kappa_5 w_{i,j}-\kappa_6 w_{j,i}\\ \square_i=(K-\kappa_3)\mathrm{T}_{,i}+(\kappa_1-\kappa_2)w_i\ ;\quad q_i=K\mathrm{T}_{,i}+\kappa_1 w_i\end{cases} \tag{4}$$

Where $\sigma_{ij}=(i,j=1,2,3)$ is the force tensor, q_{ij} is the first heat flux moment vector, Q_i is heat flux vector and q_i is the heat flux vector, a comma in the subscript denotes the spatial derivative, δ_{ij} is the Kronecker delta and e_{ij} is the strain tensor given by $2e_{ij}=\left(u_{i,j}+u_{j,i}\right)$, Introducing the following non-dimensional variables as

$$\left(x'_i,u'_i\right)=\frac{(x_i,u_i)}{l_0},w'_i=w_i l_o,\nabla^2=\frac{\nabla'^2}{l_0^2}\left(t',\tau'_q,\tau'_{\mathrm{T}}\right)=\frac{c_1\left(t,\tau_q,\tau_{\mathrm{T}}\right)}{l_0},\mathrm{T}'=\frac{\mathrm{T}}{\mathrm{T}_0}, \tag{5}$$

Where $l_{\circ}$ is standard length and c_1 is the standard velocity given by $c_1=\sqrt{(\lambda+2\mu)/\rho}$. Non dimensionalizing the governing equation and dropping primes for convenience, we get,

$$\alpha_{\circ}u_{i,jj}+\alpha_1 u_{j,ji}-H\mathrm{T}_{,i}\ =\left[\ddot{u_i}+\Omega_j u_j\Omega_i-\left(\vec{\Omega}^2\right)u_i+2\varepsilon_{ijk}\vec{\Omega}_j\dot{u}_k\right] \tag{6}$$

$$\left(1+\tau_{\mathrm{T}}\frac{\partial}{\partial t}\right)\alpha_2\mathrm{T}_{,jj}+\alpha_3 w_{j,j}=\left(1+\tau_q\frac{\partial}{\partial t}\right)\left(\dot{\mathrm{T}}+\alpha_4\dot{u}_{j,j}\right) \tag{7}$$

$$\beta_o w_{i,jj}+\beta_1 w_{j,ji}-\beta_2 T_{,i}-\beta_2 w_i-\beta_4\dot{w}_i=0 \tag{8}$$

Where $\beta_0=\dfrac{\kappa_6}{\rho l_0^3 c^3{}_1};\beta_1=\dfrac{(\kappa_4+\kappa_5)}{\rho l_0^3 c^3{}_1};\beta_2=\dfrac{\kappa_3\mathrm{T}_0}{\rho l_0 c^3{}_1}\quad \beta_3=\dfrac{\kappa_2}{\rho l_0 c^3{}_1};\quad \beta_4=\dfrac{b}{\rho l^2{}_0 c^2{}_1}\quad \alpha_2=\dfrac{\kappa}{l_0\rho c_e c_1};$

$\alpha_3=\dfrac{\kappa}{l_0\rho\mathrm{T}_0 c_e c_1};\ \alpha_4=\dfrac{\beta}{\rho c_e}\ \ \alpha_0=\dfrac{\mu}{\rho c_1^2};\ \alpha_1=\dfrac{(\lambda+\mu)}{\rho c_1^2},\ H=\dfrac{\beta\mathrm{T}_0}{\rho c_1^2}$

Displacement and micro-temperature functions could be converted in terms of potential function by following expression,

$$u_1=\left(\frac{\partial R}{\partial x}+\frac{\partial \psi}{\partial z}\right), u_3=\left(\frac{\partial R}{\partial z}-\frac{\partial \psi}{\partial x}\right), w=\nabla v \tag{9}$$

Where R and v are scalar potentials functions, vector potential ψ. Rotation is taken along the y-axis as $\vec{\Omega}=(0,\Omega,0)$, Eqs. (6-8) gives,

$$\left(\alpha\nabla^2-\frac{\partial^2}{\partial t^2}+\Omega^2\right)R-H\mathrm{T}+2\Omega_2\dot{\psi},=0 \tag{10}$$

$$(\alpha_\circ\nabla^2-\frac{\partial^2}{\partial t^2}+\Omega^2)\psi-2\Omega_2\nabla^2\dot{R}=0 \tag{11}$$

$$\alpha_2\left(\nabla^2\mathrm{T}+\tau_\mathrm{T}\nabla^2\dot{\mathrm{T}}\right)+\alpha_3\nabla^2 v-\dot{\mathrm{T}}\left(1+\tau_q\frac{\partial}{\partial t}\right)-\left(1+\tau_q\frac{\partial}{\partial t}\right)\left(\alpha_4\nabla^2\dot{R}\right)=0 \tag{12}$$

$$\sigma\nabla^2 v-\beta_2\mathrm{T}-\left(\beta_3+\beta_4\frac{\partial}{\partial t}\right)v=0 \tag{13}$$

3. Harmonic Solution of the Problem

Now let us consider that each field variable is propagating through the medium in terms of harmonic waves as,

$$\{R,\mathrm{T},v,\psi\}(x,y,t)=\left\{R^*,T^*,v^*,\psi^*\right\}(x)\exp(\omega t+iax) \tag{14}$$

Where ω is the angular frequency, a is the wave number and $R^*,\mathrm{T}^*,v^*,\psi^*$ are the amplitudes. By using equation (14) governing equations can be represented as,

$$\left(\alpha D^2+A_1\right)R^*-H\mathrm{T}^*+A_2\psi^*=0 \tag{15}$$

$$\left(\alpha_\circ D^2+A_3\right)\psi^*-A_4R^*=0 \tag{16}$$

$$\left(\alpha_2D^2\pi_1+A_5\right)\mathrm{T}^*+\left(\alpha_3D^2-\alpha_3a^2\right)v^*+\left(-\alpha_4D^2\pi_2+A_6\right)R^*=0 \tag{17}$$

$$\left(\sigma D^2-A_7\right)v^*-\beta_2\mathrm{T}^*=0 \tag{18}$$

Where $A_1=-\alpha a^2-w^2+\Omega^2$, $A_2=2\Omega_2 w$, $A_3=-\alpha_0a^2-w^2+\Omega^2$, $A_4=2\Omega w$ $\pi_1=(1+\tau_\mathrm{T}\omega)$, $\pi_2=\left(\omega+\tau_q\omega^2\right)$, $A_5=-\alpha_2a^2(1+\tau_\mathrm{T}\omega)$, $A_6=\alpha_4a^2\left(\omega+\tau_q\omega^2\right)$, $\sigma=\beta_0+\beta_1$, $A_7=\sigma a^2-\beta_3-\beta_4\omega$

For non trivial solution the determinant of the above equations need to be zero.

$$[\mathrm{D}^{10}-A\mathrm{D}^8+B\mathrm{D}^6-C\mathrm{D}^4+E\mathrm{D}^2-F]=0 \tag{20}$$

In factorized form,

$$(\mathrm{D}^2-k_1^2)(\mathrm{D}^2-k_2^2)(\mathrm{D}^2-k_3^2)(\mathrm{D}^2-k_4^2)(\mathrm{D}^2-k_5^2)\{v^*,\psi^*,R^*,T^*\}(z)=0 \tag{21}$$

Where k_i, $i=1,2,..,5$ gives the roots of equation (21)

$$A=g_1g_8;B=\left(g_8g_2+g_1g_9-g_5g_{11}\right); \qquad C=\left(g_3g_8+g_2g_9+g_1g_{10}-g_{12}g_5-g_6g_{11}\right);$$
$$D=\left(g_4g_8+g_3g_9+g_{10}g_2-g_{13}g_5-g_7g_{11}-g_{12}g_6\right);E=\left(g_4g_9+g_{10}g_3-g_{13}g_6-g_{12}g_7\right);$$
$$F=\left(g_4g_{10}-g_{13}g_7\right),\ g_1=\alpha_2\alpha_\circ\alpha; \qquad g_2=\left(A_3\alpha\alpha_2\pi_1+\alpha_2\pi_1A_1\alpha_\circ+A_5\alpha\alpha_\circ-H\alpha_4\pi_2\right);$$
$$g_3=\left(\alpha_2A_1A_3\pi_1+A_5A_3\alpha+A_2A_4\alpha_2\pi_1+A_1A_5\alpha_\circ-H\alpha_4\pi_2A3+HA_6\alpha_0\right);$$
$$g_4=\left(A_1A_3A_5+A_5A_2A_4+HA_6A_3\right); \qquad g_5=\alpha_3\alpha_0;g_6=\left(\alpha_3A_3-\alpha_3\alpha_0a^2\right); \qquad g_7=\alpha_3a^2A_3;$$
$$g_9=\left(-A_7\alpha_0+A_3\sigma\right);g_8=\alpha_0\sigma; \quad g_{10}=-A_3A_7;g_{11}=-\beta_2\alpha\alpha_0; \quad g_{12}=-\beta_2A_3\alpha-\beta_2A_1\alpha_0;$$
$$g_{13}=\left(-\beta_2A_1A_3+\beta_2A_2A_4\right),$$

The solution of equation (21), has the form

$$R^*=\sum_{n=1}^{5}M_ne^{-k_nz}\quad T^*=\sum_{n=1}^{5}H_{1n}M_ne^{-k_nz}\quad \psi^*=\sum_{n=1}^{5}H_{2n}M_ne^{-k_nz}\quad V^*=\sum_{n=1}^{5}H_{3n}M_ne^{-k_nz} \qquad (22)$$

Where M_n are some parameters to be determined,

$$H_{2n}=\frac{A_4}{\left(\alpha_\circ k_n^2+A_3\right)} \qquad H_{1n}=\frac{\left[\alpha\alpha_0k_n^4+\left(A_3\alpha+A_1\alpha_0\right)k_n^2+A_1A_3+A_2A_4\right]}{\left[H\left(\alpha_0k_n^2+A_3\right)\right]}$$

$$H_{3n}=\frac{\left(g_{11}k_n^4+g_{12}k_n^2+g_{13}\right)}{\left(g_8k_n^4-g_9k_n^2+g_{10}\right)}$$

4. The boundary conditions

The boundary conditions on the plane $z=0$ are:

(1) The surface of the half-space obeys,

$$\sigma_{zz}\left(x,0,t\right)=f\ \exp(\omega t+iax),\sigma_{zx}\left(x,0,t\right)=0 \qquad (23)$$

(2) The surface is subjected to a thermal shock, $\frac{\partial T}{\partial z}\left(x,0,t\right)=0$ (24)

(3) Normal component of heat flux moment is $q_{zz}\left(x,0,t\right)=0,q_z\left(x,0,t\right)=0$ (25)

By using equation (4) we have obtained the following set of equations,

$$\bar{\sigma}_{zz}=\sum_{n=1}^{5}Y_{1n}M_ne^{-k_nz}\quad \bar{\sigma}_{zx}=\sum_{n=1}^{5}Y_{2n}M_ne^{-k_nz}\quad \frac{\partial \mathrm{T}}{\partial z}=\sum_{n=1}^{5}Y_{3n}M_ne^{-k_nz}$$
$$q_{zz}=\sum_{n=1}^{5}Y_{4n}M_ne^{-k_nz}\quad q_z=\sum_{n=1}^{5}Y_{5n}M_ne^{-k_nz} \qquad (26)$$

Where

$$Y_{1n} = [\lambda ia\left(ia - k_n H_{2n}\right) + \left(\lambda + 2\mu\right) k_n \left(k_n + iaH_{2n}\right) - \beta \mathrm{T}_0 H_{1n}]$$

$$Y_{2n} = \left[-k_n\left(ia - k_n H_{2n}\right) - ia\left(k_n + iaH_{2n}\right)\right] \qquad Y_{3n} = [-H_{1n} k_n]$$

$$Y_{4n} = [-K_4\left(k_n^2 - a^2\right) - k_n^2\left(K_5 + K_6\right)]H_{3n} \quad Y_{5n} = [-Kk_n H_{1n} - \kappa_1 k_n H_{3n}]$$

Applying the boundary conditions Eqs. (23) – (25) and using Eqs. (26), we obtained the following matrix transform,

$$\begin{pmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} & Y_{35} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} & Y_{45} \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55} \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{pmatrix} = \begin{pmatrix} f_1^* \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

By solving the above matrix we can obtained the values of five constants $M_n,\ n = 1, 2, .., 5$.

5. Numerical Results and Discussion

The evaluated theoretical results in equations (22) are computed numerically by using the relevant parameters for the case of magnesium crystal. The relevant physical values of elastic constants and micro-temperatures are [13]

$$\rho = 1.74\times10^3\, kg\ m^{-3}, \lambda = 9.4\times10^{10}\, Nm^{-2},\ C_E = 1.04\times10^3\, NmKg^{-1}K^{-1}, T_0 = 0.298,$$

$$\mu = 4.0\times10^{10}\, Nm^{-2}, \gamma = 7.779\times10^{-8}\, N, \quad K = 1.7\times10^2\, N\sec^{-1} K^{-1}, b = 0.15\times10^9\, N$$

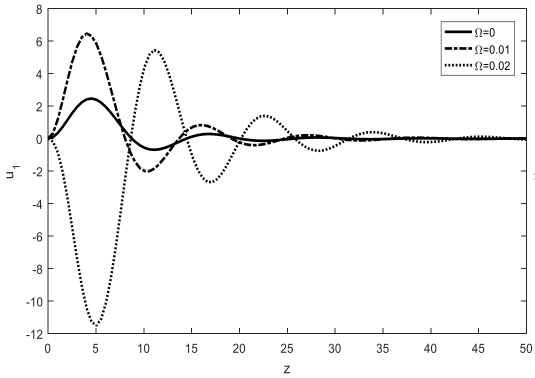
The microtemperature parameters are

$$k_1 = 0.0035Ns^{-1}, k_2 = 0.045Ns^{-1}, k_3 = 0.055NK^{-1}s^{-1},$$

$$k_4 = 0.064Ns^{-1}m^2, k_5 = 0.075Ns^{-1}m^2,\ k_6 = 0.096Ns^{-1}m^2$$

The change in amplitude of field variables against vertical component of distance for generalized thermo-elastic medium is represented graphically. Figs. 1-6 show variation in waves due to different rotational frequency of the medium. Figs. 7-12 are representing the comparison between dual phase lag equation and coupled heat conduction equation in presence and absence of rotational frequency.

Fig. 1-2 represents the components of displacement distribution function against vertical distance from the surface of medium. It is observed that, absolute value of horizontal component of displacement u_1 obtains maximum amplitude near surface and in the first mode along z axis. Highest absolute value of amplitude is found during $\Omega = 0.02$, indicating that rotation is having increasing effects. Similarly for the case of vertical component of displacement u_3, rotation increases the amplitude of wave propagating through the medium. From graphical representation, it can be seen that rotation is also increasing the harmonic nature of curves propagating though the medium and reduces the attenuation factor.



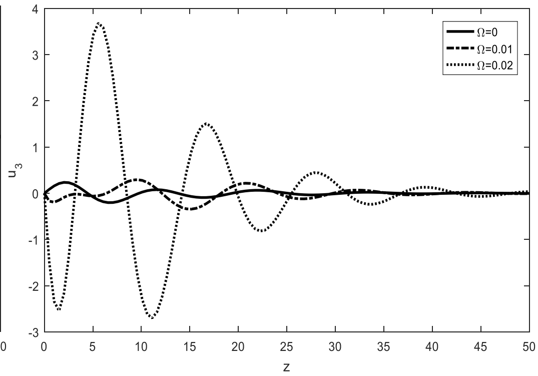
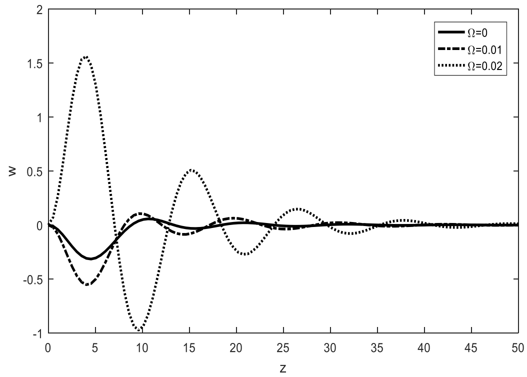


Fig. 1 Horizontal components for different Ω Fig. 2 Vertical components for different Ω

Fig. 3 depict the influence of rotation on micro-temperature vector, amplitude of wave is directly proportional to rotational frequency of the medium i.e., maximum amplitude is obtained for the case of $\Omega = 0.02$. Harmonic behavior of curve is also increasing by increasing the rotational frequency of the medium about y-axis.



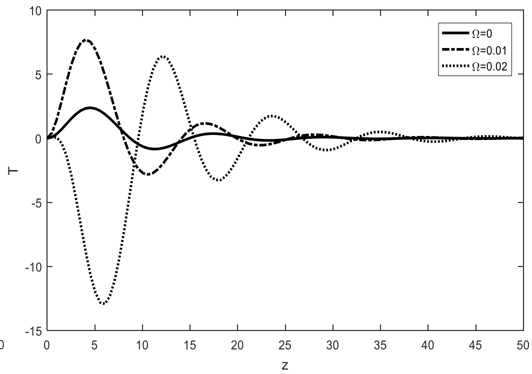


Fig. 3 Microtemperature vector for different Ω Fig. 4 Temperature for different Ω

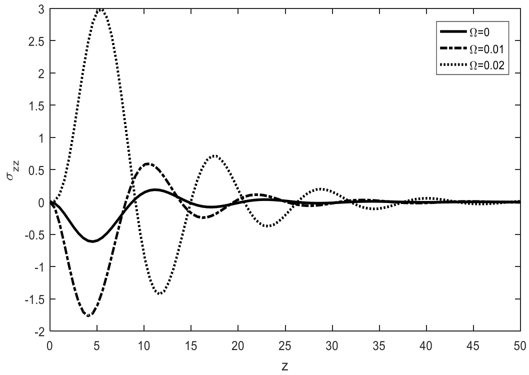


Fig. 5 Normal stress for different Ω

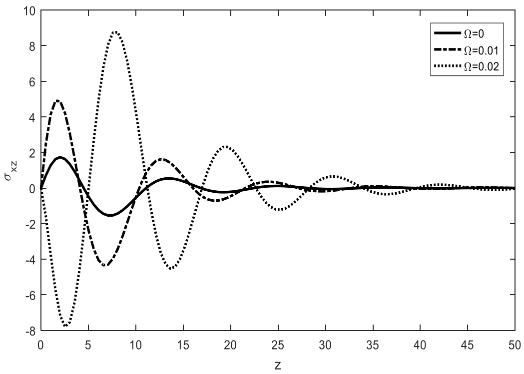


Fig. 6 Tangential stress for different Ω

Figs. 4, 5 and 6 are analyzing the behavior of waves for temperature distribution function T, normal stress σ_{zz} and tangential component of stress σ_{xz}. Curves in each graph shows that, harmonic nature of curve increases as the rotational frequency of the medium increases. Rotational frequency of mediums is having increasing effects on absolute value of amplitudes for each variable. Curves without rotation converge to zero earlier, representing that rotation decreases the factor responsible for decaying of wave.

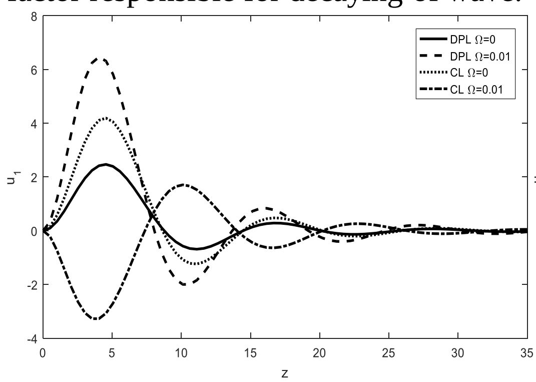


Fig. 7 Comparison of u_1 for DPL and CL theory.

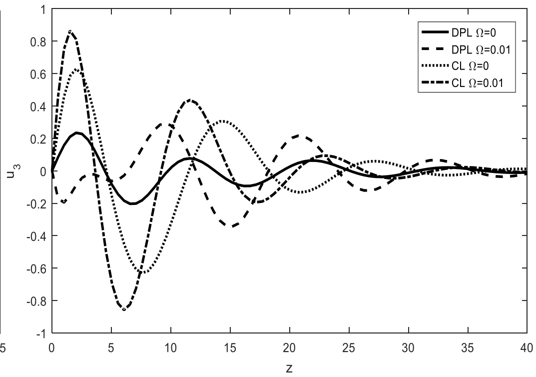


Fig. 8 Comparison of u_3 for DPL and CL theory.

Figures 7-12, gives the relation between dual phase lag (DPL) and coupled linear theory of heat conduction, and these figures also predict the effect of rotation on these two thermo elastic theories. Fig. 7 indicates that the curves obtained in context of DPL model are having high amplitudes as compared to CL model. In both the theories rotation increases the harmonic behavior of wave propagating through the medium and maximum amplitude is obtained for $\Omega = 0.01$. Relation between DPL and CL model along with rotation for vertical component of displacement is represented in fig. 8. Maximum amplitude is obtained during CL model and for $\Omega = 0.01$. Rotation increases the harmonic behavior of curves propagating through the medium and decreases the attenuation factor so the curves during $\Omega = 0.01$ will converge to zero slower.

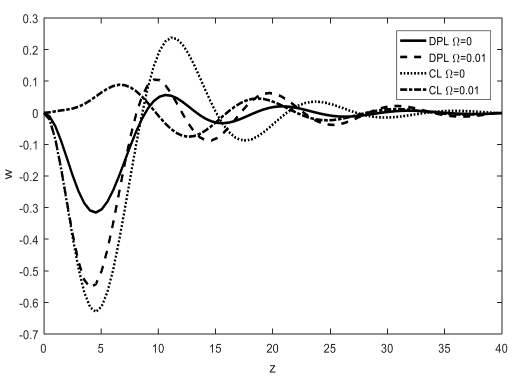


Fig. 9 Comparison of w for DPL and CL theory.

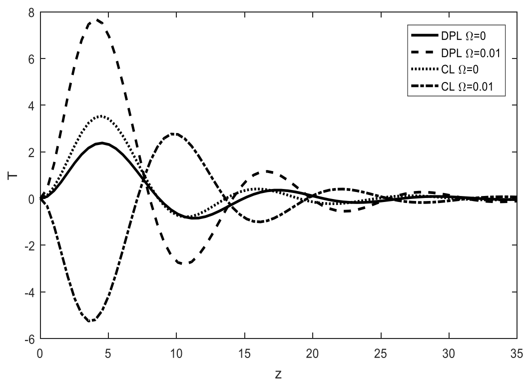


Fig. 10 Comparison of T for DPL and CL theory.

Figure 9 gives analysis of micro-temperature distribution function in which rotation increases the harmonic nature of wave propagating through the medium. Finally, all curves converge to zero as distance from the surface of the medium increases. Temperature distribution function is represented in fig. 10, maximum amplitude of curve for both models of elastic theories is obtained during the case of $\Omega = 0.01$ and maximum value of amplitude is obtained in context of DPL model.

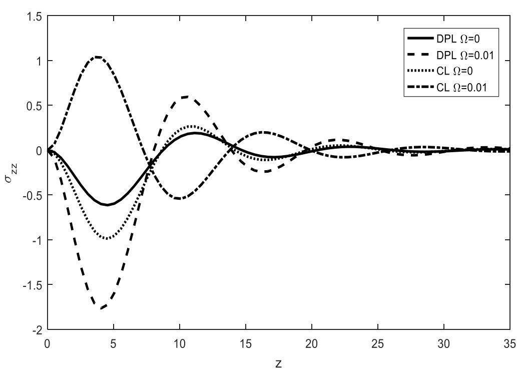


Fig.11 σ_{zz} for DPL and CL theory.

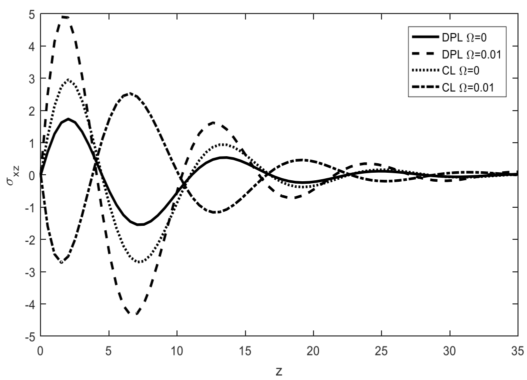


Fig.12 σ_{xz} for DPL and CL theory.

Fig. 11 gives detailed analysis of normal component of stress distribution function σ_{zz}. For $\Omega = 0$ curves for absolute values of amplitude obtained during CL model are having greater values then the curves for DPL model. During the case of $\Omega = 0.01$ the results are reversed, i.e., curves for DPL model is of high amplitude then the curves for CL model. Same as that of normal component of stress, tangential stress is having increasing effect of rotation for DPL model and decreasing effects for CL model. In presence of rotational frequency Curves obtained in context of DPL model are higher than the curves during CL model. 3D figures 13-18 are very important in depicting the response of curves along horizontal and vertical components of distance. The curves move harmonically in the form of normal modes along x-axis and in terms of decaying waves along z-axis.

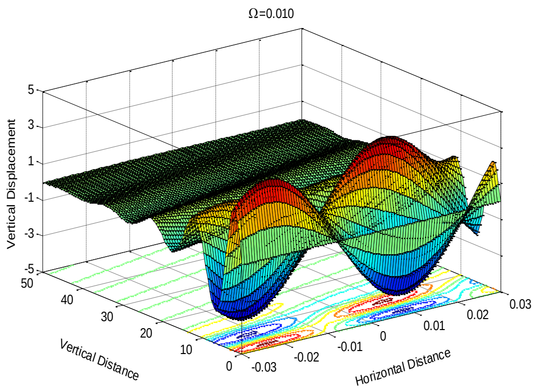


Fig. 13, 3D curves, Horizontal component of displacement

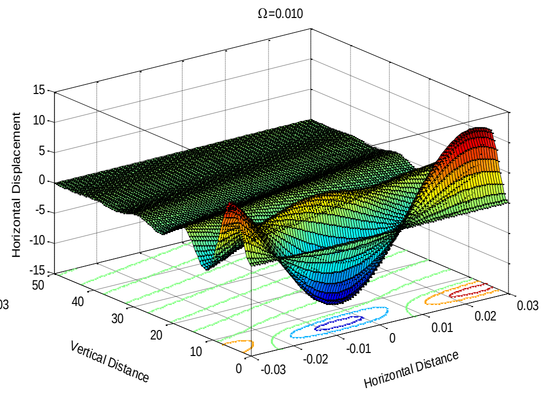


Fig. 14, 3D curves, Vertical component of displacement

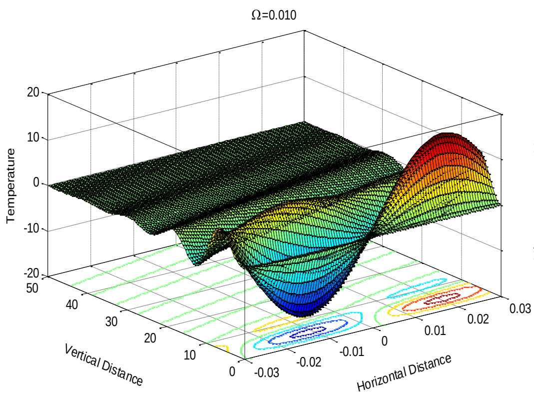


Fig. 15, 3D curves Micro-temperature

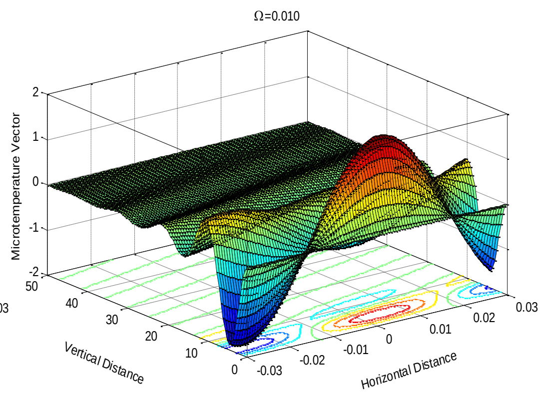


Fig. 16, 3D curves Temperature distribution

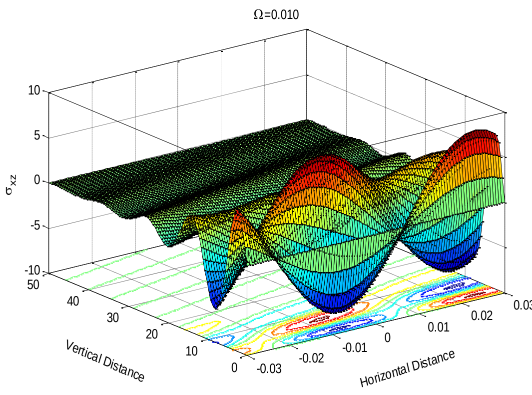


Fig. 17, 3D curves Normal Component of Stress

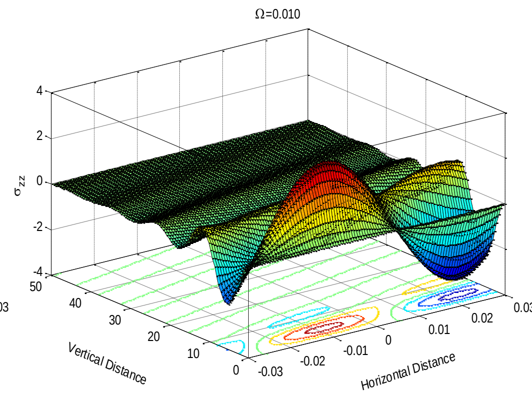


Fig. 18, 3D curves, Tangential component of stress

6. Conclusions

Following are some main points which could be concluded after considering the solutions of the problem:

1) All curves obtained converge to zero as depth from the surface of the medium increases.

2) Rotation is having increasing effect on each field variable for DPL while decreasing effect in context of CL model.

3) Rotational effect increases the harmonic behavior of waves propagating through the medium i.e., it increases the dispersive nature of medium.

4) Attenuation factor for vertical component of displacement, micro-temperature and temperature distribution function for CL model is stronger than that of DPL model. Rotation is responsible for reduction in attenuation factor for waves in both the theories of heat conduction.

5) All the curves obtained in context of DPL model are having high amplitude as compared to the curves studied by considering CL model.

6) 3D curves predicts that the curves are moving in the form of normal modes along horizontal and moves in the form of decaying waves along vertical component of distance.

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