HONEYCOMB STRUCTURES

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The natural honeycomb is the optimum structure, considering material volume, but expensive to manufacture because the shape complexity. Additive technology or 3D printing, raise the question if the presumed improved structural behavior, of the natural honeycomb, compared to the industrial variants, it is a good argument to adopt it on a large industrial scale. This study reveals the structural behavior of the natural honeycomb and two industrial variants, single layer and double layer, in the most common loading, for this type of structures, compression, and also bending, both in elastic range and beyond yield stress, the material model adopted being bilinear.

Keywords: natural honeycomb, single layer industrial honeycomb, double layer industrial honeycomb, structural behavior.

1. Introduction

Presently, the structural engineers are looking to obtain stiffer and lighter structures. They are using optimization modules from the finite element software [1], [2], [3]. With all these, there are some natural structural with ideal shapes, impossible to improve further, as honeycomb structures. Usually, they are used in compression and the industrial honeycombs [4], [5], [6] are simplified forms of the natural one. They are single layer or double layer with flat bottom. The natural honeycomb has two layers and a convex bottom, formed from three rhombi.

The natural honeycomb is the optimum solution, in terms of requiring material volume, to construct individual cells for bees. This study will also consider a few aspects of its structural behavior comparing with two industrial variants. If the natural honeycomb structural behavior is proven to be significant better than industrial ones [7], [8], it can be further largely adopted in practical applications. Until now, the main obstacle in this decision was the manufacturing technology [9].

2. Natural honeycomb

In order to describe the natural honeycomb structure we will use the best tool possible, the mathematics. The honeycomb structure requires a lot of wax and many hours of labor. The bees found the way to use minimum resources as to

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obtain the required effect – optimal structure. We will consider a volume with a fixed height. We will divide it in volumes and we will determine which shape requires the least material. The equivalent problem is to find the shape having the minimum perimeter, because the height is the same for all possible shapes. The perimeter decrease as the number of sizes increase. The shape with the minimum perimeter is the circle, considered to have an infinite number of sizes.

Considering the area *A* equal to unity, we will calculate the perimeter *P*.

The absolute perimeter values are synthetized in table 1.

Table 1

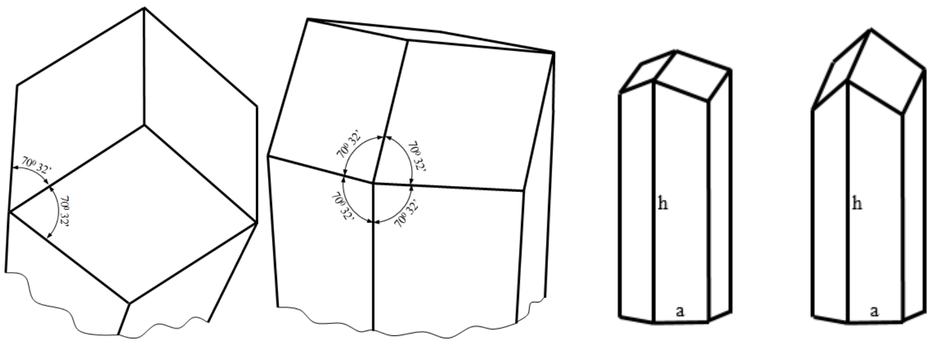
Perimeter of Different Cell Shapes for Unity Area

Cell	Area	Perimeter
Circle	1	3.54490
Hexagon	1	3.72241
Square	1	4
Triangle	1	4.55901

The optimum shape is the circle, but only in the particular situation when it is single. If we intend to use multiple cylinders we will lose a lot of space and material. Wall sharing will save material. The reducing material amount is 25% if we use octagons and 50% if we use squares or triangles.

Even from antiquity, Pappus of Alexandria had discovered that the optimum solution for multiple cells is the hexagon.

The astronomer Moraldi observed that the bottom of the honeycomb cells is convex, being composed from three rhombi surfaces, see figure 1, a, so, the optimum shape is a hexagonal prism with a trihedral base – figure 1, b.



a b

Fig. 1. Natural honeycomb bottom and entire cell

The surface of the area depends on *x*, as figure 3 reveals

$$A = 6\cdot\left(A\cdot h\right) - \frac{a\cdot x}{2} + 3a\sqrt{3}\sqrt{x^2 + \frac{a^2}{4}}\ . \tag{1}$$

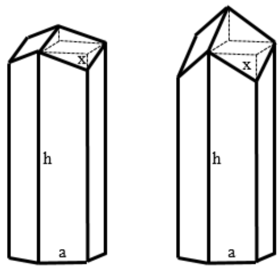


Fig. 3. The components of the honeycomb cell surface

It consists from six rectangular sides minus six triangles plus three rhombi.

Using the formula (1) in which we consider $a = 1$ and $h = 1$, we will calculate the area *A* for $x = 0$, x = 0.1, $x = 0.2$ up to $x = 1$. The values and the trend line connecting the points are presented in figure 4.

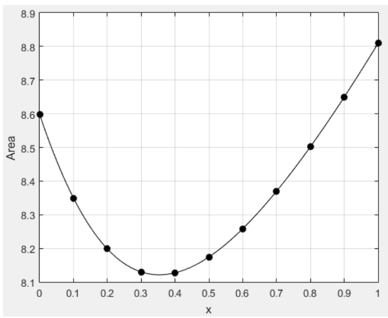


Fig. 4. The components of the honeycomb cell surface

Using the visual observation the minimum area is in the 0.3 – 0.4 interval. However, we will determine the exact value of the *x*, for which the *A* is minimum, equalling to zero the first derivative of the area function

$$\frac{d}{dx}\left[6\cdot\left(a\cdot h - \frac{a\cdot x}{2}\right) + 3a\sqrt{3}\sqrt{x^2 + \frac{a^2}{4}}\right] = 0\,, \tag{2}$$

with the solution

$$x = 0.353553\,, \tag{3}$$

For this value of *x*, the corresponding area is $A = 8.1213$.

The cells are not placed end to end point – figure 5, but, each cell from one side is placed into the space created by the three cells from the opposite side – figure 6.

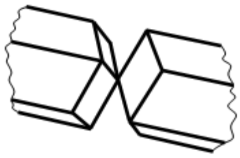


Fig. 5. End to end cells

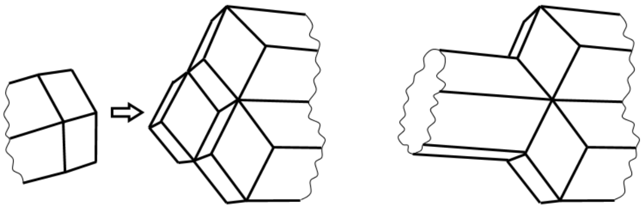


Fig. 6. Position of cells from opposite sides

If the structure is subjected to loads, the square cells do not transfer them to the adjacent ones. The triangles shear the loads but, there are members in compression which buckle. Again, the hexagonal cell structure is desirable because the stresses are exclusively tensile.

3. Industrial honeycomb

We will consider two variants of the industrial honeycomb structures. All three structures, two industrial honeycombs and the natural one have (121.2435 × 105 × 20) mm (Length × Width × Height)

The cell is hexagonal, with size $a = 5$ mm, the wall thickness $t = 0.1$ mm, height $h = 10$ mm for double industrial layer honeycomb and $h = 20$ mm for simple one. The double layer structure has flat bottom cells placed end to end.

The complete double and single layer industrial honeycomb structures, with details, are presented in figures 7 and 8.

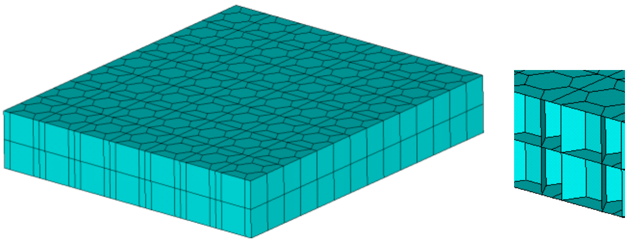


Fig. 7. The double layer industrial honeycomb with detail

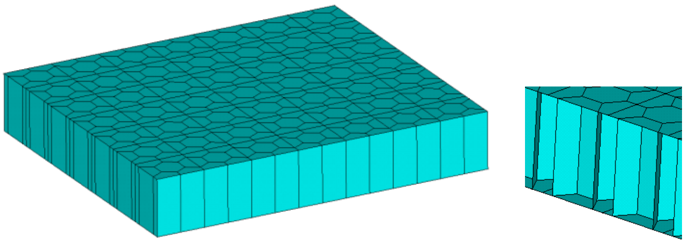


Fig. 8. The single layer industrial honeycomb with detail

We will use the same material in all presented cases. The material is aluminum with Young's modulus $E = 70000$ MPa and Poisson's coefficient $\nu = 0.33$. The material model is bilinear, with yield stress = 100 MPa and the tangent modulus 70 MPa. The model of the adopted stress – strain curve is presented in figure 9.

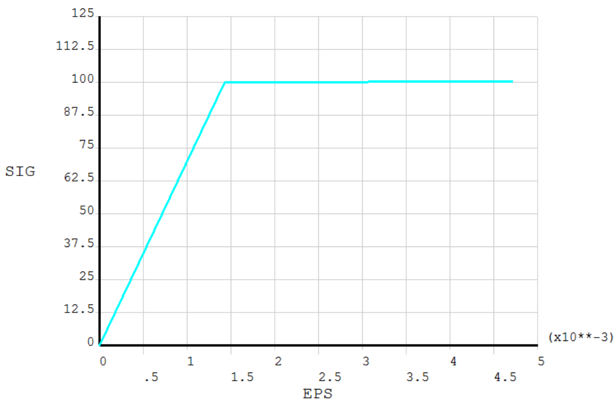


Fig. 9. The material model

3. Natural honeycomb model

This type of structure has a single model variant, presented in figure 10.

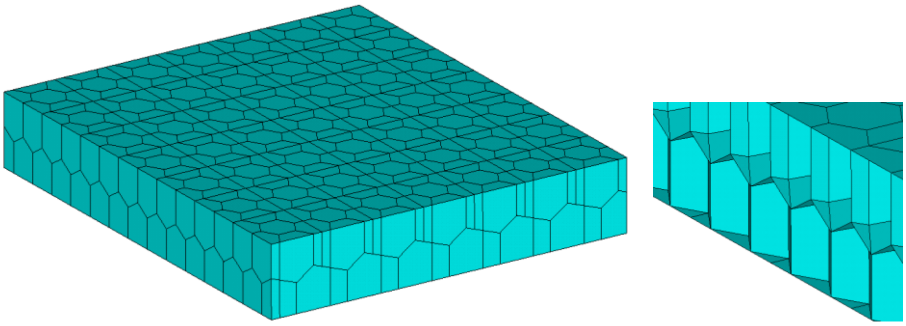


Fig. 10. The natural honeycomb structure with detail

5. The finite element method analysis

We will subject the three above mentioned structures to compression and bending.

Compression – Case A

For all three structures, the boundary condition is the embedment of the bottom surface. The load of this case, is a δ = 0.15 mm compression displacement applied on the top of structure. The von Mises stress results are presented in the figures 11, 12, 13 and table 2.

Double and single layer industrial honeycomb analysis

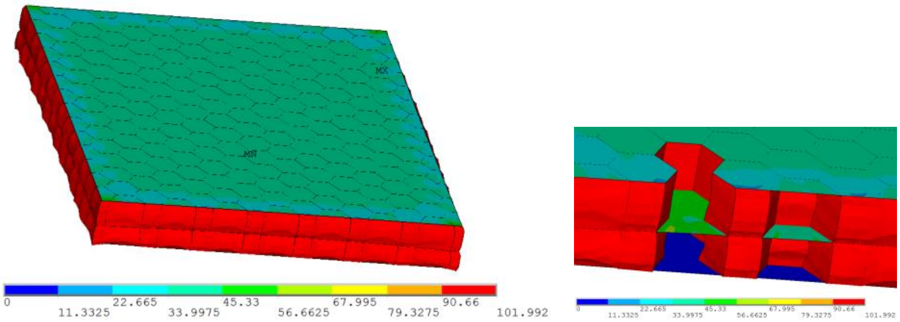


Fig. 11. The von Mises stress of the double layer industrial honeycomb with detail – compression case A

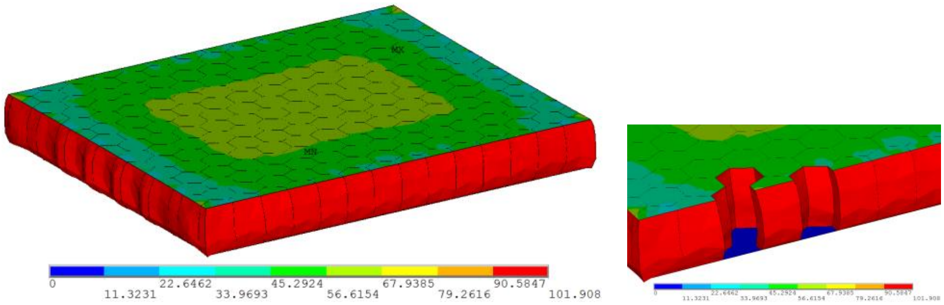


Fig. 12. The von Mises stress of the single layer industrial honeycomb with detail – compression case A

Natural honeycomb analysis

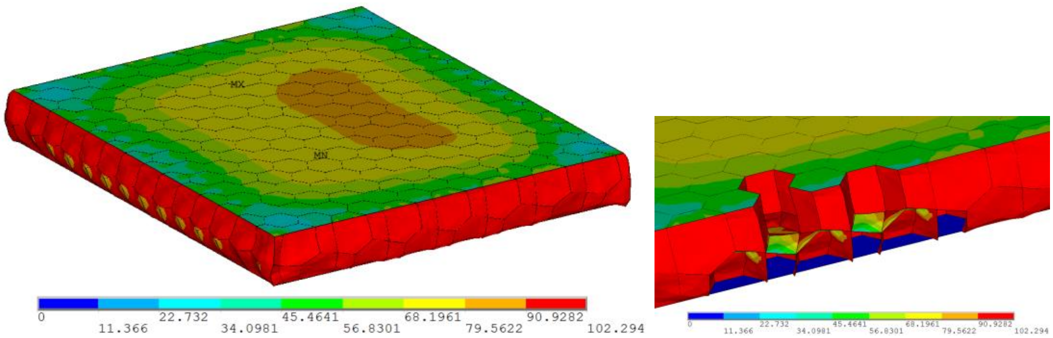


Fig. 13. The von Mises stress of the natural honeycomb with detail – compression case A

The maximum stress results are in a tight range because they are higher than the yield stress. In this domain, the material behavior model, revealed through the stress – strain curve, has a small slope – see Fig. 9. So, to better observe the differences between the structural behaviors, we will analyze a second case, in which the loads will produce maximum stresses lower than the yielding stress.

Compression – Case B

For all three structures, the boundary condition is the embedment of the bottom surface. The load is a $\delta = 0.015$ mm compression displacement applied on the top of structure. The von Mises stress results are presented in the table 2.

The maximum stress results are in a higher range than previous case, because they are lower than the yield stress and the stress – strain curve has a stiffer slope in this domain (figure 9). The structures volumes and the maximum stresses, considering von Mises criterion, are synthetized in table 2.

Table 2

Volumes of the structures and von Mises stresses for compression

Structure\Compression	Structure volume [mm^3]	Case A $\delta = 0.15$ mm	Case B $\delta = 0.015$ mm
		Von Misses stress [MPa]	
Double layer industrial honeycomb	10534	101.992	80.326
Single layer industrial honeycomb	9261.1	101.908	75.7486
Natural honeycomb	10307	102.294	82.601

Bending – Case A

For all three above mentioned structures, the boundary condition is the embedment of one side and bending with a $\delta = 10$ mm displacement on the opposite side, both on the 121.2435 mm sides. The von Mises stress results are presented in the table 3.

The maximum stress results are in a tight range, so, we will analyze a second case, with maximum stresses lower than the yielding stress.

Bending – Case B

For all three structures, the boundary condition is the embedment of one side and bending with a $\delta = 0.4$ mm displacement on the opposite side, both on the 121.2435 mm sides. The von Mises stress results are presented in the figures 14, 15 and 16.

Double and single layer industrial honeycomb analysis

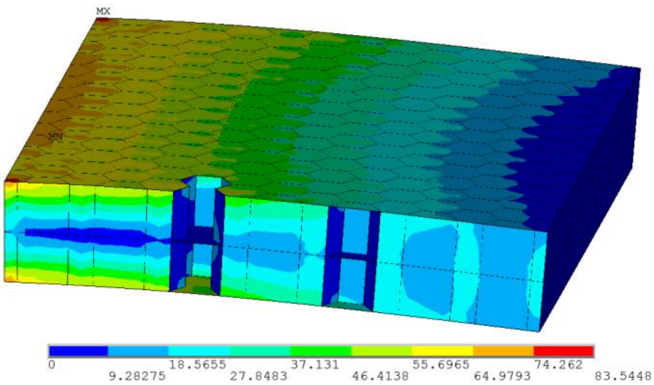


Fig. 14. The von Mises stress of the double layer industrial honeycomb – bending case B

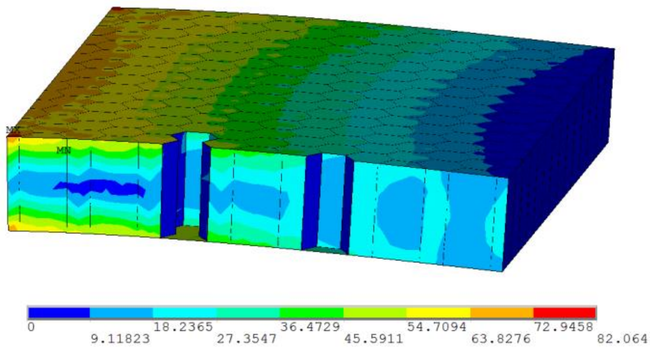


Fig. 15. The von Mises stress of the single layer industrial honeycomb – bending case B

Natural honeycomb analysis

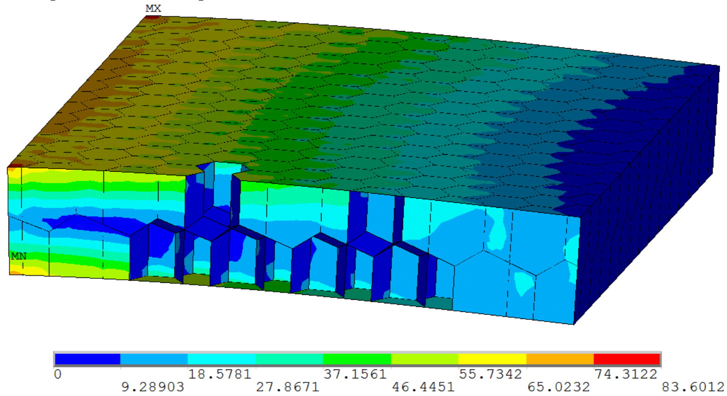


Fig. 16. The von Mises stress of the natural honeycomb – bending case B

The structures volumes and the maximum stresses, considering von Mises criterion, are synthetized in table 3.

As loading is given by imposed displacements, higher the stress is, higher the maximum load the structure is able to withstand. So, higher the stresses, better the structure behavior is.

Table 3

Volumes of the structures and von Mises stresses for bending

Structure\Bending	Structure volume [mm^3]	Case A δ = 10 mm	Case B δ = 0.4 mm
		Von Misses stress [MPa]	
Double layer industrial honeycomb	10534	109.795	83.5448
Single layer industrial honeycomb	9261.1	116.639	82.064
Natural honeycomb	10307	111.598	83.6012

6. Synthesis and result interpretation

Using data from tables 2 and 3, the volume values, relative to the natural honeycomb, are presented in table 4.

Table 4

Volumes of the structures relative to the single natural honeycomb's one

Structure\Bending	Structure volume [%]
Natural honeycomb	100
Single layer industrial honeycomb	89.85
Double layer industrial honeycomb	102.20

Using data from tables 2 and 3, the stresses values, relative to the natural honeycomb, are presented in table 5.

Table 5

The maximum von Mises stresses of the structures relative to natural honeycomb's one

Structure\Loading	Compression		Bending	
	Case A δ = 0.15 mm	Case B δ=0.015 mm	Case A δ = 10 mm	Case B δ = 0.4 mm
	Relative von Mises stress [%]			
Natural honeycomb	100	100	100	100
Single layer industrial honeycomb	99.62	91.70	104.51	98.16
Double layer industrial honeycomb	99.70	97.24	98.38	99.93

From table 4, we observe that the single layer is 10.15 % lighter and the double layer industrial honeycomb is 2.2 % heavier than natural one.

Table 5 reveals that for compression in the plastic range the differences between all three structures behavior is less than 0.4 %. For the same loading, in the elastic domain, the behavior of both industrial honeycombs has not such a good behavior than the natural one, with 8.3 % for single layer and 2.76 % for double layer. In plastic bending, single layer has a better behavior with 4.51 % and the double layer not such a good behavior with 1.62 %. In elastic bending, both industrial honeycombs have not such a good behavior as the natural one but only with 1.84 % the single layer and with 0.07 % the double layer.

7. Conclusions

Theoretically, the natural honeycomb is the best structure possible, the optimum one. The natural honeycomb structures cannot be produced by classical technologies. That is why, until now, engineers have used the industrial variants on the honeycomb, which are simplified, with one layer of cells or two layers but a flat inner bottom. The reason was the manufacturing technology limits. Presently, the engineers have the increased possibility to use the additive technology – 3D printing – which is slower and more expansive [10], [11], [12]. However, this technology offers the possibility to produce even the natural honeycomb structures. The expectation was that this type of structure will bring a significant structural behavior improvement, comparing to the industrial variants, but this assumption was invalidated by this study.

The most relevant and oddest situation, supporting the above statement, is plastic bending of the single layer industrial honeycomb. For this particular case, the single layer industrial honeycomb it is not only 10.15 % lighter than the natural one, but it also behaves better, the maximum stress, occurring in order to obtain the same displacement being with 4.51 % higher.

So, analyzing the results, we can conclude that present industrial honeycomb variants are well suited for their purpose, while, we must underline that the original purpose of the natural honeycomb was only to separate a space into cells, with less material.

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