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MULTIDIMENSIONAL EXTENICS THEORY

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În această lucrare vom "extinde" teoria Extenics de la cazul unidimensional cunoscut în prezent la cazul n-dimensional, $n > 1$*. Datorită importanței pe care această teorie o are în rezolvarea multor probleme practice, extensia pe care o realizăm în acest articol oferă domeniului aplicativ maximul de consistență tehnică pe care aceasta îl poate oferi.*

This paper will "extend" Extenics Theory from the one-dimensional case known at present to the n-dimensional case, $n > 1$*. Due to the importance of this theory in solving multiple practical problems, the extension presented in this paper brings a maximum of technical consistency to the applicative field.*

Keywords: Extenics theory, Wen indicators, dynamic systems with status indicators.

1. Introduction

For solving certain paradoxical problems which are often met in daily practice, Prof. Cai Wen developed an efficient mathematical theory which he generically named "Extenics", namely the science of extending the means of investigation used in classical mathematics[6]. The results obtained up to the present by Prof. Cai deal exclusively with problems expressable by unidimensional mathematical models. Since this restricts the area of applicability of a means of investigation which even under these conditions proved to have widespread

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[6] For further details on this matter see papers [4, 9, 10, 11], where [9] represents the early work of prof. Cai Wen to which we shall refer expressly within our article and [4, 10, 11] represent developments of the points of view formulated in [9] followed by numerous applicative examples particularly for the engineering field.

applications[7] it is only natural that one becomes concerned with its development in a way that allows a generalization of the existing theory which would comprise the practical problems that need to be mathematically expressed by multidimensional models[8]. This paper attempts to do just that. As can be seen from the content, the transition from the unidimensional case to the multidimensional is neither direct, as intuitive aspects do not function well within the generalization process, nor immediate, since reaching the general cadre of the theory necessitates a much more sophisticated mathematical apparatus.

2. Maximal extension of the notion of distance in Extenics Theory

By d we denote the euclidean distance on $\mathbb{R}^n$, i.e. $d(x,y)=\left(\sum_{k=1}^{n}(y_k-x_k)^2\right)^{1/2}$, where $x=(x_1,x_2,..,x_n)$, and $y=(y_1,y_2,..,y_n)$ represent two points from $\mathbb{R}^n$. This thing allows us to consider the relation

$$\delta(x,A)=\inf_{y\in A} d(x,y),$$

which, as is known, represents the distance from point $x\in\mathbb{R}^n$ to the set $A\subseteq\mathbb{R}^n$.

In classical mathematics the notion of distance from a given point to a certain set of points is sufficient to express whether that point belongs or not to the set considered. In Extenics theory, the relation that can exist between a point $x\in\mathbb{R}^n$ and a set $A\subseteq\mathbb{R}^n$ is to be extended, with the intention of expressing more than the simple idea that $x\in A$ or $x\notin A$. In order to obtain this result we propose replacing the indicator δ defined earlier with the indicator $\mathfrak{s}$ defined as follows:

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[7] See in this respect papers [1-7], [10 - 12].

[8] Preoccupations in this direction do not represent a topic introduced by us exclusively. The first initiative of this kind belongs to prof. Forentin Smarandache (see paper [8]) who was able to develop prof Cai Wen's theory in two particular directions: one referring to the situation in which the sets of the considered mathematical model have "central symmetry" and the second in which the problems being studied admit the so-called "attraction point principle". However, the theory presented by us in this paper fundamentally differs from the approach of prof. Smarandache and the results that we obtained are in general, distinct. When compared to the theory of prof. Smarandache, our theory represents another variant of generalizing the theory of prof. Cai Wen, which leads to other implications upon the applicative field.

$$\mathfrak{s}(x,A)=\begin{cases}\delta(x,A), & x\in\complement A\\ -\delta(x,\complement A), & x\in A\end{cases}, \tag{1}$$

where by $\complement A$ we denoted the absolute complement of A, i.e. $\complement A=\mathbb{R}^n\setminus A$.

We shall now show that this new indicator keeps all the properties of the indicator

$$\rho(x,[a,b])=\left|x-\frac{a+b}{2}\right|-\left|\frac{b-a}{2}\right|, \tag{2}$$

introduced by Cai Wen in [9] for the particular case where $x\in\mathbb{R}$ and set A is an interval of the real numbers' set of the form $[a,b]$, with $a<b$, which it also generalizes, in the sense that the restriction applied to our indicator, to the case studied by Prof. Cai, coincides with ρ. Indeed, the indicator $\mathfrak{s}$ verifies the properties detailed below:

Proposition 1. For any point $x\in\mathbb{R}^n$ and any set $A\subseteq\mathbb{R}^n$, if $x\in\overset{o}{A}$, where with $\overset{o}{A}$ is denoted the interior of the set A in topology induced by the metric d fixed earlier on the space $\mathbb{R}^n$, then $\mathfrak{s}(x,A)<0$, and reciprocally.

The proof of this sentence results directly from the definition of the indicator $\mathfrak{s}$.

Proposition 2. For any point $x\in\mathbb{R}^n$ and any set $A\subseteq\mathbb{R}^n$, we have $x\in\complement\overline{A}\Leftrightarrow\mathfrak{s}(x,A)>0$, where with $\overline{A}$ is noted the closure of set A in topology induced by metric d on space $\mathbb{R}^n$.

As earlier the proof of this sentence results directly from the definition of indicator $\mathfrak{s}$.

Proposition 3. For any point $x\in\mathbb{R}^n$ and any sets A and B in $\mathbb{R}^n$, if $\overline{A}\subset\overset{o}{B}$ then $\mathfrak{s}(x,A)>\mathfrak{s}(x,B),\forall x\in\mathbb{R}^n$.

Proof. We need to analyze three distinct cases.

Case l. $x \in A$. Let $x" \in \partial B$ so that $\delta(x, \complement B) = d(x, x")$ and $x' \in \partial A$ so that the points x, x', $x"$ be collinear, and x' be between x and $x"$, see Figure 1[9]. Under these conditions $\delta(x, \complement A) \le d(x, x') < d(x, x") = \delta(x, \complement B)$. It follows $\mathfrak{s}(x, A) > \mathfrak{s}(x, B)$.

Note. The symbols ∂A and ∂B used above denote the boundaries of sets A and B respectively, meaning $\partial A = \overline{A} \setminus \overset{o}{A}$, respectively, $\partial B = \overline{B} \setminus \overset{o}{B}$.

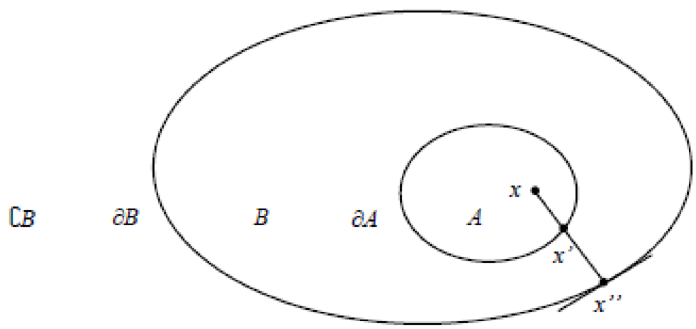

Fig. 1.

Case 2. $x \in B \setminus A$. In this case we have $\delta(x, A) > -\delta(x, \complement B)$, or equivalently, $\mathfrak{s}(x, A) > \mathfrak{s}(x, B)$. See Fig. 2.

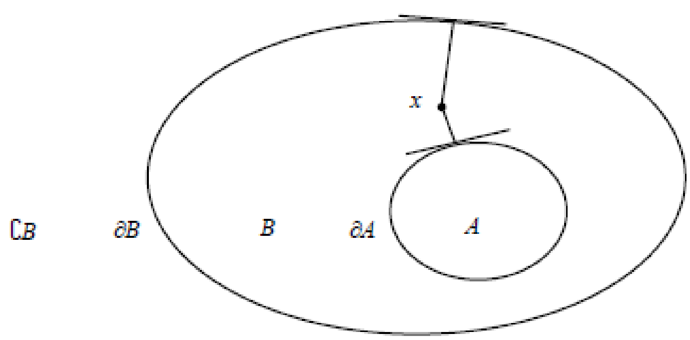

Fig. 2.

[9] Please note that both figure 1 and other figures, to which we shall refer to hereinafter, do not exhaustively cover the multitude of all situations envisaged by the demonstration (for example sets A and B must not necessarily be of domain type). The purpose of these figures is limited to only providing an intuitive visual framework meant to help fix the ideas within that demonstration.

Case 3. $x \in \complement B$. Let $x" \in \partial A$ so that $d(x,x") = \delta(x,A)$ and $x' \in [x,x"] \cap \partial B$, where $[x,x"]$ represents the set of all poinst of the straight line determined by points x and $x"$ situated between x and $x"$. See Fig. 3. With this preamble we can write $\delta(x,A) = d(x,x") > d(x,x') \geq \delta(x,B)$, which means $\mathfrak{s}(x,A) > \mathfrak{s}(x,B)$.

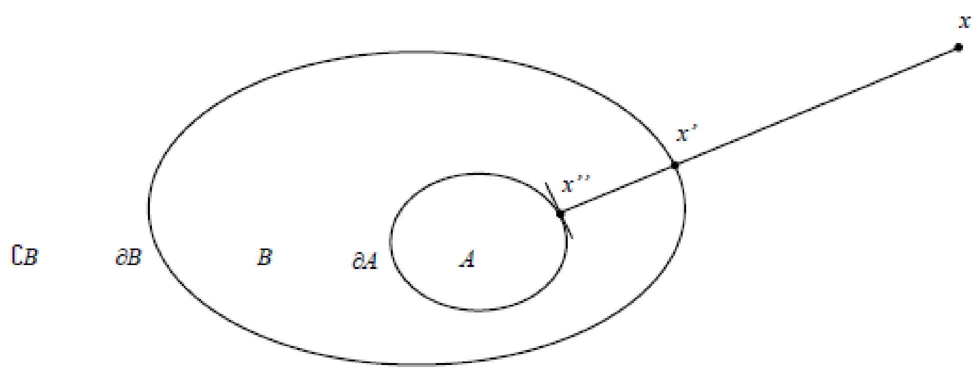

Fig. 3.

Proposition 4. In the particular case $x \in \mathbb{R}$, $A = [a, b]$, $a, b \in \mathbb{R}$, $a < b$, the indicators $\rho(x,A)$ and $\mathfrak{s}(x,A)$ coincide.

The proof to this affirmation results directly from calculation.

3. Maximal extension of the Cai Wen indicator

With the help of the $\mathfrak{s}$ indicator we defined in the previous paragraph it is possible to build a new vital indicator for Extenics Theory, namely

$$\mathfrak{S}(x,A,B) = \frac{\mathfrak{s}(x,A)}{\mathfrak{s}(x,B) - \mathfrak{s}(x,A)}, \tag{3}$$

defined for any $x \in \mathbb{R}^n$ and for any sets A and B in $\mathbb{R}^n$ for which $\overline{A} \subset \overset{o}{B}$. This indicator generalizes the indicator

$$K(x,A,B) = \frac{\rho(x,A)}{\rho(x,B) - \rho(x,A)}, \tag{4}$$

introduced by Prof. Cai in [9] for the case $x \in \mathbb{R}$, $x \in \mathbb{R}$, $A = [a_0, b_0]$, $B = (a, b)$, $a_0, b_0, a, b \in \mathbb{R}$, $a < a_0 < b_0 < b$. The indicator $\mathfrak{S}$ has all the properties detailed below.

Proposition 5. For any two sets A and B in $\mathbb{R}^n$ for which $\overline{A} \subset \overset{o}{B}$ we have $\mathfrak{S}(x, A, B) < -1$, if $x \in \complement\overline{B}$; $-1 \le \mathfrak{S}(x, A, B) < 0$, if $x \in \overline{B} \setminus \overline{A}$; $\mathfrak{S}(x, A, B) \ge 0$, if $x \in \overline{A}$, and reciprocally.

Proof. Using the properties of the indicator $\mathfrak{s}$ detailed in the previous sentences it is easy to deduce that:

In the first case where $x \in \complement\overline{B}$, we have $\mathfrak{s}(x, A) > 0$ and $\mathfrak{s}(x, B) > 0$. It follows that $-\mathfrak{s}(x, B) + \mathfrak{s}(x, A) < \mathfrak{s}(x, A)$. Taking into account that $\mathfrak{s}(x, B) - \mathfrak{s}(x, A) < 0$, (see Proposition 3) we obtain the inequality $\mathfrak{S}(x, A, B) < -1$;

In the second case where $x \in \overline{B} \setminus \overline{A}$, we have $\mathfrak{s}(x, B) \le 0$ and $\mathfrak{s}(x, A) > 0$. Consequently, $-\mathfrak{s}(x, B) + \mathfrak{s}(x, A) \ge \mathfrak{s}(x, A)$. Since additionally $\mathfrak{s}(x, B) - \mathfrak{s}(x, A) < 0$, we can usually deduce that $-1 \le \mathfrak{S}(x, A, B) < 0$.

In the last case where $x \in \overline{A}$, we have $\mathfrak{s}(x, A) \le 0$. Since $\mathfrak{s}(x, B) - \mathfrak{s}(x, A) < 0$, it can be seen immediately that $\mathfrak{S}(x, A, B) \ge 0$.

Proposition 6. In the particular case $x \in \mathbb{R}$, $A = [a_0, b_0] \subset (a, b)$, $a_0, b_0, a, b \in \mathbb{R}$, $a < a_0 < b_0 < b$, the indicators $K(x, A, B)$ and $\mathfrak{S}(x, A, B)$ coincide.

Proof. In the hypotheses above the indicators ρ and $\mathfrak{s}$ coincide (see Proposition 4).

4. Aplication

Many of the state of the art technical installations such as those which emit high intensity radiation bring about areas which are dangerous for humans. We shall assume that we wish to secure such an installation through a centralized system using electronic sensors which oversees the danger areas and depending on the gravity of the situation can send warning signals for users or even stop the system. For this we shall note with X the area in the surrounding space (mathematically modeled through $\mathbb{R}^3$) inside which the radiation is over the admitted safety level and with X_0, ($\overline{X_0} \subset \overset{o}{X}$) that area inside X in which the level of radiation is unacceptable (fatal) for humans. The sensors mounted in the areas $\mathbb{R}^3 \setminus X$, $X \setminus X_0$ and X_0 send to the central monitoring and control unity the spatial coordinates of all persons implicated in the activity. The software application which interprets the data received from the sensors must accomplish the following functions: 1) When the spatial coordinates $x = (x_1, x_2\ x_3)$ of a user belong to the area $\mathbb{R}^3 \setminus X$, the installation is allowed to function unimpeded; 2) When the spatial coordinates $x = (x_1, x_2\ x_3)$ of a user belong to the area $X \setminus X_0$, the overseer system must send warning signals; 3) When the spatial coordinates $x = (x_1, x_2\ x_3)$ of a user belong to the area X_0, the overseer system must stop the installation.

Producing such a software application is much simplified by using the indicator $\mathfrak{S}(x, X_0, X)$ which was defined earlier in relation (3) in which $A = X_0$ and $B = X$. Indeed, according to Proposition 5, if $\mathfrak{S}(x_a(t), X_0, X) < -1$, for any $a \in \mathcal{A}$, where by $\mathcal{A}$ we have denoted the set of employees serving the installation which we refer to and by $x_a(t)$ the spatial coordinates of the employee a at the moment t, the monitoring and control system will not send out a warning signal – the installation is left to function at normal capacity; if $\mathfrak{S}(x_a(t), X_0, X) \in [-1, 0)$ for at least an $a \in \mathcal{A}$, then the monitoring and control system sends out warning signals for the installation users, but the installation is allowed to continue functioning; if $\mathfrak{S}(x_a(t), X_0, X) \geq 0$ for at least an $a \in \mathcal{A}$, then the centralized command system stops the installation unconditionally.

The problem analyzed previously referred to the simple case of a technological installation with only one risk factor. The solution presented can be extended to the general case of installations containing multiple risk factors. To exemplify this we shall analyze the case of a similar installation, but with $n > 1$ sources of radiation. The intended goal is the same as in the case previously studied: designing a monitoring system which will warn of approach into dangerous areas and if necessary interrupt the use of those radiation sources which could endanger humans. In order to fix the ideas we shall suppose $X_1, X_2, .., X_n$ to be the danger areas distributed for each of the n sources of the installation which emit radiation and $X_{01}, X_{02}, .., X_{0n}$ ($\overline{X_{01}} \subset \overset{o}{X}_1, \overline{X_{02}} \subset \overset{o}{X}_2, .., \overline{X_{0n}} \subset \overset{o}{X}_n$) to be the areas in their immediate vecinity of their sources, where the danger for personnel is maximum. Once these zones are established, we may then consider, for each of the alarms systems of the installation, an indicator

$$\mathfrak{S}\left(x, X_{0k}, X_k\right)=\frac{\mathfrak{s}\left(x, X_{0k}\right)}{\mathfrak{s}\left(x, X_k\right)-\mathfrak{s}\left(x, X_{0k}\right)},\ k=1,2,..,n, \tag{5}$$

of the form considered earlier in the case of a one-source installation. The n alarm systems are designed to function according to the same principle as in the case of only one alarm system, but independently from one another, thus, if at a certain amount of time t all indicators $\mathfrak{S}\left(x_a(t), X_{0k}, X_k\right)$, $k=1, 2,.., n$, are strictly smaller than -1 for any $a \in \mathcal{A}$, where $\mathcal{A}$ is the set of personnel, and $x_a(t)$ are the spatial coordinates of employee a at time t, then the technological process is left unimpeded; if at a certain time t there is $a \in \mathcal{A}$ for which one or more of the indicators $\mathfrak{S}\left(x_a(t), X_{0k}, X_k\right)$, $k=1,2,..,n$, have values between -1, inclusively, and 0, exclusively, then the monitoring system or systems in that area will send out warning signals; and if at a certain time t there is $a \in \mathcal{A}$ for which one or more of the indicators $\mathfrak{S}\left(x_a(t), X_{0k}, X_k\right)$, $k=1,2,..,n$, have values greater or equal to 0 then the monitoring and control system or systems in those areas will automatically stop this or these sources of radiation emission.

In the case of the system with more degrees of freedom in regards to the possibility of producing an unwanted incident, situations may arise which are yet more complicated to monitor. For example, in an installation with n, $n>1$ risk factors, if stopping a certain subsystem of the installation is impossible (for technical reasons or those relating to disaster prevention) unless the entire installation is stopped, then the n monitoring and control systems of the installation, which supervise (each of them) the n sources of potential danger, will have to dispose of a relative independence of action only, a central unit for control and monitoring being required to synthesize the information from the local monitoring systems. Moreover, this central unit must have the power to override the local systems when the situation requires an overall interruption of the installation. In order to realize such a monitoring system we would propose that the n local monitoring and control systems are equipped with an indicator of the form (5), and the central unit with a temporal indicator of the form:

$$S(t)=\sup\left\{\mathfrak{S}\left(x_a(t),\ X_{0k},\ X_k\right) \mid 1 \le k \le n,\ a \in \mathcal{A}\right\}.$$

Parameter t appearing above designates the moment in time at which the monitoring is made.

The functioning principle of the protection system: As long as indicator $S(t)$ is strictly negative, the n local monitoring systems are allowed to act independently (meaning they will signal, whenever necessary, the presence of personnel in the moderate risk areas $X_k \setminus X_{0k}$, $k = 1, 2, .., n$). However, when a critical situation arises, where the indicator $S(t)$ would have a value greater or equal to 0, the centralized safety system would command the interruption of the entire installation.

5. Comments

The applicable examples of the indicators $\mathfrak{S}(x, X_0, X)$ and $S(t) = \sup\left\{\mathfrak{S}\left(x_a(t), X_{0k}, X_k\right) \mid 1 \le k \le n, a \in \mathcal{A}\right\}$ respectively, given in the previous paragraph suggest the introduction of a new concept in the theory of dynamic systems, that of "dynamic system with status indicator"[10]. By this notion an important class of systems can be delimitated within dynamic systems, which, by the status indicators they are endowed with, can benefit from special methods to solve a great number of specific issues, such as those related to the real-time quality control process regarding their own functioning.

REFERENCES

[1] *He Bin, Zhu Xuefeng,* "Hybrid Extension and Adaptive Control", Control Theory & Applications, 2005, 22 (2), 165-170.

[2] *Zhi Chen, Yongquan Yu,* "To Find the Key Matter-Element Research of Extension Detecting", Int. Conf. Computer, Communication and Control Technologies (CCCT) , Florida, USA, 2003, 7.

[3] *Wong Chingchang, Chen Jenyang,* "Adaptive Extension Controller Design for Nonlinear Systems", Engineering Science, 2001, 3(7), 54-58.

[4] *Yang Chunyan*, *Cai Wen*, "Extension Engineering", Science Press, Beijing, 2002.

[5] *Yang Chunyan, Li Weihua, Li Xiaomei,* "Recent Research Progress in Theories and Methods for the Intelligent Disposal of Contradictory Problems", Journal of Guangdong University of Technology, 2011, 28 (1), 86-93.

[6] *Guan Feng-Xu, Wang Ke-Jun,* "Study on Extension Control Strategy of Pendulum System ", Journal of Harbin Institute of Technology, 2006, 38 (7), 1146-1149.

[10] In a very abstract manner the notion of dynamic system endowed with status indicator assumes, by definition, the existence of a set $(\Sigma, \mathfrak{I})$ made up of a dynamic system Σ and an indicator $\mathfrak{I}$ of the states which the system Σ passes through during its functioning.

[7] *Chen Jenyang, Wong Chingchang,* “Extension Controller Design via Sliding Mode Control”, Engineering Science, 2001，3 (7), 54-58.
[8] *Florentin Smarandache*, “Generalizations of the Distance and Dependent Function in Extenics to 2D, 3D, and *n*-D”, viXra.org, http://vixra.org/pdf/1206.0014v1.pdf, 2012.
[9] *Cai Wen*, “Extension Set and Non-Compatible Problems”, Advances in Applied Mathematics and Mechanics in China, Peking: International Academic Publishers, 1990,1-21.
[10] *Cai Wen,* “Extension Theory and Its Application”, Chinese Science Bulletin，1999, 44 (17), 1538-1548.
[11] *Cai Wen, Shi Yong*, “Extenics, its Significance in Science and Prospects in Application”, Journal Of Harbin Institute of Technology, 2006, 38 (7): 1079-1086.
[12] *Zhao Yanwei*, “Study of Conceptual Design of Extension for Mechanical Products”, Engineering Science，2001, 18 (6), 68-71.