HIGH PRECISION EXPRESSIONS FOR DETERMINING THE MAXIMUM POWER POINT COORDINATES OF THE SOLAR CELLS (IDEAL MODEL)

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In this work are presented four sets of high precision expressions for estimation of the maximum power point coordinates of the solar cells. The proposed expressions are in explicit-algebraic form and have the advantage that allows the direct calculation of the maximum power point coordinates and the fill factor. The presented relations are the most accurate algebraic-form expressions presented in the literature that can be used to calculate the maximum power point coordinates for the ideal solar cell model.

Keywords: maximum power point, solar cells, single diode model, fill factor, Lambert W-function

1. Introduction

The precise identification of the maximum power point is one of the most important aspects of the practical use of the solar cells. In practice, the determination of the maximum power point is done by implementing at the solar inverters level of the maximum power point tracking algorithms (MPPT algorithms). For common commercial inverters the most used MPPT algorithms are Perturb & Observe (P&O) and Incremental Conductance (IC). The research in recent years has identified many new MPPT algorithms such as Fuzzy Logic Control (FLC) [1], Particle Swarm Optimization (PSO) [2], Artificial Neural Network (ANN) [3].

To understand how the properties of the materials and the technological processes involved in the manufacturing and operating of the real solar cells will affect the electrical behavior of the solar cells, it is necessary to extend and to complete the physical model. A mandatory step in the developing process of a complete reference model is to identify the most accurate expressions that can derive from the physical model.

Identifying of high precision relations (in algebraic form) for the maximum power point coordinates determination is a real challenge and a subject for many works in literature. The difficulty of this issue is due the implicit-

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transcendental equations involved in this process, equations that are without closed-form solutions. Due to the special form of basic equations, the determination of high precision algebraic expressions is even more difficult.

For the case of the ideal single diode model, the most familiar algebraic expression is that presented by M. A. Green in [4]. The proposed expression ensures estimation of the fill factor (*FF*) with an accuracy of up to 4 decimals:

$$FF = \frac{I_{\max} \cdot V_{\max}}{I_{sc} \cdot V_{oc}} = \frac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 1} \tag{1}$$

The expression (1) only refers to fill factor (which is a product between coordinates of the maximum power point) and does not give any clue regarding to the real values of I_{max} and V_{max}.

In [5] is proposed a set of relations for the maximum power point coordinates determination. Unlike the Green expression, the advantage of these relations is given by the fact that allows the calculation of the maximum power point coordinates and implicit the calculation of the fill factor. Although the study presented in [6] indicates that proposed relations are close to the precision of the Green relation (1) but the precision is net in favor of the Green's relationship.

Due to the relatively good precision of the Green relation, after the publication of M.A. Green's relationship, the research on identifying an algebraic relationship for ideal model has diminished. Most recent works are focusing on fill factor (*FF*) determination for the single-diode model with parasitic resistances.

2. Mathematical model of the solar cells

The simplest equivalent electric circuit that models the PV cells is the single diode model (*Fig.1*).

The equation that results from the single diode model is based on the Shockley diode equation [4], [7]:

$$I = I_L - I_0 \cdot [e^{\left(\frac{V}{V_T}\right)} - 1] \tag{2}$$

Where: I_L – the photo-generated current, I_0– the junction saturation current, m – the junction ideality factor, k – Boltzmann constant, q – Elementary charge, T –the cell temperature in Kelvin.

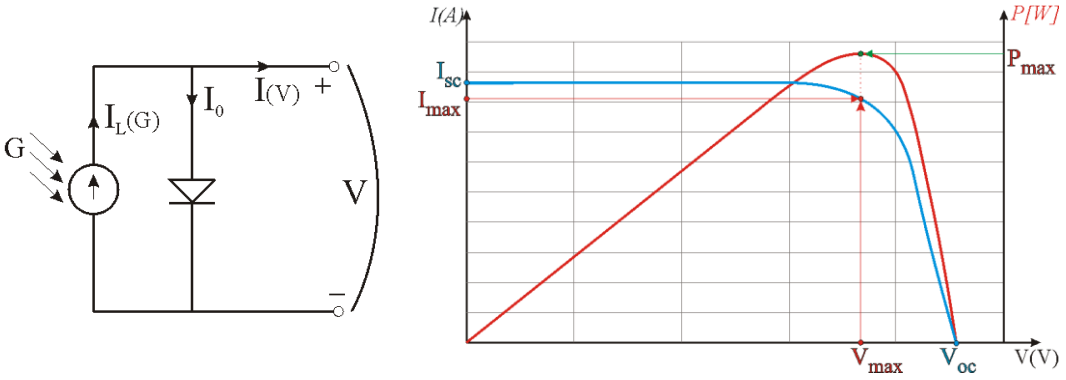

Fig. 1. PV cell electrical equivalent circuit and *I-V*, *P-V* characteristics

The equation (2) is known as the three-parameter model of PV cells. The three parameters are: I_L, I_0 and m, the cell temperature (T) is assumed to be known.

On the characteristic curves (*Fig.1*) can be identified three important points: open-circuit point ($V=V_{OC}$, $I=0$), short-circuit point ($V=0$, $I=I_{SC}$) and maximum power point ($V=V_{max}$,$I=I_{max}$).

2.1 Normalized forms of the characteristic equations

In this paper it is proposed the using of the characteristic equations in normalized forms. For this purpose the following normalizations of the main variables will be made:

$$\bar{I} = \frac{I}{I_{sc}}; \quad \bar{V} = \frac{V}{V_{oc}} \tag{3}$$

Using of the equations in normalized forms presents some advantages:

- Simpler forms of the equations;
- The new variables ($\bar{V}$ and $\bar{I}$) will by characterized by subunit values.

Another advantage of the normalized forms is that the maximum power (written in normalized form) it is, in the fact, the fill factor (FF):

$$\bar{P}_{\max} = \bar{I}_{\max} \cdot \bar{V}_{\max} = \frac{I_{\max}}{I_{sc}} \cdot \frac{V_{\max}}{V_{oc}} = FF \tag{4}$$

Considering that the junction saturation current is very small relative to the photo-generated current the following approximations can be made:

$$I_L + I_0 \cong I_L \;\; ; \;\; I_{SC} = I_L - I_0 \cong I_L \tag{5}$$

Using the approximations above (5) and the adopted notations (3), the *I-V* and *V-I* characteristic equations can be written in normalized forms as below:

$$\bar{I} = 1 - e^{v_{oc}\cdot\left(\bar{V}-1\right)} \tag{6}$$

$$\bar{V} = 1 + \frac{\ln\left(1-\bar{I}\right)}{v_{oc}} \tag{7}$$

Where the v_{oc} notation is known as the normalized open voltage [4]:

$$v_{oc} = \frac{V_{oc}}{V_T} \;\; ; \;\; V_T = \frac{m\cdot k\cdot T}{q} \tag{8}$$

Where V_T is known as the thermal voltage

3. The maximum power point determination

For the practical applications, the most important point is P_{max}. The importance of this point derives from the fact that the operation of the solar cell at this point will ensure the maximum efficiency of the photovoltaic conversion.

The maximum power point coordinates can be obtained by solving the equations resulted by zeroes of the power function ($P=V\cdot I$) derivatives.

Based on normalized forms (6) and (7), the derivatives of the power function can be written as bellow:

$$\begin{cases} \left.\dfrac{d\bar{P}}{d\bar{V}}\right|_{(\bar{V})} = \bar{I} + \bar{V}\cdot\dfrac{d\bar{I}}{d\bar{V}} = 1 - \left(v_{oc}\cdot\bar{V}+1\right)\cdot e^{v_{oc}\cdot\left(\bar{V}-1\right)} \\ \left.\dfrac{d\bar{P}}{d\bar{I}}\right|_{(\bar{I})} = \bar{V} + \bar{I}\cdot\dfrac{d\bar{V}}{d\bar{I}} = 1 + \dfrac{\ln\left(1-\bar{I}\right)}{v_{oc}} - \dfrac{\bar{I}}{v_{oc}\cdot\left(1-\bar{I}\right)} \end{cases} \tag{9}$$

So, the maximum power point coordinates can be obtained by solving the next equations:

$$\left.\frac{d\bar{P}}{d\bar{V}}\right|_{(\bar{V})} = 0 \;\Leftrightarrow\; v_{oc}\cdot\bar{V} + 1 = e^{v_{oc}\cdot\left(1-\bar{V}\right)} \tag{10}$$

$$\left.\frac{d\bar{P}}{d\bar{I}}\right|_{(\bar{I})} = 0 \;\Leftrightarrow\; \left(v_{oc}+1\right)\cdot\bar{I} - v_{oc} = \left(1-\bar{I}\right)\cdot\ln\left(1-\bar{I}\right) \tag{11}$$

It can be seen that the both equations (10) and (11) have implicit-transcendental form. The exact solutions of these equations cannot be expressed in algebraic forms. The solutions of these equations can be determined using numerical methods, by approximations, empirical or by combined methods.

4. Solving methods of the equations (10) and (11)

In this chapter methods for approximate solving of the (10) and (11) are presented. The proposed solving methods will result in four sets of algebraic equations useful for high precision determination of the maximum power point coordinates.

4.1 Solving method based on graphical method

The graphical representation of the (11) is presented on *Fig.2.* From this graphical representation it can be observed that the solution of (11) is the *M* point. The *M* point is the intersection point between the line: $y=(v_{oc}+1)\cdot\bar{I}-v_{oc}$ and the curve: $y=(1-\bar{I})\cdot\ln(1-\bar{I})$.

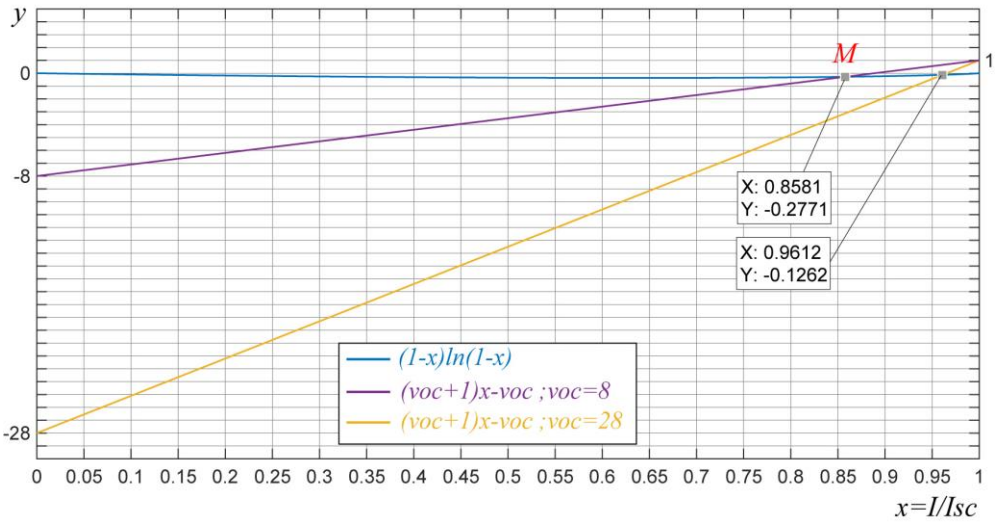

Fig. 2. Graphical representation of the equation (11)

For commercial silicon-based PV cells, the open circuit voltage at 25°C is around 0.6-0.7 V. Thus, for a level of solar radiation intensity: (100-1000) W/m^2 and for operating temperatures: (-20,+80)°C, the normalized voltage (v_{oc}) values can be considered to be in the range: $v_{oc}\in(8-28)$. For this v_{oc} interval, based on

(11), it can be determined that the values interval for the *x*-coordinate of the intersection point (*M*) is:

$$x = \bar{I} \in \approx [0.86 - 0.96] \tag{12}$$

4.1.1 Linear approximation

For the mentioned range (12) of the $\bar{I}$, the following linear approximation of the right term of the (11) is proposed:

$$(1-\bar{I})\cdot \ln(1-\bar{I}) \cong 1.44 \cdot \bar{I} - 1.52 \tag{13}$$

Substituting (13) in (11), the $\bar{I}_{\max}$ will be expressed by solving the resulting first-degree equation:

$$(v_{oc} - 0.44)\cdot \bar{I} - (v_{oc} - 1.52) = 0 \tag{14}$$

The $\bar{V}_{\max}$ will be expressed using (7). Thus, are obtained the first set of algebraic expressions for the maximum power point coordinates determination:

$$\begin{cases} \bar{I}_{\max} = \dfrac{v_{oc} - 1.52}{v_{oc} - 0.44} \\ \bar{V}_{\max} = 1 + \dfrac{1}{v_{oc}} \cdot \ln\left(\dfrac{1.08}{v_{oc} - 0.44}\right) \end{cases} \tag{15}$$

4.1.2 Quadratic approximation

For the mentioned range (12) of the $\bar{I}$, the following quadratic approximation of the right term of the (11) is proposed:

$$(1-\bar{I})\cdot \ln(1-\bar{I}) \cong 5.83 \cdot \bar{I}^2 - 9.17 \cdot \bar{I} + 3.3 \tag{16}$$

Substituting (16) in (11), the $\bar{I}_{\max}$ expression can be obtained by solving the resulting second-degree equation:

$$5.83 \cdot \bar{I}^2 - (v_{oc} + 10.17)\cdot \bar{I} + 3.3 + v_{oc} = 0 \tag{17}$$

The $\bar{V}_{\max}$ expression will be determinate using (7). Finally, are obtained the second set of algebraic expressions for the maximum power point coordinates determination:

$$\begin{cases} \bar{I}_{\max} = \dfrac{(v_{oc}+10.17)-\sqrt{(v_{oc}-1.49)^2+24.253}}{11.66} \\ \bar{V}_{\max} = 1+\dfrac{\ln\left[(1.49-v_{oc})+\sqrt{(v_{oc}-1.49)^2+24.253}\right]-\ln(11.66)}{v_{oc}} \end{cases} \tag{18}$$

4.2 Solving method based on Lambert W-function

The Lambert *W*-function is, by definition, the inverse function of the relation $f(x)=x\cdot e^x$:

$$W(x)=f^{-1}(x\cdot e^x) \tag{19}$$

The *W*-function is defined for complex number space and is not an injective function. On the real domain are defining two main branches of the *W*-function: one for positive range (injective branch) and one for negative range.

The definition of the Lambert *W*-function makes it useful in solving various types of exponential/logarithmic equations.

The Lambert *W*-function is a multi-valued function and cannot be expressed in an algebraic form. For estimation of the *W*-function, often are used numerical algorithms of successive approximations such as Newton-Raphson method or the Halley method [8].

The Lambert *W*-function values calculation can be done using some specialized mathematical software that has implemented functions for this kind of evaluation (e.g. Maple, MatLab).

Some of the main proprieties of the *W*-function are [8]:

$$x=W\left\{x\cdot e^x\right\}\ ;\ \ x\cdot e^x=A\Leftrightarrow x=W\{A\} \tag{20}$$

$$x=W\{x\}\cdot e^{W\{x\}}\ \ \Leftrightarrow\ \ e^{W\{x\}}=\frac{x}{W\{x\}} \tag{21}$$

The equation (10) can be written as follows:

$$\left(v_{oc}\cdot\bar{V}+1\right)\cdot e^{\left(v_{oc}\cdot\bar{V}\right)}=e^{v_{oc}} \tag{22}$$

Multiplying both terms with e^1 will be obtained:

$$\left(v_{oc}\cdot\bar{V}+1\right)\cdot e^{\left(v_{oc}\cdot\bar{V}+1\right)}=e^{(v_{oc}+1)} \tag{23}$$

According to the property of the *W*-function (20), the solution for (23) is:

$$v_{oc} \cdot \bar{V}_{\max} + 1 = W\left\{e^{(v_{oc}+1)}\right\} \tag{24}$$

From (24) the $\bar{V}_{\max}$ expression can be written as bellow:

$$\bar{V}_{\max} = \frac{W_0 - 1}{v_{oc}} \tag{25}$$

Where: $W_0 = W\left\{e^{(v_{oc}+1)}\right\}$ (26)

Substituting (25) in (6) is obtained:

$$\bar{I}_{\max} = 1 - \frac{e^{W_0}}{e^{(v_{oc}+1)}} \tag{27}$$

Using propriety (21) in (27), $\bar{I}_{\max}$ expression becomes:

$$\bar{I}_{\max} = 1 - \frac{1}{W_0} \tag{28}$$

Expressions similar with (25) and (28) are presented in [9], with the difference that the I_{max} expression is multiplied by a factor (f_0) that compensates the approximations (4) used in this work. Even the author mentions that usually this coefficient can be considered equal to the unit.

Based on (25) and (28), an important observation can be made: for determining of the maximum power or for determination of the fill factor (*FF*) it is sufficient to know only one of the coordinates of the maximum power point.

4.2.1 W_0 parameter approximation

Expressions (25) and (28) are the exact explicit forms solutions of the (10) and (11). Because of the W_0 parameter, these equations cannot be considered as algebraic forms. As previously mentioned, the *W*-function is a not an algebraic function. Fortunately, for well defined ranges, *W*-function values can be approximated, with very good accuracy, using some algebraic expresios.

In this paper is proposed the next precise approximation of the W_0:

$$W_0 = W\left\{e^{(v_{oc}+1)}\right\} = (v_{oc}+1) - \frac{v_{oc}}{v_{oc}+1} \cdot \ln(v_{oc}+1) \tag{29}$$

The (29) expression was obtained based on the inequality (2.5) presented in [10]. This approximation assures a relative error, for W_0 estimation, less than 0.035% for v_{oc}values greater than 8.

The simulations performed with (25) and (28) using the W_0 approximation (29), indicated that the obtained coordinates define a point which is not exactly on the *I-V* curve (*Fig. 3*).

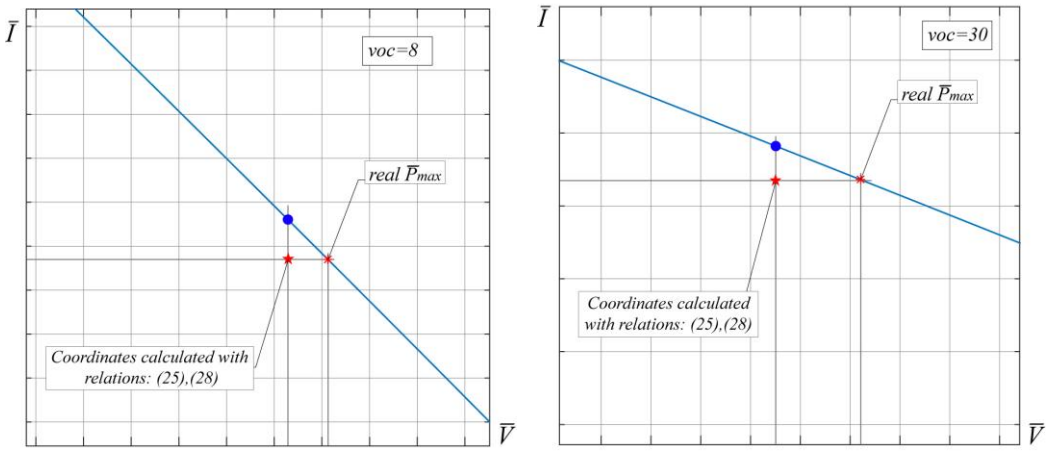

Fig. 3. Deviation from the I-V curve of the point determined by relations (25) and (28) using the W_0 approximation (29)

This mismatch occurs because (29) is an approximation of W_0 and is not the exact value of this parameter. Due this mismatch, two equations will be deduced for the maximum power point coordinates:

$$\begin{cases} \bar{I}_{\max} = v_{oc} \cdot \dfrac{(v_{oc}+1)-\ln(v_{oc}+1)}{(v_{oc}+1)^2 - v_{oc}\cdot\ln(v_{oc}+1)} \\ \bar{V}_{\max} = 1 - \dfrac{\ln\left[(v_{oc}+1)^2 - v_{oc}\cdot\ln(v_{oc}+1)\right]-\ln(v_{oc}+1)}{v_{oc}} \end{cases} \tag{30}$$

$$\begin{cases} \bar{V}_{\max} = 1 - \dfrac{\ln(v_{oc}+1)}{v_{oc}+1} \\ \bar{I}_{\max} = 1 - \left(\dfrac{1}{v_{oc}+1}\right)^{\left(\frac{v_{oc}}{v_{oc}+1}\right)} \end{cases} \tag{31}$$

5. The accuracy estimation of the proposed expressions

The accuracy of the proposed equations: (15), (18), (30) and (31) was evaluated based a numerical simulation using a script developed in MatLab.

The evaluation for each proposed relation was done for 2201 points for *voc* range: [8,30] using a 0.01 discretization step.

As an accuracy indicator was used the relative deviation from the maximum power reference values:

$$\Delta\bar{P}_{\max} = \frac{\left|\bar{I}_{\max}\cdot\bar{V}_{\max} - \bar{I}_{\max\,ref}\cdot\bar{V}_{\max\,ref}\right|}{\bar{I}_{\max\,ref}\cdot\bar{V}_{\max\,ref}} \tag{32}$$

Similarly, where estimated the $\Delta\bar{I}_{\max}$ and $\Delta\bar{V}_{\max}$ indicators.

As mentioned above, the exact solutions of (10) and (11) are given by relations (25) and (28). Consequently, relations (25) and (28) were used for calculating of the references values: $\bar{V}_{\max\,ref}$ and $\bar{I}_{\max\,ref}$. To compute the W_0 parameter as accurately as possible was used the default function implemented in MatLab: "lambertw".

To have an additional reference, the simulations were performed also for the M.A.Green relation (1). For the global evaluation of the accuracy, were made estimation of the mean values and the standard deviation values.

The obtained values for the accuracy indicators are centralized in the *Table 1* and *Table 2*.

Table 1

Values of the $\Delta\bar{P}_{\max}$ accuracy indicator

	Mean	Std. dev.	Max	Range
Proposed relations (15)	1.130E-06	9.225E-07	5.013E-06	5.013E-06
Proposed relations (18)	1.962E-08	2.575E-08	1.078E-07	1.067E-07
Proposed relations (30)	2.404E-09	2.331E-09	7.229E-09	7.061E-09
Proposed relations (31)	4.050E-07	2.080E-07	7.374E-07	7.164E-07
Relation (1) [4]	1.2083E-04	9.992E-05	6.0312E -04	6.016E-04

Table 2

Values of the $\Delta\bar{I}_{\max}$ and $\Delta\bar{V}_{\max}$ accuracy indicators

	$\Delta\bar{I}_{\max}$ (Mean)	$\Delta\bar{V}_{\max}$ (Mean)
Proposed relations (15)	3.291E-04	3.291E-04
Proposed relations (18)	3.472E-05	3.471E-05
Proposed relations (30)	1.623E-05	1.623E-05
Proposed relations (31)	2.174E-04	2.177E-04
Relation (1) [4]	-	-

The simulations results, for all four sets of the proposed expressions and for (1), are presented in graphically form in the Figs. 4-8.

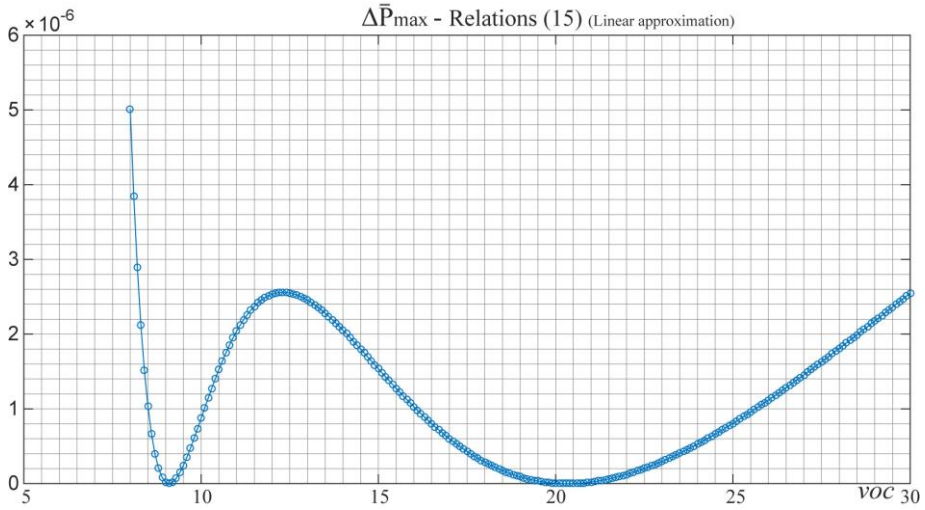

Fig. 4. $\Delta\overline{P}_{max}$ variations for the relations (15)

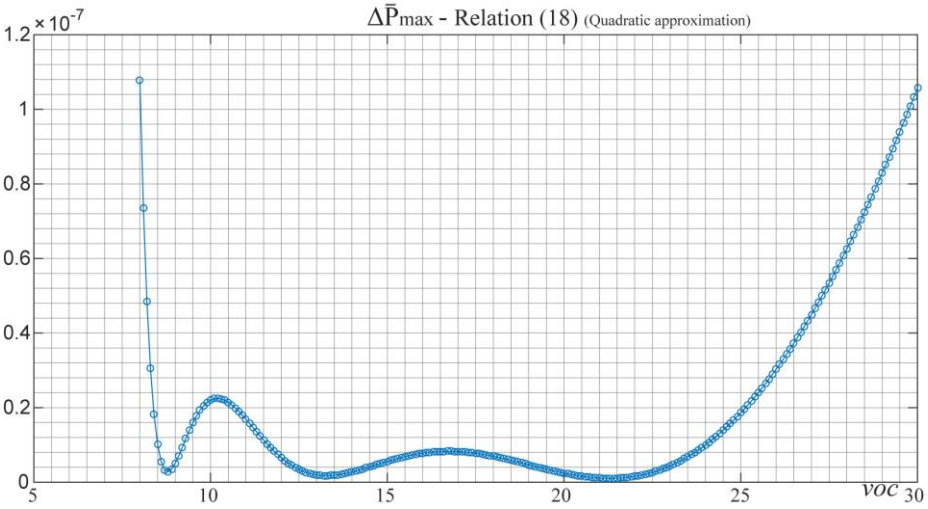

Fig. 5. $\Delta\overline{P}_{max}$ variations for the relations (18)

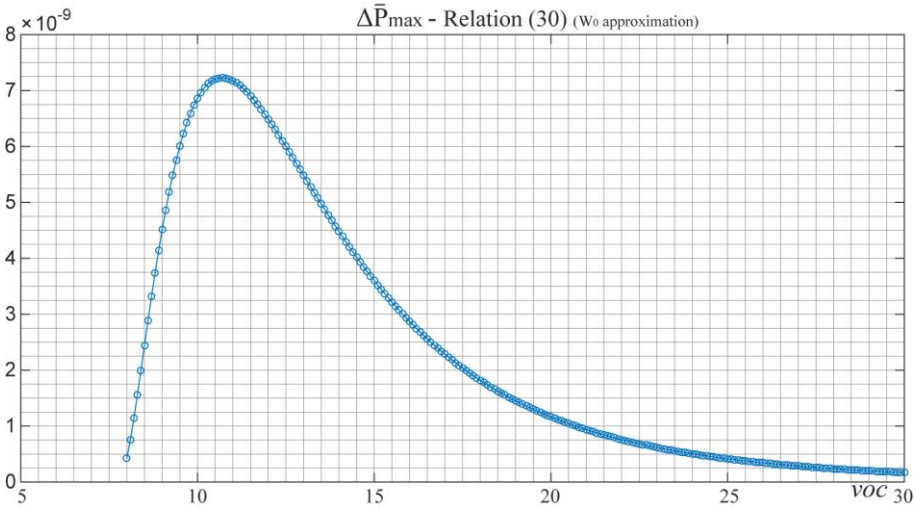

Fig. 6. $\Delta\overline{P}_{\max}$ variations for the relations (30)

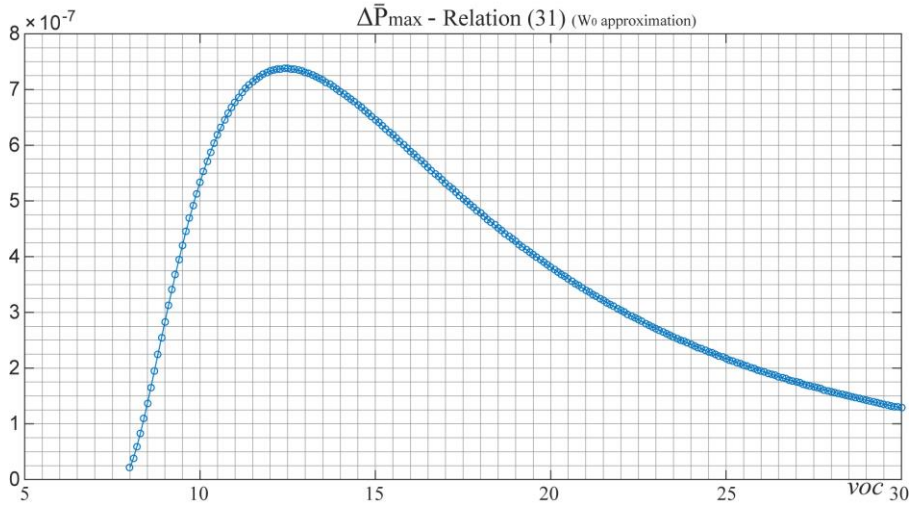

Fig. 7. $\Delta\overline{P}_{\max}$ variations for the relations (31)

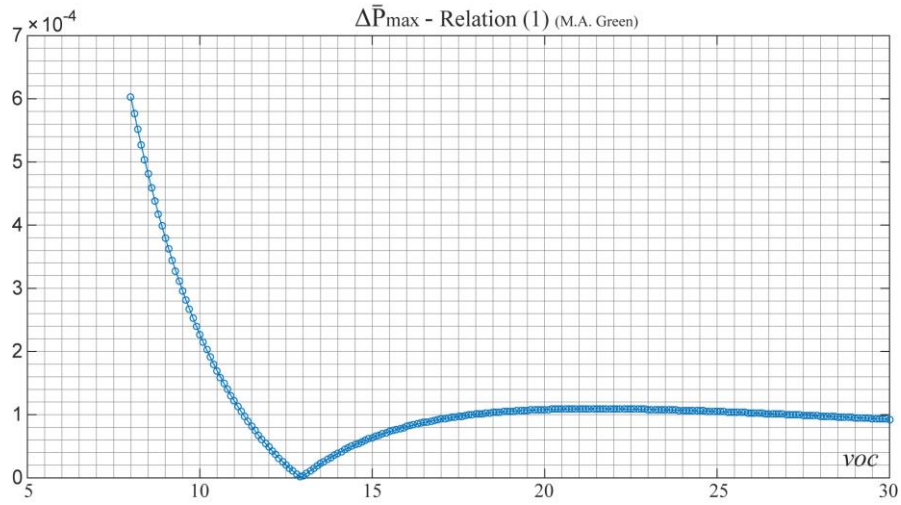

Fig. 8. $\Delta\bar{P}_{max}$ variations for the relation (1)

According to the simulation results, it can be noticed that all proposed relations have an exceptional precision; the proposed relationships have a much better accuracy than well-known relation (1).

The proposed relationships have the advantage that allowing the calculation of the fill factor and also of the maximum power point coordinates ($\bar{V}_{max}$ and $\bar{I}_{max}$). According to the simulation results the maximum power point coordinates determination is also very precise.

6. Conclusions

In this paper are presented four sets of expressions in explicit-algebraic form which are useful for the precise calculation of the maximum power point coordinates for the solar cells (ideal single diode model).

The proposed expressions provide extremely precise calculation of the maximum power point coordinates V_{max} and I_{max}. The relations (30) ensures the determination of fill factor (FF) with an accuracy better of 8 decimal places and the relations (18) with a precision better of 7 decimal places, which makes these relations to be the most accurate relations (in algebraic form) presented in literature.

The approach presented in this paper provides the basis for future research to identify similar expressions for the determination of the maximum power point coordinates for the case of the solar cell model that includes the series parasite resistance.

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