OPTIMIZATION OF MONITORING METHODS WITH LAMB WAVES OF THE COMPOSITE MATERIALS USING PIEZOCERAMICS PATCHES

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Acest articol prezintă o nouă tehnică de monitorizare a stării de integritate a materialelor compozite prin utilizarea unor patch-uri de control. Metoda este rapidă, cu productivitate ridicată, adaptată condiţiilor de măsurare in situ. În prima parte sunt prezentate aspectele teoretice ale modelarii şi simulării propagării undelor Lamb în materialele compozite, utilizând formalismul matematic al seriei Debye. Sunt discutate apoi diferite aspecte legate de sensibilitatea de detectare a defectelor, prin evidenţierea influenţei parametrilor acustici urmărindu-se optimizarea rezultatelor obţinute experimental. În final sunt prezentate concluziile şi perspectivele pentru dezvoltarea metodei.

This article presents a new monitoring technique of the composite materials' integrity status using control patches. The method is quick, with high productivity, adjusted to the "in situ" measurements' conditions. In the first part are presented theoretical aspects of the modeling and simulation of the Lamb wave propagation into composite materials, using mathematical formalism of the Debye's series. Then, different aspects related to defects' detection sensibility are discussed, by highlighting the influence of the acoustic parameters aiming optimization of the results obtained experimentally. At the end conclusions and perspectives for the method development are presented.

Keywords: Lamb waves, piezoelectric active sensors, piezoelectric patch, damage detection, wavelet-based signal processing, Debye's series

1. Introduction

Monitoring of the composite materials with Lamb waves is a developing domain due to multiple sensitive structures, of major importance, that contains materials used in aeronautic industry, aerospace industry, civil engineering, auto industry, nuclear industry by production electric energy [1] (especially for protection the concrete walls of the safety enclosure of the reactor).

Monitoring of the integrity status of the composite structures [2] is a new method which permits permanent assessment of them by measuring some critical parameters (in the presented case acoustical parameters). These ones offer

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information about changes of the undamaged condition of the material in real time.

At present, there are many nondestructive evaluation methods of the materials, different techniques to identify the localized defect, assessment of the integrity status of the structures etc. Among them, the methods that use ultrasound as the information bearer agent about the controlled test have a special place.

The control with ultrasounds is a control method similar to radar type or sonar type technique. Classically, this consists of emission of a short term impulse in the sample and observation of the reflected or diffused echoes on the possible encountered discontinuities [3].

Appearance and development of composite materials led to improvement of the control methods with classical ultrasounds which were often in the situation to be unadjusted to the specific control needs of such a structure. For this purpose is constituted this intercession which aims to identify the most appropriate control method for the control of composite material layers [4].

The most important problems of the control methods with classical ultrasounds are related to: accessibility (to be able to work, for example, in transmission the sample should have been many times disassembled), problems with acoustic coupling (when a coupling gel is used results reproductibily may be inaccurate or in case of working with high temperatures or with porous materials, coupling through classical methods practically is impossible, control techniques with ultrasounds generated by laser or air transmitter/transducer were developed [5]), difficulties when dimensions of the pieces become important (interferometry or acoustic microscopy resolves the problem of the irreproducible coupling, but this can be applied only in case of pieces with large size).

In conclusion, the choice of a certain control method with ultrasounds is made in close correlation with the specific application that will be used.

In developing control systems of composite materials integrity [6] the most encountered practical problems were related to the control of layers of small thickness, such as structure of airplane wings, the walls of different reservoirs, floating structure of boats, different pipes etc. Consequently, the majority of models led to the use and study of their own vibration modes of a solid layer in vacuum. These particular modes of vibration in vacuum (or in the air in a quite good approximation from the acoustic point of view) are Lamb waves.

The choice of Lamb waves is justified by numerous advantages which are offered to specific study. Unlike the surface waves (Rayleigh waves) whose amplitude decrease with increasing the depth of penetration, Lamb waves have the capacity to detect internal discontinuities regardless of the depth to which they are situated. Also, Lamb waves are spread through the length of the composite material layers without a significant energy loss although composite materials have a high acoustic attenuation coefficient. In fact, the use of vibration modes with relative low frequency which limit the influence of intrinsic diffusion is preferred. In addition, because of the fact that propagation occurs in the interior of the limited domain with relatively reduced dimensions in thickness, the loss of the amplitude caused by diffraction is lower than in the case of volume waves.

The structure of a composite material is heterogeneous, because it consist of fibers fabric more or less compact, integrated in a matrix, usually epoxidic resin. From the literature and taking into consideration working frequencies situated in the interval between 1-6 MHz, , in a first approximation and with a good reflection in reality, the whole structure could be assimilated as being homogenous. Instead of it the presence of fibers make the composite material to have a strong anisotropic character [7].

Thus, Lamb waves open the way to their use in the integrate systems of control to long distance [8] without the need of a very big number of transducers and hence to increase the complexity of the evaluation system.

This method presents some disadvantages which make its regular use difficult. Wavelength and structure dimension (thickness of the plate, cylinder diameter, disc height) that propagates should have same dimensions; otherwise the propagated waves become pure surface waves. Lamb waves have a pronounced multimodal character, respectively at certain frequency several waves may coexist, each being spread with different speeds [9]. The dispersive character of the Lamb waves refers to the fact that for a certain mode of propagation (usually it is used one fundamentally symmetric or anti-symmetric of zero order) the speed of waves' propagation depends on the frequency. Theses disadvantages lead to a signal complex which may be received in the frequency band of the transducer receptor. Interpretation of them is relatively difficult and that's why currently the nondestructive control methods that use Lamb waves are limited [10].

This article applies monitoring methods with Lamb waves of the composite materials using piezoceramics patches for a specific application, respectively, layer of composite material, with protective role against ionizing radiation of the safety enclosure of atomoelectric reactors.

Specific restrictions that exist in case of nondestructive control with ultrasounds of the protective walls of the composite material of the safety enclosure of a nuclear reactor are as follows: difficult and unilateral access of the piece that needs to be tested; important damages in case of structure deterioration; economical waste being out of operation during the test control; danger of harm of the health and safety of the operator that is doing the investigation; human and ecological catastrophe in case of nuclear accident.

In this study such problems are solved by setting up an intelligent control structure by adding transducers, transmitters and receptors on the wall of the composite material (affixing composite material patches, then these ones are connected to the installation that will generate/receive Lamb waves into/from the interior of the structure). Beside transducers, there is a need of classical equipments for any control process, respectively ultrasound generator, signal amplifier, oscilloscope for signal visualization, informatics equipment with specialized software for data registering and processing, automatic alert systems, warning alert, actions in case of accident.

2. Lamb waves

In elastic solid and with hypothesis of small deformations, tensor relation among tensions T_{ij} and deformations S_{kl}, may be expressed as follows:

$$T_{ij} = \sum_{k=1}^{3}\sum_{l=1}^{3} C_{ijkl} S_{kl}\,, \tag{1}$$

where coefficients C_{ijkl} are the components of elastic rigidity tensor and are named rigidity constants.

Writing the equation of wave propagation in a solid of infinite dimensions allows deduction of its characteristics in the phase and polarization matter. Neglecting the forces of gravity and inertia from the interior of elementary volume of solid matter, the fundamental principle of dynamics may be written as follows:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial T_{ij}}{\partial x_j},\ i, j = 1, 2, 3 \tag{2}$$

Introducing the value of tensor T_{ij} in the above written equation and taking into consideration the deformation tensor expression S_{lj}, the following equation system will be obtained:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = C_{ijkl} \frac{\partial^2 u_l}{\partial x_j \partial x_k},\ i, j, k, l = 1, 2, 3 \tag{3}$$

where: ρ is the density of the propagation medium; u_p, $p = i, l$ – components of the displacement vector; x_p, $p = j, k$ – components of the position vector; t - propagation time; C_{ijkl} - is the elastic rigidity tensor of 4th grade.

For an infinite propagation medium we are looking for solutions under the form of plane waves, progressive, which propagate with the phase speed c, to the defined unitary vector direction $\vec{n}$, coordinates n_i, n_j, n_k. For the components of the unitary displacement vector the relation will result in form of:

$$u_i = A \cdot P_i \exp\left(i\omega\left(t - n_j x_j / c\right)\right),\ i, j = 1,2,3 \tag{4}$$

where: A is the wave amplitude; P_i – wave polarization vector; ω - pulsation; t - time; x_j – vector position of the point where the evaluation is made; n_j - unitary vector which indicates the direction of the wave propagation; c phase speed of the wave.

Under other form the previous relation may be as follows:

$$u_i = U_i \exp\left(t - \frac{n_j x_j}{c}\right),\ i, j = 1,2,3 \quad (5)$$

where the systematic term $e^{i\omega}$ has been omitted.

Replacing in the propagation equation relation, the Christoffel equation is obtained:

$$\rho \cdot c^2 u_i = C_{ijkl} n_j n_k n_l,\ i, j, k, l = 1,2,3 \quad (6)$$

In plane waves' regime, monochromatic, solutions of the propagation equation are the eigen vectors, for polarization and eigen values, for phase speed of the tensor Christoffel Γ_{il}:

$$\Gamma_{ij} = C_{ijkl} n_j n_k, \quad (7)$$

In a general case, of one triclinic material with propagation to the direction of a vector $\vec{n}$, it will result coexistence of three plane waves orthogonally polarized, every second: one quasi-longitudinal wave (the direction of polarization is close to direction of propagation) and two quasi-transversal waves (rapid one and slow one with almost perpendicular propagation waves, orthogonal with each other).

The values for these three phase speeds according to these three waves are given by the following equation (Christoffel's equation):

$$\left|\Gamma_{ij} - \rho \cdot c^2 \delta_{il}\right| = 0, \quad (8)$$

If the solid, in which the waves propagate, is limited by two parallel faces (the case of a solid slate in vacuum), than two types of surface waves may propagate, without any interaction with each other as long as the thickness of the slate is big enough comparative to the acoustic wavelength.

When the dimensions of the thickness of the slate are comparable with the wavelength, Lamb waves emerge discovered for the first time in 1917[11]. The waves named and as layer waves, are dispersive and have the feature of propagation through the whole thickness of the layer. There are two modes of propagation of the Lamb waves: symmetric and anti-symmetric mode.

The study of Lamb wave's propagation shows the calculation of the dispersion curves which represents the profiles of the phase speeds depending on the relation frequency-thickness of the layer.

To calculate the dispersion curves of the Lamb waves in a solid layer of isotropic composite material, Viktorov [12] decompose the acoustic field from the interior of the layer in sum of the scalar sum $\boldsymbol{\Phi}$ and rotational vector potential $\overrightarrow{\boldsymbol{\Psi}}$.

$$\vec{u} = \vec{\nabla}\Phi + \vec{\nabla} \wedge \vec{\Psi}, \quad (9)$$

The studded layer has finite dimensions to x_1 and x_2 directions of the triaxial orthogonal mark from the Fig. 1 and the thickness d, finite to x_3 direction.

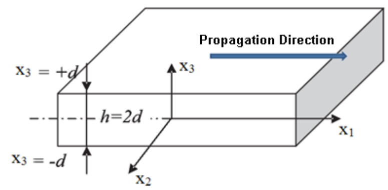


Fig. 1. Tridimensional model of homogenous and isotropic solid layer in vacuum.

A plane wave, non-uniform by propagated to the x_1 direction is considered. Potentials are invariant translating to the x_2 direction, thus all the physical dates have partial derivative as zero in relation with x_2 variable.

Scalar potential $\boldsymbol{\Phi}$ and vector potential $\vec{\boldsymbol{\Psi}}$**,** will have the following form:

$$\Phi = \varphi\left(x_3\right)e^{i\left(kx_1 - \omega t\right)}; \quad \vec{\Psi} = \vec{\psi}\left(x_3\right)e^{i\left(kx_1 - \omega t\right)}. \tag{10}$$

where: $\boldsymbol{\Phi}$ - amplitude of the scalar potential; $\vec{\boldsymbol{\Psi}}$ - amplitude of the vector potential; k - wave number; ω – wave pulsation; t – time.

Hereinafter, the phase term $e^{i(kx_1 - \omega t)}$ will be omitted, because we are interested only in variation of the amplitudes $\boldsymbol{\Phi}$, $\boldsymbol{\psi_1}$, $\boldsymbol{\psi_2}$ and $\boldsymbol{\psi_3}$ of the particular displacement on the length of axe x_1.

For a wave that propagates with wave number k and pulsation ω, the components of the displacement vector $\vec{u}$ may be calculated with the relation:

$$u_1 = ik\varphi - \frac{\psi_2}{\partial x_3}; u_3 = ik\psi_2 + \frac{\partial \varphi}{\partial x_3}; u_2 = -ik\psi_3 + \frac{\psi_1}{\partial x_3}. \tag{11}$$

It can be noticed that first two equations are coupled and depend only on scalar and vector potentials $\boldsymbol{\Phi}$ and $\boldsymbol{\psi_2}$. They describe Lamb wave which propagates through the polarized layer in sagittal (vertical) plan.

The third equation is independent and describes the transversal horizontally polarized wave on the length of axe x_2, named HT wave (horizontally transversal).

Replacing first two equations of the Lamb wave propagation in relation that describes fundamental principle of the dynamics (2), the following expressions will be obtained:

$$C_{11}\nabla^2\varphi - \rho\frac{\partial^2\varphi}{\partial t^2} = 0; C_{55}\nabla^2\psi_2 - \rho\frac{\partial^2\psi_2}{\partial t^2} = 0. \tag{12}$$

If the wave number of the longitudinal and transversal plane waves will be noted as k_L and k_T which propagate through the layer with the phase speed c_L and c_T, the following expressions will result:

$$k_L = \omega\sqrt{\frac{\rho}{C_{11}}} = \frac{\omega}{c_L}; k_T = \omega\sqrt{\frac{\rho}{C_{55}}} = \frac{\omega}{c_T}. \tag{13}$$

Replacing afterwards in relation (12), these ones will take the form mentioned bellow:

$$\frac{\partial^2 \varphi}{\partial x_3^2} + p^2 \varphi = 0; \quad \frac{\partial^2 \psi_2}{\partial x_3^2} + q^2 \psi_2 = 0. \tag{14}$$

where: $p^2 = k_L^2 - k^2$ and $q^2 = k_T^2 - k^2$ are material constants; ϕ- scalar potential amplitude ; $\boldsymbol{\psi_2}$ - vector potential amplitude corresponding to the component of the length of the axis x_2.

The solutions of the Lamb waves propagation equations' should satisfy the boundary conditions such as cancelling of the normal and shear stresses on the free surfaces:

$$T_{33}(x_3 = \pm\frac{d}{2}) = T_{13}(x_3 = \pm\frac{d}{2}) = 0\,, \tag{15}$$

where: *d* is the thickness of the layer; T_{33} – normal stress on the interface; T_{13} – tangential stress on the interface.

With these restrictions the following solutions for the equations will be obtained (10):

$$\varphi = B\cos\left(px_3 + \xi\right); \quad \psi_2 = A\sin\left(qx_3 + \xi\right). \tag{16}$$

where: *B* is scalar potential amplitude ; *A* - vector potential amplitude; $\xi = 0$ or $\xi = \pi/2$, dephasage.

For a certain value given to ξ, potentials ϕ and $\boldsymbol{\psi_2}$ depending on x_3 are in opposition of phase.

If we will replace the previous relations in the restrictions system, for each ξ value a linear system of two equations with two unknown *A* and *B* will be obtained. This system admits nonzero solutions if and only if its determinant is zero, respectively if the following expression is satisfied:

$$\left(k^2 - q^2\right)^2 \tan\left(q\frac{d}{2} + \xi\right) + 4k^2 pq\tan\left(p\frac{d}{2} + \zeta\right) = 0\,, \tag{17}$$

It may be noticed that if $\xi = 0$, the component u_1 (respectively u_3) of displacement is a pair function, respectively impair for x_3. In this case, a symmetric mode of propagation or compression mode, is described (fig. 2).

When $\xi = \pi/2$, component u_1 is pair while u_3 is impair. Thus the anti-symmetric mode of propagation is defined (fig. 2).

For each of these two possible values of ξ, dispersion equations for the symmetric mode will be obtained, when $\xi = 0$ and anti-symmetric when $\xi = \pi/2$:

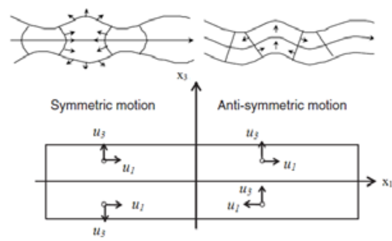


Fig. 2. Lamb propagation modes: symmetric and anti-symmetric.

$$\left(k^2 - q^2\right)^2 \tan\left(q\frac{d}{2} + \xi\right) + 4k^2 pq \tan\left(p\frac{d}{2} + \zeta\right) = 0;$$
$$\left(k^2 - q^2\right)^2 \cot\mathrm{an}\left(q\frac{d}{2} + \xi\right) + 4k^2 pq \cot\mathrm{an}\left(p\frac{d}{2} + \zeta\right) = 0. \tag{18}$$

Solving the equations (18) permits determination of waves numbers k of symmetric and anti-symmetric propagation modes depending on elastic constants C_{11} and C_{55} (they are in the parameters expressions p and q and thickness d of the layer).

Analytically, this is only possible in particular cases when $\omega = 0$, or when $\omega = \infty$. In general, these equations may be solved numerically with the help of specific software and may represent the evolution of wave number k depending on pulsation ω, on phase speed c_p, or depending on multiplication between frequency and thickness fd.

The authors have elaborated a software using the platform BorlandC++ Builder6, with which dispersion curves have been traced depending on the phase speed of the first propagation modes of the Lamb waves in a silicon layer with following characteristics: $C_{11} = 78{,}5\ [GPa]$, $C_{13} = 16{,}1\ [GPa]$, $C_{55} = (C_{11} - C_{13})/2 = 47{,}3\ [GPa]$, $\rho = 2{,}20\ [g/cm^3]$, (fig. 3).

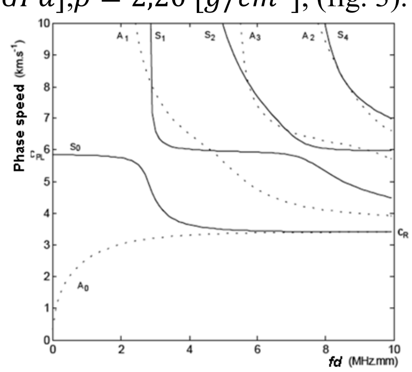


Fig. 3. Dispersion curves of the Lamb waves for the symmetric propagation mode (continuous line) and anti-symmetric (discontinuous line) in the silicon crystal

The obtained results were compared with those presented in the literature and a good agreement was noticed, the differences being under 5% thus, it can be concluded that elaborated model is correct.

3. Plane wave reflexion/refraction. Mathematical formalism of Debye's series

Hereinafter it is presented the simplified case of reflexion and refraction of heterogeneous plane wave with unitary amplitude on the origin, in isotropic layer presumed absorbent immersed in a fluid with known acoustic properties.

Acoustic displacement field in the interior of the layer, in the stationary regime, will be given by the formula:

$$\vec{U}(M.\omega,t)=\Re\left\{\sum_{p=1,2}\sum_{m=L,T}{}^{*}X_{pm}\vec{P}_{pm}\exp i\left(\omega t-{}^{*}\vec{K}_{pm}\vec{M}\right)\right\}, \tag{19}$$

where: ${}^{*}X_{pm}$ - wave amplitude, the sign „ * ” denotes the fact that the value may be complex; $\vec{P}_{pm}$ - wave's polarization vector; ${}^{*}\vec{K}_{pm}$ - wave vector; $m = L,T$ – wave type that propagates (L – longitudinal, T – transversal); $p = 1,2$ – interface where diffraction of the waves occurs (1 – superior, 2 – inferior);

In the immersion fluid the field of analog will be given by relations:

$$\begin{aligned}\vec{U}_1(M,\omega,t)&=\Re\left\{{}^{*}\vec{P}_1\exp i\left(\omega t-{}^{*}\vec{K}_1\vec{M}\right)+{}^{*}X_{1R}\,{}^{*}\vec{P}_{1R}\exp i\left(\omega t-{}^{*}\vec{K}_{1R}\vec{M}\right)\right\};\\ \vec{U}_2(M,\omega,t)&=\Re\left\{{}^{*}X_{2R}\,{}^{*}\vec{P}_{2R}\exp i\left(\omega t-{}^{*}\vec{K}_{2R}\vec{M}\right)\right\}.\end{aligned} \tag{20}$$

In order to be able to calculate the global coefficients of reflexion/refraction of the layer, to find the amplitude expression for all the excited waves in the solid material layer and fluids each layer's interface will be considered as an acoustic diopter between two semi-infinite mediums situated on a distance of +/-d/2 of it's median plane [13]. The problem then is to calculate three amplitudes corresponding to five elementary possible cases, separately considered (fig. 4).

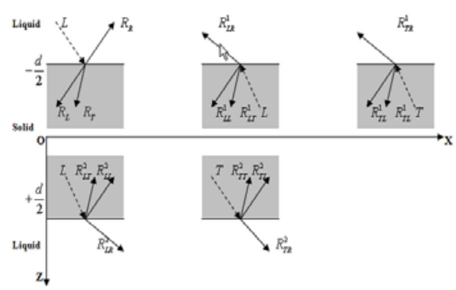


Fig. 4. Simplified model to calculate the reflexion/refraction coefficients in composite material layer

In the figure with discontinuous line is represented the incidental wave and with continuous line is indicated reflected wave and/or transmitted wave.

Each of these cases is a classical problem of propagation of four waves and the solution is resolved in the literature [14]. Solutions obtained should take into account the fact that interface on which reflection/refraction take place is situated on the distance of +/-d/2 regarding the centered mark in median plane of the layer.

Thus if ${}^{*}X_{ps}^{0}$ (*p,s* – conversion coefficient of the incidental wave *p* type transmitted/reflected in wave *s* type) is the amplitude of evaluated wave placing the origin of the cartesian mark on the interface, the expression of this amplitude on the distance z will be given by relation

$$ {}^{*}X_{ps} = {}^{*}X_{ps}^{0} \exp\left(-i\,{}^{*}\phi_{m}\right), \tag{21} $$

where: ${}^{*}\phi_{m} = {}^{*}K_{m} z \cos {}^{*}\theta_{m}$; exponential $\exp\left(-i\,{}^{*}\phi_{m}\right)$ is named phase factor, and $\exp\left(-i\,{}^{*}\phi_{m}\right)$ is named dephasage.

Three vectors will be defined composed from reflexion and refraction coefficients of the layer, as follows:

$$ {}^{*}\overrightarrow{X}_{0} = \left\{ {}^{*}R_{R}, {}^{*}R_{L}, {}^{*}R_{T} \right\}^{T}; \quad {}^{*}\overrightarrow{X}_{p} = \left\{ {}^{*}X_{pR}, {}^{*}X_{pL}, {}^{*}X_{pT} \right\}^{T}. \tag{22} $$

where: ${}^{*}R_{m}\ (m = R, L, T)$ are the components of the vector $\overrightarrow{X}_{0}$ from relation (22), respectively, reflexion coefficients *R*, longitudinal transmission *L* and transversal transmission *T*, on the separation interface (diopter), fluid-solid, for an incidental wave in fluid; ${}^{*}\overrightarrow{X}_{p}$ - are the components of the vector, relative to the excited waves in solid on the separation interface *p*, more detailed quantities ${}^{*}X_{pm}$, with *p = 1,2* and *m = L,T*.

For the case when an incidental wave (discontinuous) is considered to propagate in the fluid in the sense of axe Oz, vector $\overrightarrow{X}_{0}$ components will be obtained from the Fig. 4.

Similar for the rest of the cases, when it is analyzed the situation of longitudinal and transversal waves incidental on the interface *1*, coming from the solid (cases 2 and 3 on top of axe *Ox*), then that of the incidence on the surface 2 of the same waves types coming from the solid into the increasing sense of the axe *Oz*, the components of the four reflexion/refraction vectors on each interface will result.

For each of the evaluated cases the amplitude of the incidental wave is considered known and equal with unity.

In turn, theses permit definition of two reflexion/refraction matrices on each considered interface, as follows:

$$[R_i] = \begin{bmatrix} 0 & R^i_{LR} & R^i_{TR} \\ 0 & R^i_{LL} & R^i_{TL} \\ 0 & R^i_{LT} & R^i_{TT} \end{bmatrix}, \tag{23}$$

Without getting into calculation details, all reflexion/refraction coefficients present in the expression (23) are determined writing the continuity equations in a point on the considered interface [14].

Let [T] be double reflexion matrix in the interior of the solid layer:

$$[T] = [R_1][R_2], \tag{24}$$

Developed, this one will have the form:

$$[T] = \begin{bmatrix} 0 & T_{LR} & T_{TR} \\ 0 & T_{LL} & T_{TL} \\ 0 & T_{LT} & T_{TT} \end{bmatrix}, \tag{25}$$

where the 6 nonzero coefficients of the double reflexion matrix *[T]*, are given by the formula:

$$T_{ps} = R^1_{pL} R^2_{Ls} + R^1_{pT} R^2_{Ts}, \tag{26}$$

This double reflexion matrix is the ratio of the geometrical series (27), named Debye's series:

$$\vec{X}^n_1 = \left([1] + [T] + [T^2] + [T^3] + ... + [T^n]\right)\vec{X}_0, \tag{27}$$

The coefficients vector on the second interface is obtained with relation:

$$\vec{X}^n_2 = [R_2]\vec{X}^n_1, \tag{28}$$

The vector that gives transmission and reflexion coefficients will be obtained taking into consideration infinity of successive reflexion in the interior of the composite material layer, which is the limit of the previous series when n is the number of successive reflexions, increase unlimited.

The obtained relation relative to first interface will have the form:

$$\vec{X}_1 = \left([1] - [T]\right)^{-1}\vec{X}_0, \tag{29}$$

For the second interface the immediate obtained result is:

$$\vec{X}_2 = [R_2]\vec{X}_1, \tag{30}$$

It is important to emphasize that all these coefficients calculated till now take into consideration the wave propagation through only one acoustic diopter that separate two mediums considered semi-infinite fluid-solid, though itdefines propagation of the acoustic waves in the interior of a layer with two separation interfaces fluid-solid.

4. Results

For the performed experimental studies there were built structures of composite material with a thickness of *5* [*mm*] on whose surface transmitter and receptor transducers of Lamb waves were affixed.

The basic used principle was processing of cylinder type defect with a known distance regarding the transmitter patch situated outside of the axis that connects transmitter with the closest receptor transducer with an angle $\theta = 15^o$ and variable depth (fig. 5).

In the first stage, simulated propagation of Lamb waves in this type of complex structure was configured mathematically and numerically and generation and reception of Lamb waves in its interior was analyzed. Each layer (basic composite material, adhesive film, control composite patch) may be considered separated as a monoclinic crystal with Ox_1, Ox_2 symmetry plan.

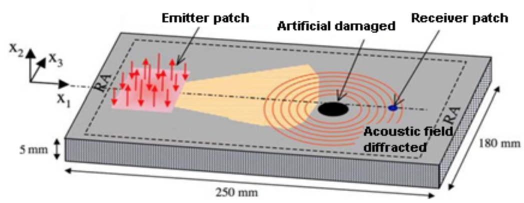


Fig. 5. Schematic model of a composite with control patches.

Lamb waves are guided waves which have many propagation modes through the whole thickness of a thin layer. The modes properties are given by dispersion curves by presenting pulsation ω, depending on the wave number k. Afterwards with simple known mathematical relations, the relation of the phase speed depending on the frequency may be deduced.

To calculate theoretical dispersion curves in a multilayer structure of composite material the method of the transfer matrix with the formalism of the Debye's series was used.

In Fig. 6 are presented dispersion curves drawn for CFRP type a composite. Adjacent layers are disposed each other orthogonally, with observational direction situated on angle of $\varphi = 30^o$. Test thickness is 5 mm, number of layers – 12 placed symmetrically regarding the median plan. Simulation was realized with our own software. These curves are important because they allow identification of the propagation modes excited in experimental conditions. For example, in relative low frequency domain, used usually for non-destructive control, only three propagation modes exists: two fundamental propagation modes A_0, S_0 and horizontal transversal mode SH_0, that are not cut out from Lamb waves in this case.

In order to study the influence of the direction of propagation in relation with orientation of the composite material fibers, simulations were performed on the similar material type previously discussed and the evolution of the ratio between maximum and minimum of the focalizing factor in polar coordinates were analyzed.

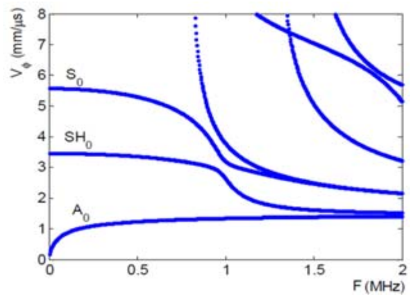


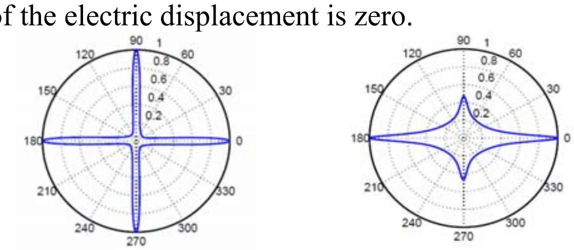
Fig. 6. Theoretic dispersion curves for a composite with adjacent layers placed orthogonally (CFRP type) observational direction on angle φ = 30º, test thickness 5 mm, number of layers 12 placed symmetrically regarding the median plan.

It was ascertained for the fundamental propagation mode A_0 and S_0, so for each made a focalization of the acoustic fascicle exists in the fibers direction. Though, the ratio between the maximum and minimum of the factor is 8% for S_0 and 23,5% for the mode A_0. Consequently, in the high anisotropic structures, to avoid "blind" zones where defects detection with the integrated system control previously described would be impossible, it will be used fundamental propagation mode anti-symmetric A_0, less sensitive to the anisotropy increase of the material than S_0 mode (fig. 7).

The piezoelectric disc affixed to the composite material layer was configured as being of thickness and of diameter . The thickness of the composite material layer stratified is .

If the dimensions of the components respect conditions: $h_{PZT} << h_{plate}$ and $R >> h_{PZT}$ the hypothesis that stresses T_{33} will be canceled may be made.

If it is considered that electric impedance of the measurement circuit is very high comparative with that one of the piezo disc may be presumed that component of the electric displacement is zero.



a b

Fig. 7. Focalization factor , with frequency of 300 kHz in a composite material with fibers placed orthogonal symmetric with 12 layers; s = 5 mm: a – for the fundamental propagation mode S_0; b – for the fundamental propagation mode A_0.

Simplifying constitutive equations of the material leads to relation:

$$E_3 = -\widetilde{h}_{31}\left(S_{11} + S_{22}\right), \tag{31}$$

where: E_3 represents electrical field to direction O$_3$, $\tilde{h}_{31}$ – piezoelectric coefficient of the transducer; S_{ij} – is the deformation tensor.

The acoustic displacement determined by an incidental plane wave is given by relation:

$$u(x,t) = U_0 e^{j(kx_1 - \omega t)}, \tag{33}$$

The created volume dilatation may be calculated with:

$$S_{11} + S_{22} = div(u), \tag{34}$$

The measured tension V is averaged on the whole piezo disc surface according to the relation:

$$V = -\widetilde{h}_{31}\frac{h_{PZT}}{\pi R^2}\int_s div(u)ds\,, \tag{35}$$

If the Stockes' theorem is applied to the relation (35) it will be obtained:

$$V = -2j\widetilde{h}_{31}\frac{h_{PZT}}{\pi R}U_0 e^{-i\omega t}J_1(kR), \tag{36}$$

where J_1 is Bessel's function of the first cause and first order.

Consequently the tension on the piezoelectric disc terminals respects the proportionality relation with the form:

$$\left|J_1(kR)\right| = \left|J_1(\pi D/\lambda)\right|, \tag{37}$$

Thus, incidental plane wave was detected optimally for the wavelength λ equal with 1.5 D, when the maximum of the Bessel function is obtained and was not detected for the wavelength λ equal with 0.8 D, when first zero of Bessel function is obtained.

The behavior of the piezoelectric disc as transmitter was determined by the answer of the level of electric impulse that this one has after applying a tension of a certain structure (modulation) on its terminals. The study was realized by simulating with a software that uses the formalism of the finite elements (Disperse) comparing the simulated results with the one obtained experimentally. The 2D the finite element model is similar with the one presented previously specifying that one layer of absorbent composite material (patch that contains transducers) was added. The influence of adhesive layer that assure the adherence of the patch on the support layer was neglected. We revealed that shear stresses are transmitted to the inferior layer structure especially on the periphery of the piezoelectric disc [15]. The action of the transmitter was configured by a point force applied on the transducer extremity.

Simulations were performed in the domain of specter frequency and converted then in function λ_{A_0}, using the curb of wavelength deduced from the dispersion curves of the composite material layer. The simulation results obtained with finite elements [16] were compared with experimental measurements realized with composite patches as specified previously (fig. 8). The position of the cut wavelength is obvious and corresponds with numeric results. As presumed, the cut wavelengths are the same in situation of the receptor transducer as those considered for piezoelectric transducer in transmitter regime.

This study revealed the necessity of adjusting piezoelectric disc diameter with wavelength of Lamb modes which will be generated or received from the support structure of composite material.

To validate all the results obtained previously, in presented situation, were traced, in parallel, theoretical and experimental transmission coefficients of the energy of Lamb waves for the fundamental propagation modes symmetrical S_0, anti-symmetrical A_0 and anti-symmetrical of first order 1, A_1 (fig. 9).

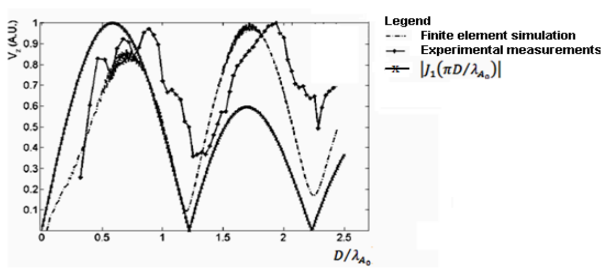


Fig. 8. Normal speed depending on the ratio from the surface of composite material with thickness of 5 mm excited by transducer of 10 mm in diameter submitted to a fusillade of signs sinusoidal type with central frequency of 300 kHz

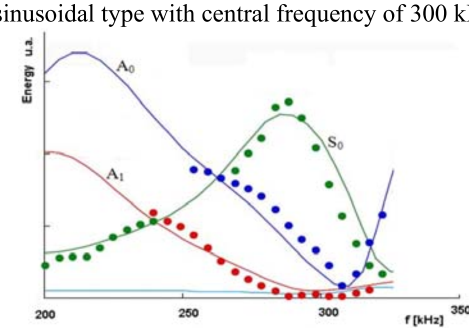


Fig. 9. Transmission coefficients of the energy of Lamb waves for the fundamental propagation modes anti-symmetric A_0 and A_1, respectively symmetric S_0; continuous line – simulated values, discontinuous line- measured values

5. Conclusions

We demonstrated with this study that the use of control sensors with Lamb wave patch type for stratified anisotropic composite materials is justified from physical reality point of view.

Also, definition and programming of mathematical device using the formalism of Debye's series shortens very much the calculation time which could be obtained previously with traditional simulation methods with finite elements.

Particular application, for which the program was created to be used for the composite materials study, can be easily extended to other materials and to other concrete situations excepting control patches, due to the fact that this software contains modules easy to change end to use for any other domain's application.

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