ENHANCING INVERTED PENDULUM CONTROL USING THE ULTIMATE PERIOD METHOD UNDER RANDOM DISTURBANCES

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Inverted pendulums play an important role in automation and mechatronic systems since it is widely applied in products like self-balancing vehicles. Inspiring its mechanism, this paper presents a method for tuning the motion controller using the critical cycle method to address and smooth the response of inverted pendulums. Firstly, theoretical model of this system is established and its controller is proposed to manipulate whole pendulum. To imitate the real-world platform, external forces can be introduced as unwanted disturbances which might reduce system performance. By ensuring stable operation under the effects of disturbances, the ultimate period method is able to drive this system robustly and effectively. From these results of simulations, our method promises significant improvement in control performance under various conditions.

Keywords: stable mechanism, motion control, optimization, mechanical stability, theory control.

1. Introduction

Inverted pendulum systems have long been fundamental models in control theory and robotics due to their inherent instability and nonlinear dynamics [1-3]. They serve as quintessential benchmarks for testing and validating control algorithms before deploying them on more complex and practical systems. The mathematical representation of inverted pendulums is relatively straightforward, yet they encapsulate the challenges inherent in balancing and stability control, making them invaluable for both theoretical exploration and practical application.

Over the years, significant advancements have been made in controlling inverted pendulum systems. Classical control methods, such as Proportional-Integral-Derivative (PID) controllers, have been widely used for their simplicity and effectiveness in linear systems [4]. However, these methods often struggle with the nonlinearities and external disturbances present in real-world scenarios. To

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address these limitations, various advanced control strategies have been proposed, including fuzzy logic controllers [5, 6], neural network approaches [7], sliding mode control [8], and model predictive control techniques [9].

The challenge of controlling inverted pendulum systems is further compounded when random disturbances are introduced. Real-world applications often involve unpredictable environmental factors and uncertainties that can adversely affect system performance. Robust and adaptive control methods have been developed to mitigate these issues, focusing on maintaining stability and performance in the presence of such disturbances [10-13]. Despite these efforts, achieving optimal control under random disturbances remains a significant challenge in the field.

The ultimate period method has emerged as a promising approach for system identification and controller tuning, particularly in dealing with nonlinear systems [14]. By analyzing the system's oscillatory behavior, this method facilitates the design of controllers that can adapt to changing dynamics and disturbances. However, its application to inverted pendulum systems under random disturbances has not been extensively explored, presenting an opportunity for further research.

Inverted pendulum models are not only theoretical constructs but also have practical implementations across various domains. They are instrumental in the development of wheeled robots, providing insight into balance and navigation [15]. In obstacle avoidance algorithms, the principles derived from inverted pendulum control contribute to more efficient and responsive systems [16]. Additionally, these models are utilized in the design and control of humanoid robots, where maintaining upright posture is critical [17]. Beyond robotics, inverted pendulum systems serve educational purposes, offering a tangible means to teach control theory concepts, and even find applications in the entertainment industry, such as in balancing toys and interactive exhibits.

The primary aim of this paper is to enhance the control of inverted pendulum systems under random disturbances by utilizing the ultimate period method. We propose a novel control strategy that integrates the ultimate period method with adaptive control techniques to improve system stability and performance. The specific objectives of this study are:

1. **To develop a comprehensive control framework** for inverted pendulum systems that accounts for nonlinear dynamics and random disturbances.
2. **To analyse the effectiveness of the ultimate period method** in tuning controllers for inverted pendulum models subjected to unpredictable environmental factors.
3. **To validate the proposed control strategy** through simulations and experimental setups, demonstrating its superiority over conventional control methods in terms of stability and response time.

2. Problem Statement

In general, the inverted pendulum is a classic control system widely used in teaching and research at institutes or universities around the world. This system is considered an ideal model to validate and develop both nonlinear and linear control algorithms, especially in restoring and maintaining stability. It is a typical SIMO (Single Input Multi Output) system, with one input being the force acting on the motor, and multiple outputs including the position and driving angle of the pendulum that needs to be controlled to maintain a stable state.

In previous studies, the authors of [12] compared the effectiveness of the PID algorithm and the neural fuzzy controller for controlling an inverted pendulum system. While the PID controller was highlighted for its simplicity and effectiveness in maintaining balance and position control, researchers in [13] noted the challenges posed by disturbances and external forces, which the PID controller must handle simultaneously. Investigators in [14], along with scholars in [15], explored the control of an inverted pendulum system using the PID algorithm enhanced with genetic optimization techniques.

Those researches have indicated the potential of PID controllers in controlling inverted pendulums, but have also shown that improvements are needed to enhance control efficiency under disturbed conditions. In our work, this investigation proposes to use a new approach in fine-tuning the PID controller to improve the ability of system to respond to disturbances. Owing to the method of adjusting PID parameters via Nichol & Zieglers law [16], it does not only enhance the effectiveness of the PID controller in ideal environments but also in complex real-life conditions.

3. Design of mechanical architecture

The mathematical model of the system is launched due to the mechanical physics of Newton law since system model is needed to compute the parameters of controller and simulate its performance. In the initial state, when considering the angular coordinate of the inverted pendulum system as shown in Fig. 1, its angle would be treated as $\varphi=\pi$. Then, the challenge to transfer angle φ to 0 degrees is defined as the swing-up phenomenon. Besides, the trouble of oscillating an inverted pendulum around the equilibrium point at $\varphi=0$ is termed as the equilibrium topic. It is considered that our target is to balance an inverted pendulum under random disturbance conditions. Later, this controller would cause the response of the inverted pendulum to oscillate around the position from 0° to 90°.

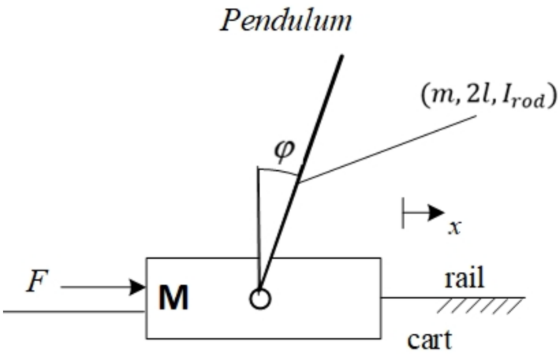

Fig. 1. Theoretical principle of the inverted pendulum (b).

We assume that the inverted pendulum is uniform and has its CoG (Center of Gravity) at the center of the bar. Releasing the link and analyzing the impact dynamics, the acting forces in the system can be shown in Fig. 2. By ignoring the friction force, we analyze the forces on the cart and the pendulum and obtain the equations of motion in the horizontal direction:

$$M\ddot{x} + N = F \tag{1}$$

where: N is the acting force in the horizontal direction, F is the external force and x is the position of cart. Summing up the horizontal forces, we achieve

$$N = m\ddot{x} + ml\ddot{\varphi}\,cos(\varphi) - ml\dot{\varphi}^2\,sin(\varphi) \tag{2}$$

From equation (1) and (2), we have

$$(M + m)\ddot{x} + ml\ddot{\varphi}\,cos(\varphi) - ml\dot{\varphi}^2\,sin(\varphi) = F \tag{3}$$

(a) Acting forces on the cart (b) Acting forces in the inverted pendulum system

Fig. 2. Analysis of the acting forces on the cart (a) and in the inverted pendulum system (b).

Considering that the acting force is perpendicular to the pendulum as Fig. 2

$$Psin(\varphi) + Ncos(\varphi) - mgsin(\varphi) = ml\dot{\varphi} + m\ddot{\varphi}\cos(\varphi) \tag{4}$$

Total moment at the center of the bar, we have

$$-Plsin(\varphi) - Nlcos(\varphi) = I\ddot{\varphi} \tag{5}$$

where I: inertial moment which is measured by the mass centre of this system

From equation (4) and (5):

$$(I + ml^2)\ddot{\varphi} + mglsin(\varphi) = -ml\ddot{x}\cos(\varphi) \quad (6)$$

Equation (3) and (6) describe the relative motion in this system. Angle φ is assumed to be small, above equations could be approximated as below

$$cos\varphi \approx 1$$

$$sin\varphi \approx \varphi$$

$$\dot{\varphi}^2 \approx 0$$

At that time, the system of equations can be represented as follows

$$(I + ml^2)\ddot{\varphi} + mgl\varphi = -ml\ddot{x} \quad (7)$$

$$(M + m)\ddot{x} + ml\ddot{\varphi} = u \quad (8)$$

To obtain the transfer function of the linearized system equations, we take the Laplace transform of the system equations with the assumption of zero initial conditions.

$$(I + ml^2)\Phi(s)s^2 + mgl\Phi(s) = -mlX(s)s^2 \quad (9)$$

$$(M + m)X(s)s^2 + ml\Phi(s)s^2 = U(s) \quad (10)$$

Solving the first equation for $X(s)$:

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Solving the first equation for $X(s)$:

$$X(s) = \left[\frac{I+ml^2}{ml} - \frac{g}{s^2}\right]\Phi(s) \quad (11)$$

Substituting equation (11) into equation (10)

$$(M+m)\left[\frac{I+ml^2}{ml}-\frac{g}{s^2}\right]\Phi(s)s^2+ml\Phi(s)s^2=U(s) \tag{12}$$

We have the transfer function

$$\frac{\Phi(s)}{U(s)}=\frac{\frac{ml}{q}s^2}{s^4-\frac{(M+m)mgl}{q}s^2} \tag{13}$$

Hence,

$$q=[(M+m)(I+ml^{2)}-(ml)^2] \tag{14}$$

From above transfer function, both zeros and poles locate at origin, it can be re-written as

$$P_{pend}(s)=\frac{\Phi(s)}{U(s)}=\frac{\frac{ml}{q}s}{s^3-\frac{(M+m)mgl}{q}s} \qquad \left[\frac{rad}{N}\right] \tag{15}$$

The transfer function with cart position X(s) as output can be derived in a similar way, we get

$$P_{cart}(s)=\frac{X(s)}{U(s)}=\frac{\frac{(I+ml^2)s^2-gml}{q}}{s^4-\frac{(M+m)mgl}{q}s^2} \qquad \left[\frac{m}{N}\right] \tag{16}$$

On the other hand, the modeled system can be represented as a state space $\dot{\boldsymbol{x}}=\boldsymbol{A}\boldsymbol{x}+\boldsymbol{B}u+f(t)$ with

$$\boldsymbol{x}=[x \quad \dot{x} \quad \varphi \quad \dot{\varphi}\,]^T$$

$$\boldsymbol{A}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)}{D} & \frac{m^2gl^2}{D} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{ml}{D} & \frac{mgl(M+m)}{D} & 0 \end{bmatrix}$$

$$\boldsymbol{B}=\begin{bmatrix} 0 & \frac{I+ml^2}{D} & 0 & \frac{ml}{D} \end{bmatrix}^T$$

with $D = I(M+m) + Mml^2$

where $f(t)$: random disturbance acting on this system in the range of δ_f, u: control signal from cart.

4. The proposed approach

A. Design of PID controller

Generally, a Proportional Integral Derivative (PID) controller is a feedback loop mechanism widely used in industrial control systems as shown in Fig. 3. It is one of the most favorite controllers in the feedback control systems. The PID scheme computes the error by taking the difference between the measured value of the variable parameter and the desired value. By adjusting these parameters, the PID controller tries to reduce that error to a minimum value.

The computational algorithm of this controller includes three separated parameters, and is therefore sometimes named as three-stage control: proportional (P), integral (I), and differential/derivative (D). The proportional value affects the current error, the effect of integral value is on the sum of previous errors, and the derivative value impacts the rate of change of the error. The combination of these three influences is used to regulate the process through the control element. This clarifies the timing relation such that P stage depends on the current error, I stage is subject to the accumulation of past errors, and D stage predicts future errors based on the current rate of change.

In detail, PID controller can only control one parameter of the system. To manipulate both the pendulum angle and the position of vehicle at the same time, two PID controllers must be used. In which, one is considered as main controller and directly controls the motor torque, while the other parameter applies to the impact of the reference point of the main parameter. Two input signals are fed into the PID controller and the output signal is the force acting on the vehicle. To ensure the pendulum is stable, it is necessary to use a feedback controller. This adds system information to the output data.

The proportional, integral, and derivative stages are added together to evaluate the output of the PID controller. The output of the controller $u(t)$ is defined as the final expression of the PID algorithm
where

Proportional gain (Kp): The larger the value is, the faster the response is. Hence the larger the error is, the larger the proportional compensation achieves. A proportional gain value that is too large, would lead to process instability and oscillation.

Integral gain (Ki): The larger the value, the faster the steady-state error is eliminated. However, this results in a greater overshoot: any negative error integrated during the transient response must be compensated by positive error before reaching a steady state.

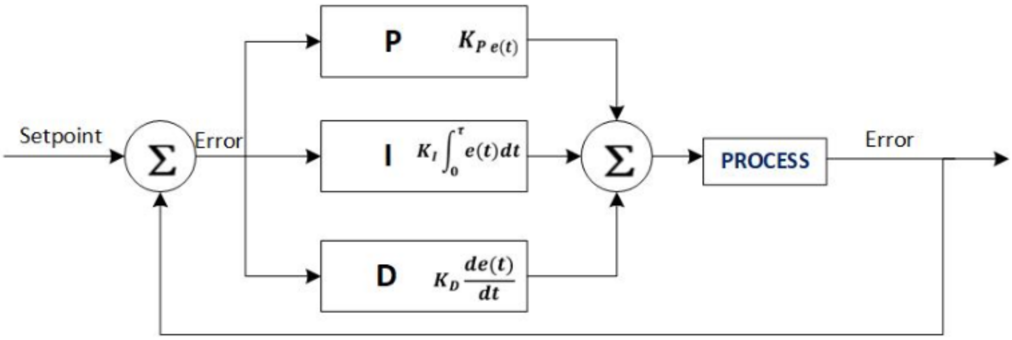

Fig. 1 Block diagram of PID controller

Derivative gain (Kd): The larger the value, the more it reduces overshoot, but it also slows down the transient response and can lead to instability due to the amplification of noise in the differentiation of the error signal.

B. Design of the ultimate period

In the early 1940s, the critical period method was first introduced by Nichols and Ziegler [17]. The term ultimate is utilized with this method since it requires determining the ultimate gain $\boldsymbol{K_U}$ (or ultimate proportional band $\boldsymbol{PB_U}$). It is the maximum gain (or maximum proportional band) at which the process remains stable. The ultimate period $\boldsymbol{P_U}$ is the oscillating period of the process with the gain or proportional band at its highest value (critical value). To identify the ultimate gain $\boldsymbol{K_U}$ or the ultimate proportional band $\boldsymbol{PB_U}$, the controller must be operated in proportional mode (P-mode). Fig. 4 shows the response of a control system at the value $\boldsymbol{K_U}$ or $\boldsymbol{PB_U}$. This diagram exhibits the measured variable oscillating at a fixed amplitude and frequency which is named the critical frequency.

A 360° phase shift exists in the control system. The amplitude of oscillation is constant. Therefore, the system gain is 1 approximately, which is the ultimate gain (or ultimate proportional band). In other words, it is the time from when the controlled variable upsurges to reach its maximum amplitude until it falls back and then rises again to reach the maximum amplitude. These two pieces of information are then deployed in formulas to compute the P, I, and D parameters for the controller in different modes. To find the value of $\boldsymbol{K_U}$ and $\boldsymbol{PB_U}$, the system must be measured, and the process characteristics should be observed.

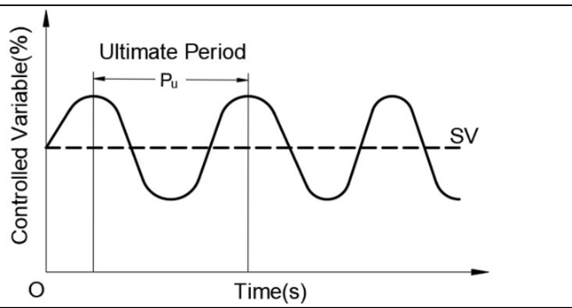

Fig. 2 Description of the control system at the ultimate gain $\boldsymbol{K_U}$ or ultimate proportional band $\boldsymbol{PB_U}$

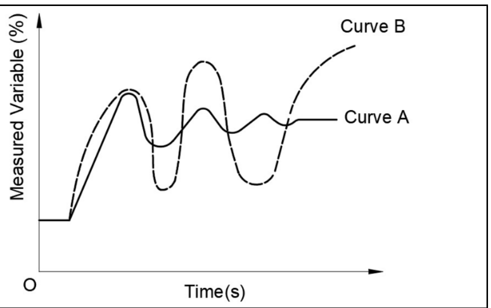

Fig. 3 Response of the measured variable in respect to disturbance

Algorithm 1:	
1:	Switch the controller to manual control mode (MANUAL)
2:	Set the parameters of controller in proportional mode, it means that $K_P = minimum\ value;\ T_I = \infty\ or\ RPM = 0\ and\ T_D = 0)$
3:	Insert the setting value SV
4:	Adjust MV in order to stabilize PV with the value of SV at step 3
5:	Switch the controller to automated control mode (AUTOMATIC)
6:	Establish disturbance by increasing SV about 5%

By observing the response of the measured variable PV, its performance may fail to one of three cases as below:

Case 1: if PV variable continuously goes up as curve B shown in Fig. 5 then decrease the proportional gain K_P. Consequently, this system is controlled by updating K_P in the procedure from step 1 to step 5 until the response of PV variable has a sinusoidal oscillation shape.

Table 1

List of parameters for PID controller by using the ultimate period method

Control mode	K_P	P_B	T_I	RPM	T_D
P	$0.5K_U$	$2PB_U$	∞	0	0
PI	$0.45K_U$	$2.2PB_U$	$\frac{P_U}{1.2}$	$\frac{1.2}{P_U}$	0
PD	$0.6K_U$	$1.66PB_U$	0	-	$\frac{P_U}{8}$
PID	$0.6K_U$	$1.66PB_U$	$\frac{P_U}{2}$	$\frac{2}{P_U}$	$\frac{P_U}{8}$

RPM (repeats per minute)

Case 2: If the oscillation of the controlled variable is gradually decreasing as curve A shown in Fig. 5, it means that this system would be stabilized. Thus, the proportional gain K_P needs to be augmented. At that moment, control the system with the new K_P value from steps 1 to 5 until the response of PV takes on a sinusoidal shape.

Case 3: If the response of the measured variable has a sinusoidal oscillation as shown in Fig. 4, the value of the proportional gain K_P is restored. This value is the ultimate gain $\boldsymbol{K_U}$, and the ultimate period $\boldsymbol{P_U}$ is also identified from the graph. Due to these values of both $\boldsymbol{K_U}$ and $\boldsymbol{P_U}$, we can compute the controller settings as Table 1.

It is noted that the formulas for estimating these settings do not produce perfect results. However, using those raw values as a basis for fine-tuning the parameters would achieve the best results for the controller.

5. Results of study

To validate the effectiveness and feasibility of our approach, several numerical simulations have been conducted. The system parameters for theoretical model of the inverted pendulum are launched in Simulink/Matlab as Table 2. Our host computer includes CPU Intel Core i7 gen 10 up to 5.4GHz, 16 core 24 thread, 24MB cache, 16GB 3200MHz DDR4, OS Windows 10 Professional. The system response of the cart regarding its position and the system response of the pendulum regarding its rotation angle are sampled. In both cases, the proposed algorithm is deployed to drive the pendulum from its initial position at $\varphi = \pi$ to oscillate around the equilibrium point $\varphi = 0$ within 15 seconds, and then move to the position $\varphi = \pi/2$ in the next 15 seconds.

Table 2

List of the system parameters for the inverted pendulum

Description	Value
Weight of pendulum $[m, kg]$	0.1
Weight of cart $[M, kg]$	1
Distance from the center of pendulum to origin $[l, m]$	0.25
Inertial moment of pendulum $[I, kgm]$	0.0021
Gravitational acceleration $[g, m/s^2]$	9.81

Additionally, to imitate the practical scenario, the noise condition is added when simulating. In the initial stage, several simulations of the response, control signal values, and phase trajectories are presented in the absence of external disturbances. Under the influence of random external forces within a 20% range, the response of inverted pendulum must suffer them. Similar to the case without noise, the initial position of the pendulum is subjected to the proposed algorithm to bring the pendulum to the equilibrium position oscillating around $\varphi = 0$.

5.1 Results of preliminary simulations

PID control mode: $\boldsymbol{K_U}$ = 25% and $\boldsymbol{P_U}$ = 76s then this system reaches to the stable condition after 2,5 second with offset = 0 as Fig. 6.

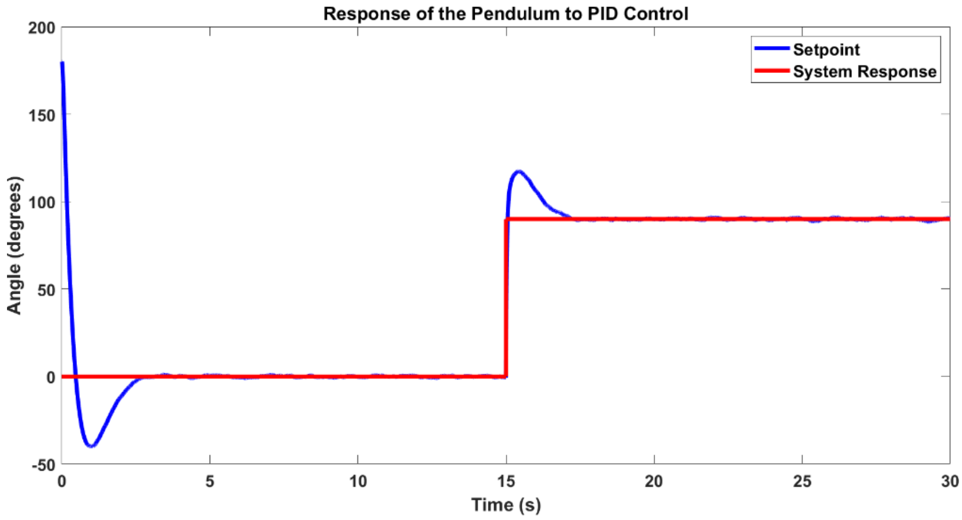

Fig. 4 Result of system response with PID control mode

PD control mode: In the same situation such $\boldsymbol{K_U}$ = 25% and $\boldsymbol{P_U}$ = 76s, our pendulum still stabilizes but high value of offset, angular error is approximately 10 degrees in the first 15 second and 30 degrees in next 15 second as Fig. 7.

PI control mode: Correspondingly, $\boldsymbol{K_U}$ = 25% and $\boldsymbol{P_U}$ = 76s, the proposed system is stabilized absolutely as Fig. 8.

5.2 Tuning procedure

Since the PID control mode produces the desired results, it means that the pendulum is stable at 0° and 90° during the simulation period. As a result, there is no need to tune the controller in this mode. However, the simulation results in PD mode indicate that the steady-state error is still high, so tuning the parameters of this controller is necessary to achieve better results.

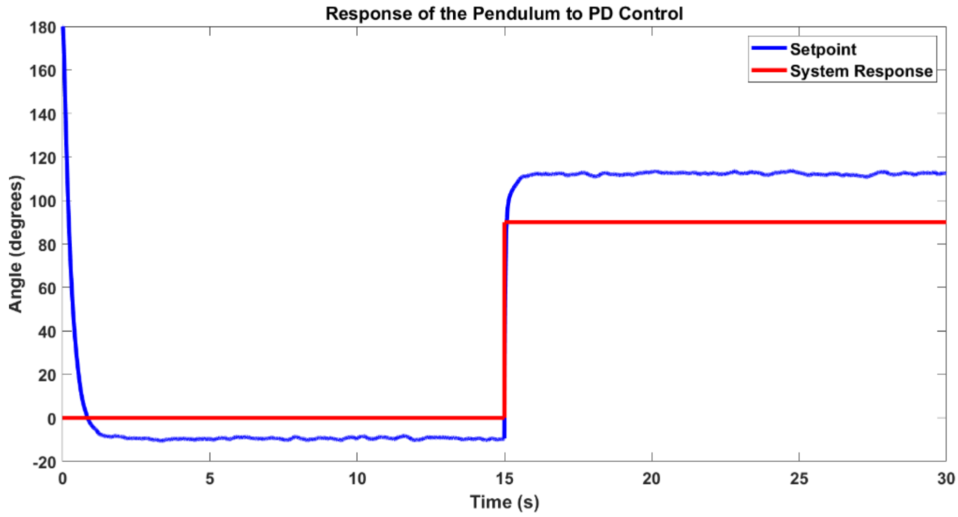

Fig. 5 Result of system response with PD control mode

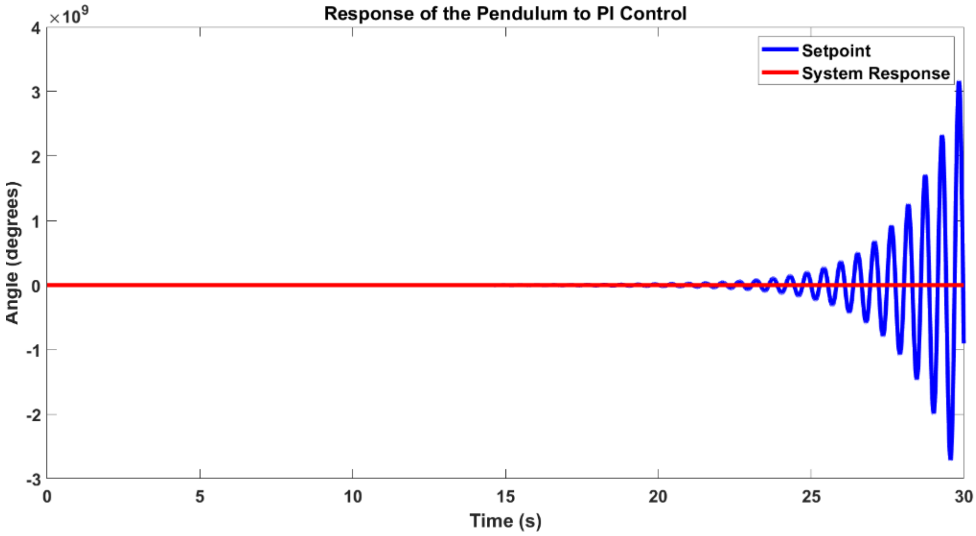

Fig. 6 Result of system response with PI control mode

Table 3

List of tuning parameters for our controller

No.	K_U	K_P	K_D	offset	t_{steady} (s)
1	25	41.5	9.5	≈ 25°	≈ 1
2	50	83.0	9.5	≈ 10°	≈ 1
3	75	124.5	9.5	≈ 7°	≈ 1
4	100	166.0	9.5	≈ 5°	≈ 1

Due to the fact that the steady-state error (offset) is mainly influenced by the gain factor $\boldsymbol{K_U}$, only K_P is tuned while maintaining the unchanged value of derivative term K_D. The parameters for simulation after tuning the PD controller are shown in Table 3

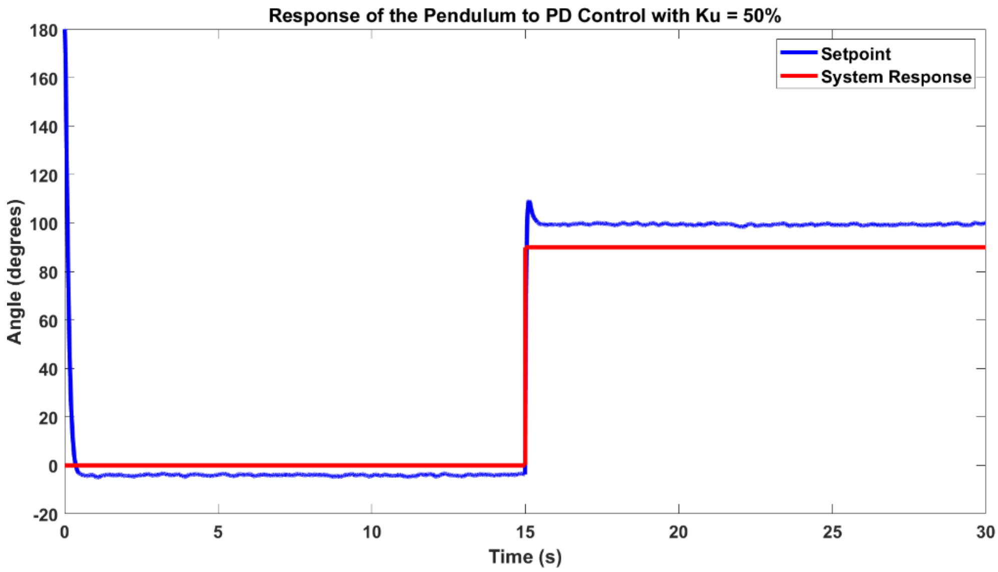

Fig. 7 Result of system response with PD control mode (tuning $\boldsymbol{K_U}$ = 50%)

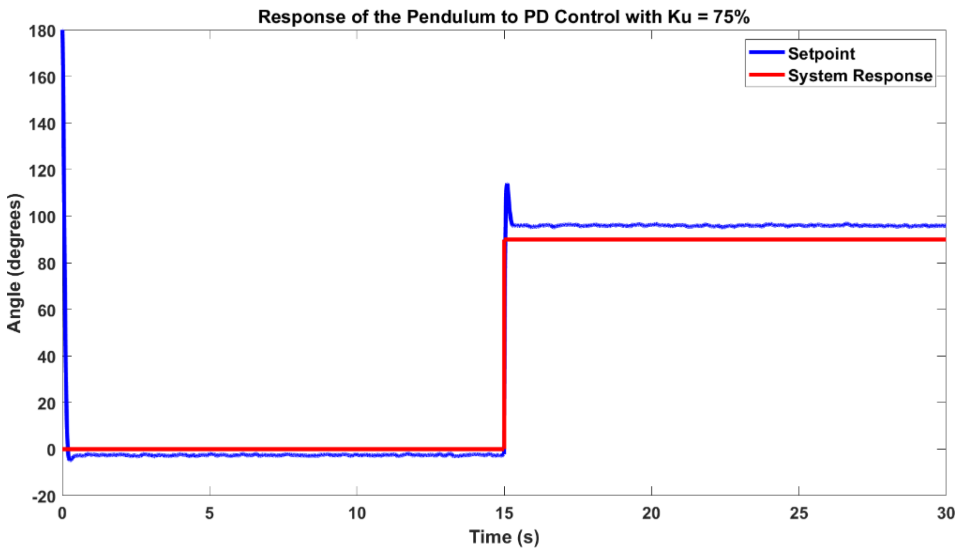

Fig. 8 Result of system response with PD control mode (tuning $\boldsymbol{K_U}$ = 75%)

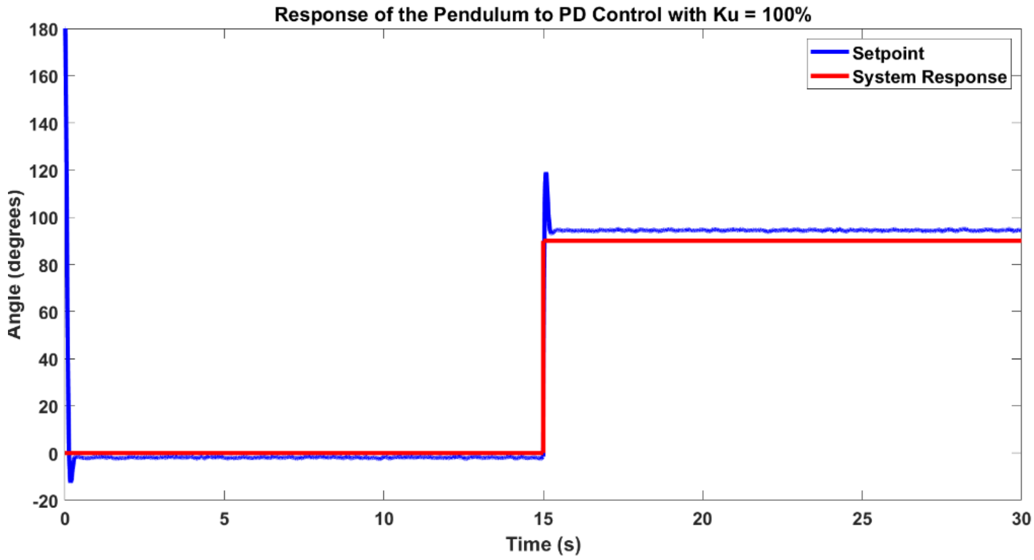

Fig. 9 Result of system response with PD control mode (tuning $\boldsymbol{K_U}$ = 100%)

From the provided table 3 and the responds of the system shown at Fig. 9, Fig. 10, and Fig. 11, we can observe that the parameter K_U increases progressively from 25 to 100, while the corresponding offset decreases from approximately 25° to 5°. This suggests that as K_U increases, the system's error (represented by the offset) becomes smaller, indicating improved accuracy in the control response. Despite the changes in K_U the time to reach steady-state (t_{steady})remains constant at approximately 1 second for all cases. This indicates that increasing K_U improves the precision of the system by reducing the offset without affecting the speed of stabilization.

6. Conclusions

In this paper, we investigated a new method for controlling an inverted pendulum system on a cart by integrating a PID control law with the ultimate period method. Utilizing linear matrix inequalities, the pendulum system was modeled and linearized based on small oscillation angles within a range of ±10°. Simulation results demonstrated that the controlled system responds stably under both PID and PD control modes, even in the presence of random disturbances. The tuning of the PID controller parameters in PD mode was performed to optimize performance and minimize control errors. These findings provide a strong foundation for applying this method to real-world systems, especially in noisy environments.

Limitations of the Study

Despite the promising results, the study has certain limitations. The linearization of the pendulum system was based on small oscillation angles, which may not accurately capture the system's behavior under larger deviations. Additionally, the simulations were conducted under ideal conditions, and factors such as model uncertainties, parameter variations, and external disturbances were not fully explored. The control method's performance in a real-world setting with hardware imperfections and sensor noise remains to be validated.

Prospects for Research Development

For future research, it would be beneficial to extend the proposed control strategy to accommodate larger oscillation angles and to enhance its robustness against a wider range of disturbances and model uncertainties. Implementing the control approach on an actual inverted pendulum system will provide valuable insights into its practical applicability and effectiveness. Furthermore, exploring the integration of adaptive or nonlinear control techniques with the ultimate period method could lead to improved performance in more complex and uncertain environments. Investigating the scalability of this method to multi-degree-of-freedom systems or other nonlinear control applications could also open new avenues for research and development in advanced control systems.

Acknowledgement

We acknowledge the support of time and facilities from Ho Chi Minh City University of Technology (HCMUT), VNU-HCM for this study.

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