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LINEAR MODELS AND CALCULATION OF AEROELASTIC FLUTTER

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În această lucrare se propune un model aerodinamic îmbunătățit destinat analizei fenomenului complex de flutter aeroelastic. Modelarea cât mai apropiată de fenomenul real este esențială, întrucât determinarea condițiilor de undă (viteza şi frecvență de propagare a undei) depind în mod semnificativ de modelul aerodinamic adoptat. În lucrare se prezintă un studiu comparativ, elaborat în termeni de aerodinamică, pentru care s-au considerat modelul forțelor aerodinamice cvasi-staționare şi respectiv modelul periodic nestaționar. Pe baza rezultatelor obținute, se arată că se pot folosi modele relativ mai simple, cu o bună precizie, precum şi faptul că sistemul oscilatorului armonic propus de Theodorsen aproximează mai bine realitatea decât modelul cvasi-staționar.

In this paper the authors present an improved aerodynamic model dedicated to analyzing the intricate phenomenon of aeroelastic flutter. The appropriate modelling of the real phenomenon is crucial, since the determination of the wave conditions (i.e. the wave velocity and frequency) is significantly dependent on the aerodynamic model. The paper presents a comparative study, expressed in terms of aerodynamics, based on the quasi-steady aerodynamic forces model and the unsteady periodic model. According to the results obtained by the authors, it is shown that simpler models are proven accurate and the fact that the harmonic oscillator system approximates the real phenomenon better than the quasi-steady model.

Keywords: Linear models, typical section, flutter analysis.

1. Introduction

The aeroelastic flutter is an intricate phenomenon and for its analysis is used the *V-g* method, while for the determination of the unsteady aerodynamic forces the Theodorsen model and the quasi-steady model are used [1]. The mathematical models are based on the concept of linear aeroelastic typical section with two and three degrees of freedom.

Since the aerodynamic forces are those which introduce energy into the system and their value depends on the speed for a given configuration

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(characteristic mass, elastic and geometric structure) it is possible to calculate the critical flutter speed, speed which if exceeded, the system becomes unstable dynamically and virtually destroyed. Consequently, the critical wave speed is defined as the speed at which the motion is harmonic structure and oscillation damping (structural and aerodynamic) is zero. Determination of wave conditions (wave speed and frequency associated) is significantly dependent on the wind model adopted, a harmonic oscillator system (proposed by Theodorsen) approximating reality better than a quasi-stationary. In the following, in terms of aerodynamics will be waving this study both for simplified cases based on the study of the quasi-stationary aerodynamic forces as well as periodic non-stationary[1]. Aeroelastic problems of light weight structures of modern aerospace vehicles are the result of interactions between aerodynamics, structural and inertial forces.

The mathematical model of the aeroelastic problem is based on the Lagrange equations of motion for the structural dynamics and on a quasi-steady approach of the generalized unsteady incompressible aerodynamic forces [1], [2], [3]. Aeroelastic phenomena of aircraft structures appear as a result of interactions between deformations of the elastic structure and the aerodynamic forces induced by the structure deformations. They have a strong influence on the structural dynamics and dynamic flight stability and also on the overall performance and controllability of the aircraft. Undoubtedly, the most important aeroelastic phenomenon is flutter, i.e. a self-excited oscillation of the elastic structure under the action of the aerodynamic loads. Flutter instabilities often exhibit an explosive behavior that causes a sudden change in stability despite only a small change in flight condition. Further, the aeroelastic vibrations that occur at all flight regimes have a strong impact on the fatigue life of the structure. In this paper we will refer only to aeroelastic phenomena which can be avoided or kept under control by some active measures [3].The concept active flutter suppression appears as part of a rather new technology in aviation, and it means controlling by some active devices - typically through activated controls - the natural instabilities of the aeroelastic system, in order to make this flutter-free beyond its "nominal" flutter boundary. However, currently there is no vehicle in production that uses active flutter suppression and much remains to be done before one can consider the routine incorporation of such systems in production aircraft. Further, the concept of active flutter suppression is strongly related to the much more realistic problem of controlling the conventional aeroelastic structural vibrations. Consequently, the active structural control is basically investigated and designed both for flutter prevention and structural load alleviation [3].

2. Flutter Equation of the Typical Section Model

The phenomenon of flutter (flutter) is the phenomenon of dynamic instability, at a certain speed (called the wave speed) which partially elastic structure in turbulent plane passes. Flutter occurs in the interaction between elastic forces, aerodynamics and mass, so that ultimately results in increasing exponentially with time of the periodic motion amplitude [1],[2].To explain this phenomenon one considers an elastic bearing surface fixed in a stream of air flowing speed. Suppose that the bearing surface of the elastic links enables two types of movements, namely: a shift in the direction perpendicular to the airflow and a rotation that changes to the original scope. Movement in the air stream bearing surface takes place without external interference, but it is necessary for the structure to receive an initial external perturbation whose nature does not matter. Initially, the two movements are supposed synchronous on bearing surface, respectively, both moving and start turning the zero position, reach a maximum after returning to their original position zero [3].The energy introduced into the system by the aerodynamic forces on the first quarter period is removed from the system the second quarter period. The same happens in the second half of the period. Since the whole energy cycle introduced in the system is zero, the initial amplitude cannot increase. If, however, between the displacement and rotation there is a lag of one quarter of the period, the entire motion cycle work will be done by aerodynamic forces and bring positive energy into the system which leads obviously to increase the amplitude of the initial data.

The phenomenon occurs for any gap between the rotation and displacement provided that the gap is less than half a rotation period and with that precedes the phase shift difference. If rotation is delayed behind the movement, a phase difference of no more than half of the period, the phenomenon that occurs is a continued consumption of energy by buoyancy, a continuous removal of energy from the system and therefore, rapidly damping motion [1], [2], [3].

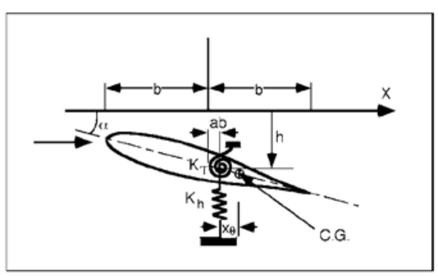

Fig.1 The typical section of an airplane wing

For the typical section shown in Fig. 1, one can provide details about the model, such as: the model has a translational spring with the stiffness K_h and a torsion spring, characterized by the stiffness K_T. These springs are attached to the airfoil at the shear center. The distance h is measured at the shear center (which is crossed by the elastic axis). Therefore, there is two degrees of freedom (h,α). The downward displacement of any other point on the airfoil is:

$$z = h + x\alpha \tag{1}$$

Where x is a distance measured from the shear center [1].

The strain energy and the kinetic energy are respectively given by

$$U = \frac{1}{2}K_T\alpha^2 + \frac{1}{2}K_h h^2 \tag{2}$$

$$T = \frac{1}{2}\int \rho \dot{z}^2 dx = \frac{1}{2}\left(\dot{h}^2\int \rho dx + 2\dot{h}\dot{\alpha}\int \rho x dx + \dot{\alpha}^2\int \rho x^2 dx\right) = \frac{1}{2}m\dot{h}^2 + m x_\theta \dot{h}\dot{\alpha} + \frac{1}{2}I_\theta \dot{\alpha}^2 \tag{3}$$

where ρ is the mass per unit length of the airfoil.
The virtual work due to the unsteady aerodynamic forces is

$$\delta W_a = \int \Delta p \delta z dx = \int \Delta p \left\{\delta h + x\delta\alpha\right\} dx = Q_h \delta h + Q_\alpha \delta\alpha \tag{4}$$

Lagrange's equations provide the equations of motion of the airfoil [1].

$$\frac{d}{dt}\left(\frac{\partial(T-U)}{\partial \dot{q}}\right) - \frac{\partial(T-U)}{\partial q} = Q_q \tag{5}$$

$$\begin{bmatrix} 1 & \overline{x}_\theta \\ \overline{x}_\theta & \overline{r}_\theta^2 \end{bmatrix}\begin{Bmatrix} \ddot{h}/b \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} \varpi_h^2 & 0 \\ 0 & \varpi_\theta \overline{r}_\theta^2 \end{bmatrix}\begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} = \begin{Bmatrix} F/mb \\ M/mb^2 \end{Bmatrix} \tag{6}$$

$$h = h_0 e^{i\omega t}, \alpha = \alpha_0 e^{i\omega t} \tag{7}$$

$$-\varpi^2\begin{bmatrix} 1 & \overline{x}_\theta \\ \overline{x}_\theta & \overline{r}_\theta^2 \end{bmatrix}\begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} + \begin{bmatrix} \varpi_h^2 & 0 \\ 0 & \varpi_\theta \overline{r}_\theta^2 \end{bmatrix}\begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} = \begin{Bmatrix} F/mb \\ M/mb^2 \end{Bmatrix} \tag{8}$$

The unsteady aerodynamic force and moment will be

$$F = \pi\rho b^3\omega^2\left[\frac{h}{b}L_h + \alpha\left(L_\alpha - \left(\frac{1}{2} + a\right)L_h\right)\right] \tag{9}$$

where the reduced frequency is

$$k = \frac{\omega b}{V} \tag{10}$$

$$L_h = 1 - i2C\frac{1}{k} \tag{11}$$

$$L_\alpha = \frac{1}{2} - i\frac{1+2C}{k} - \frac{2C}{k^2} \tag{12}$$

Likewise,

$$M = \pi\rho b^4 \omega^2 \left[\left\{ M_h - \left(\frac{1}{2}+a\right) L_h \right\} \frac{h}{b} + \left\{ M_\alpha - \left(\frac{1}{2}+a\right)\left(L_\alpha + M_h\right) + \left(\frac{1}{2}+a\right)^2 L_h \right\} \alpha \right] \tag{13}$$

$$M_h = \frac{1}{2}, M_\alpha = \frac{3}{8} - i\frac{1}{k} \tag{14}$$

Then, the equation of motion can be rewritten as

$$-\varpi^2 \begin{bmatrix} 1 & \bar{x}_\theta \\ \bar{x}_\theta & \bar{r}_\theta^{\,2} \end{bmatrix} \begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} + \begin{bmatrix} \varpi_h^{\,2} & 0 \\ 0 & \varpi_\theta \bar{r}_\theta^{\,2} \end{bmatrix} \begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} =$$

$$= -\frac{\varpi^2}{\mu} \begin{bmatrix} L_h & L_\alpha - \left(\frac{1}{2}+a\right) L_h \\ M_h - \left(\frac{1}{2}+a\right) L_h & M_\alpha - \left(\frac{1}{2}+a\right)\left(L_\alpha + M_h\right) + \left(\frac{1}{2}+a\right)^2 L_h \end{bmatrix} \begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} \tag{15}$$

Let's define

$$\Omega^2 = \frac{\omega^2}{\omega_\theta^2} \tag{16}$$

$$R^2 = \frac{\omega_h^2}{\omega_\theta^2} \tag{17}$$

Then

$$-\Omega^2 \begin{bmatrix} 1 & \bar{x}_\theta \\ \bar{x}_\theta & \bar{r}_\theta^{\,2} \end{bmatrix} \begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} + \begin{bmatrix} R^2 & 0 \\ 0 & \bar{r}_\theta^{\,2} \end{bmatrix} \begin{Bmatrix} h/b \\ \alpha \end{Bmatrix}$$

$$= \frac{\Omega^2}{\mu} \begin{bmatrix} L_h & L_\alpha - \left(\frac{1}{2}+a\right) L_h \\ M_h - \left(\frac{1}{2}+a\right) L_h & M_\alpha - \left(\frac{1}{2}+a\right)\left(L_\alpha + M_h\right) + \left(\frac{1}{2}+a\right)^2 L_h \end{bmatrix} \begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} \tag{18}$$

3. *V-g* method for flutter analysis

Let's express the above flutter equation in the following matrix form [1],[2].

$$\left[K\right] \begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} = \Omega^2 \left[\frac{1}{\mu} A + M \right] \begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} \tag{19}$$

where $\left[K\right]$ is the stiffness matrix, M mass matrix, and A is the aerodynamic matrix. Note that the aerodynamic matrix is a function of the reduced frequency, k.
V-g method assumes first the artifial structural damping, g.

$$\left[K\right]=\left(1+ig\right)\left[K\right] \tag{20}$$

For a given reduced frequency, $k=\dfrac{\omega b}{V}$, a complex eigenvalue problem appears [4].

$$\frac{\left(1+ig\right)}{\Omega^2}\left[K\right]\begin{Bmatrix} h/b \\ \alpha \end{Bmatrix}=\left[\frac{1}{\mu}A+M\right]\begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} \tag{21}$$

The eigenvalue is

$$\lambda=\frac{1+ig}{\Omega^2} \tag{22}$$

From this eigenvalue we have

$$\frac{\omega_i^2}{\omega_\theta^2}=\frac{1}{\mathrm{Re}\left(\lambda\right)} \tag{23}$$

$$g=\frac{\mathrm{Im}\left(\lambda\right)}{\mathrm{Re}\left(\lambda\right)} \tag{24}$$

The complex eigenvalue problem is solved beginning with large values of k and then decreasing k until a flutter velocity is found. If there is no actual damping in the system, when the artificial damping, g, first becomes positive, flutter will occur.

A comparison between the numerical results obtained by the authors by using the harmonic oscillator model versus the quasi-steady model is shown in Figs.2. - 5.

In Fig. 6 and Fig.7 there the variations of the functions $g=g(V)$ and $\varpi=\varpi\left(V\right)$ are shown. The damping of the flutter velocity will be acquired when the function g will change its sign.

The calculation of flutter was performed with *V-g* method, for the study case considered and experienced by the authors; the result, i.e. the variation of the flutter velocity is shown in Figure 8.

Traditional methods of calculation (e.g. the Theodorsen and the panel methods) lead to relatively good estimations in terms of dynamic response of the system, with slight overestimation of the effects (i.e. the displacements of the aeroelastic system).

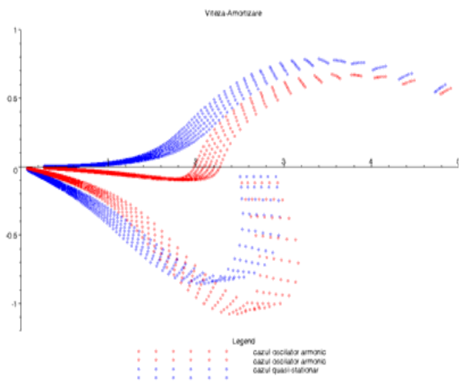

Fig. 2 The aerodynamic forces and moments

- red curves = the Theodorsen method (unsteady case).
- blue curves = the variation of *g=g(V)* for the quasi-steady case.

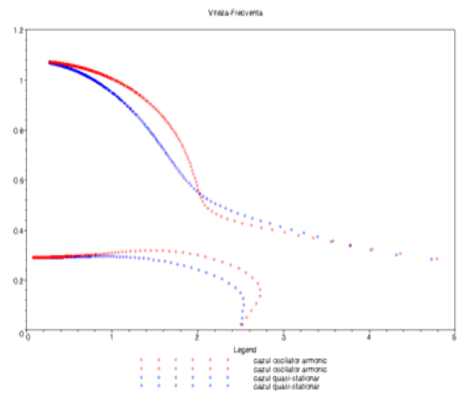

Fig. 3 The frequencies

- red curves = the Theodorsen method (i.e. the unsteady case).
- blue curves = the variation for the quasi-steady case.

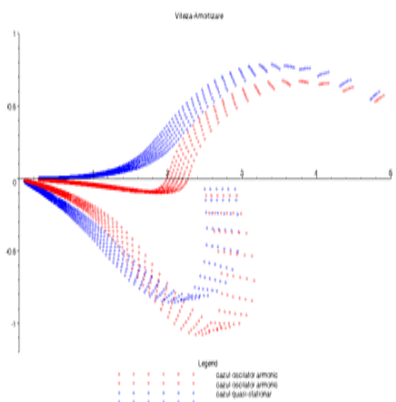

Fig. 4 The critical wave speed versus the flight altitude

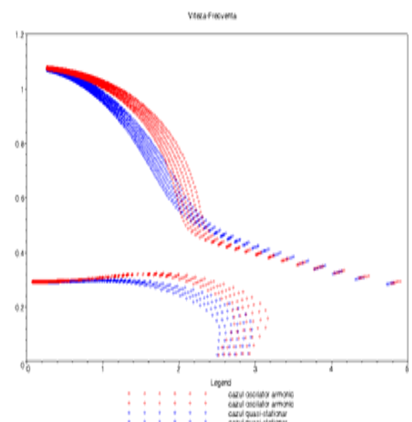

Fig. 5 The critical wave speed versus the flight velocity

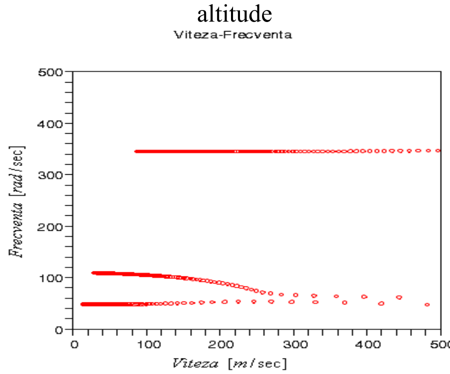

Fig. 6 The variation of *g=g(V)*

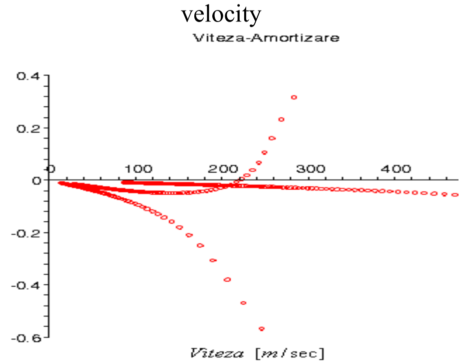

Fig. 7 The variation of $\varpi = \varpi\left(V\right)$

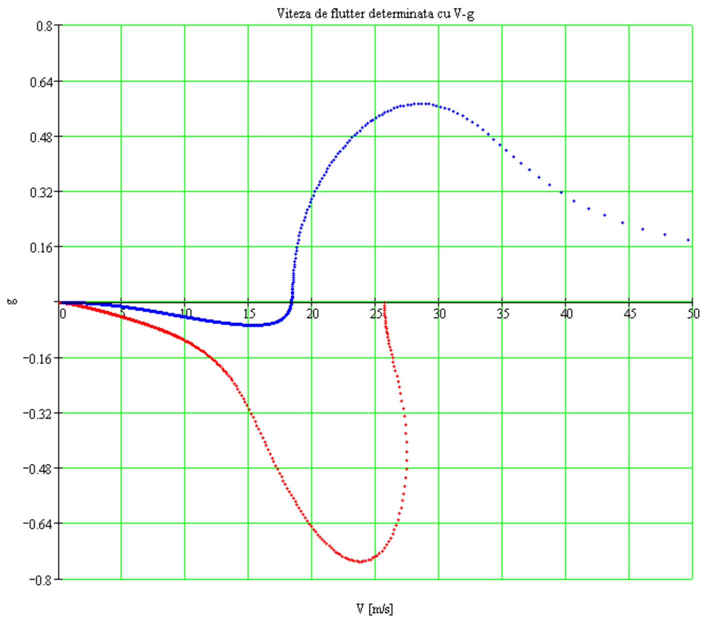

Fig.8 Flutter speed determined with the *V-g* method

Table 1

The results of the numerical simulations

Elastic axis position	V flutter [m/sec]	V flutter [m/sec]	V flutter [m/sec]
	Case1	Case 2	Case 3
0%	36	42	47
20%	24	28	25
40%	17.7	21	15.5
60%	22	29.5	16
80%	41	68	23.5

The results of numerical simulations with the *V-g* method centralized in Table1, prove the achieving of the flutter speed in the range of interest for most cases. Boxes shaded in green in the table correspond to configurations that achieve a speed of flutter within the experimental capabilities. Flutter calculations were performed for 15 different configurations (inertial characteristics, features flexible, elastic axis position) and the results prove the achievement of flutter speed in the range of interest for most cases. Although the structural analysis carried out, is static, we provided some results on the behavior of wing-fastening system. The obtained deformations have maximum values of 9.5 mm for clamping to 0% and 11 mm for fixing 80% of chord. The results indicate a low rigidity clamping system (considering that the requests were overstated by imposing how to load and enhance the safety coefficient of 1.5).

4. Conclusions

In order to make numerical comparisons, it was considered as the **study case**, the typical section model, Fig. 1, with the following characteristics:

$$x_\theta = 0,2\,;\; r_\theta = 0,5\,;\; \mu = 20\,;\; a = -0,1\,;\; b = 1\,;\; R = 0,3\,. \tag{25}$$

In Figure 2 are represented by red curves the corresponding cases where the aerodynamic forces and moments have been calculated with the method proposed by Theodorsen (i.e. the unsteady case). The blue curves represent the $g = g(V)$ quasi-steady case. The wave speed (in non-dimensional form) for the two cases is the intersection curve (red respectively blue) axis $g = 0$. There is an important difference between the two determined values of critical wave speed.
If the difference between the two speeds is small, we conclude that the phenomenon modeling in quasi-steady state is a conservative one. Since, this is not the case, it is clear that a more realistic approximation of the phenomenon is necessary to calculate aerodynamic forces and moments in harmonic oscillatory regime. Further, in Fig. 4 is plotted the critical wave speed dependence on the altitude of flight. The calculation was done at altitudes 0, 1000, 2000, 3000, 4000, 5000 meters.

In Figs. 3 and 4 the frequencies associated with each case were represented. As in the previous case, the unsteady and quasi-steady cases have been represented in red and blue contours. One can notice the increasing of the critical flutter speed with the increasing altitude flight. Like the case presented earlier, as obviously expected, the critical wave speed calculation taking into account a quasi-steady model gives unsatisfactory results. The comparative study carried on in this paper is based on the genuine experiment data (acquired through experiments done by the authors).

The analysis methods were based on the quasi-steady aerodynamic forces model and the unsteady periodic model. According to the results obtained by the authors, it is shown that simpler models are proven accurate and the fact that the harmonic oscillator system approximates the real phenomenon better than the quasi-steady model.

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